Straight Lines

1. Distance Formula

(i) The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

(ii) The distance of a point $A(x, y)$ from the origin $O(0, 0)$ is given by $OA = \sqrt{x^2 + y^2}$.

Note *Three points A, B and C are collinear i.e. in same straight line, if* $AB + BC = AC$ *or* $AC + CB = AB$ *or* $BA + AC = BC$.

2. Section Formulae

(i) The coordinates of the point which divides the joining of (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally, is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$ and externally is $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$.

(ii) The coordinates of the mid-point of the joining of (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

3. Area of a Triangle

(i) If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC, then area of ΔABC.

$$= \frac{1}{2} |[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]|$$

$$\text{or } \Delta = \frac{1}{2} \left\| \begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix} \right\|$$

(ii) If the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear, then $x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$.

Note *The coordinates of centroid of the triangle whose vertices are* $(x_1, y_1), (x_2, y_2)$ *and* (x_3, y_3), *is given by* $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

4. Shifting of Origin

Let the origin be shifted to a point $O'(h, k)$. If $P(x, y)$ are coordinates of a point referred to old axes and $P'(X, Y)$ are the coordinates of the same points referred to new axes, then $x = X + h, y = Y + k$.

5. Locus of a Point

The curve described by a moving point under given geometrical conditions is called the locus of that point.

6. Slope or Gradient of a Line

If θ is the angle of inclination of a line l, then $\tan\theta$ is called the slope or gradient of the line l and it is denoted by m.

i.e. $$m = \tan\theta$$

Note *The slope of X-axis is zero and slope of Y-axis is not defined.*

7. Slope of a Line Joining Two Points

The slope of a line passing through points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $m = \tan\theta = \left|\frac{y_2 - y_1}{x_2 - x_1}\right|$

8. Angle between Two Lines

The angle θ between two lines having slopes m_1 and m_2 is

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

(i) If two lines are **parallel**, then their slopes are equal i.e. $m_1 = m_2$.

(ii) If two lines are **perpendicular** to each other, then product of their slopes is -1, i.e. $m_1 m_2 = -1$.

Note *(i) If* $\tan\theta$ *is positive, then* θ *will be an acute angle.*

(ii) If $\tan\theta$ *is negative, then* θ *will be an obtuse angle.*

9. Various Forms of the Equation of a Line

(i) If a line is at a distance a and parallel to X-axis, then the equation of the line is $y = \pm a$.

(ii) If a line is parallel to Y-axis at a distance b from Y-axis, then its equation is $x = \pm b$.

(iii) **Point-slope form** The equation of a line which passes through the point (x_1, y_1) and has the slope m is given by $y - y_1 = m(x - x_1)$.

(iv) **Two points form** The equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

(v) **Slope-intercept form** The equation of line with slope m and making an intercept c on Y-axis, is $y = mx + c$.

(vi) **Intercept form** The equation of a line which cuts off intercepts a and b respectively on the X and Y-axes is given by $\frac{x}{a} + \frac{y}{b} = 1$

i.e. $$\frac{x}{x\text{-intercept}} + \frac{y}{y\text{-intercept}} = 1$$

(vii) **Normal form** The equation of a straight line upon which the length of the perpendicular from the origin is p and angle made by this perpendicular to the X-axis is α, is given by $x\cos\alpha + y\sin\alpha = p$.

10. General Equation of a Line

Any equation of the form $Ax + By + C = 0$, where A and B are simultaneously not zero, is called the general equation of a line.

Different forms of $Ax + By + C = 0$ are

(i) **Slope-intercept form** $y = \frac{-A}{B}x - \frac{C}{B}$, $B \neq 0$

(ii) **Intercept form** $\frac{x}{-C/A} + \frac{y}{-C/B} = 1$, $C \neq 0$

(iii) **Normal form** $x\cos\alpha + y\sin\alpha = p$

where, $\cos\alpha = \pm\frac{A}{\sqrt{A^2 + B^2}}$

$\sin\alpha = \pm\frac{B}{\sqrt{A^2 + B^2}}$ and $p = \pm\frac{C}{\sqrt{A^2 + B^2}}$

Note *Proper choice of signs to be made so that p should be always positive.*

11. Angle Between Two Lines, having General Equations

Let general equations of lines be $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$, then slope of given lines are

$m_1 = -\frac{A_1}{B_1}$ and $m_2 = -\frac{A_2}{B_2}$.

Let θ be the angle between two lines, then

$$\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1m_2}\right)$$

$$= \pm\left(\frac{-\frac{A_2}{B_2} + \frac{A_1}{B_1}}{1 + \frac{A_1}{B_1} + \frac{A_2}{B_2}}\right)$$

12. Distance of a Point from a Line

The perpendicular distance d of a point $P(x_1, y_1)$ from the line $Ax + By + C = 0$ is given by

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

13. Distance between Two Parallel Lines

The distance d between two parallel lines $y = mx + C_1$ and $y = mx + C_2$ is given by $d = \frac{|C_1 - C_2|}{\sqrt{1 + m^2}}$ and if lines are $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$,

then, $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.