DCAM classes
Dynamic Classes for Academic Mastery

Chapter 10. Straight Lines

Question-1

Determine the equation of the straight line passing through the point (-1, -2) and having slope 4/7.

Solution:

The point – slope form is $y - y_1 = m(x - x_1)$

$$y + 2 = (4/7)(x + 1)$$

$$7y + 14 = 4x + 4$$

$$4x - 7y = 10$$

Question-2

Determine the equation of the line with slope 3 and y – intercept 4.

Solution:

The slope – intercept form is $y = mx + c$.

Therefore the equation of the straight line is $y = 3m + 4$.

Question-3

A straight line makes an angle of 45° with x – axis and passes through the point (3, -3). Find its equation.

Solution:

$m = \tan 45^\circ = 1$

The slope – intercept form is $y = mx + c$.

$(-3) = 1(3) + c$

$c = -6$

Therefore the equation of the straight line is $y = x - 6$.

Question-3

A straight line makes an angle of 45° with x – axis and passes through the point (3, -3). Find its equation.

Solution:

$m = \tan 45^\circ = 1$

The slope – intercept form is $y = mx + c$.

$(-3) = 1(3) + c$

$c = -6$

Therefore the equation of the straight line is $y = x - 6$.

Question-4

Find the equation of the straight line joining the points (3, 6) and (2, -5).

Solution:

The equation of a straight line passing through two points is $\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$.

Substituting the points (3, 6) and (2, -5),

$\frac{y-6}{6+5} = \frac{x-3}{3-2}$

$\frac{y-6}{11} = \frac{x-3}{1}$

$y - 6 = 11(x - 3)$

$y - 6 = 11x - 33$

$11x - y = 27$ is the required equation of the straight line.

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Substituting the points (3, 6) and (2, -5),

$\frac{y-6}{6+5} = \frac{x-3}{3-2}$

$\frac{y-6}{11} = \frac{x-3}{1}$

$y - 6 = 11(x - 3)$

$y - 6 = 11x - 33$

$11x - y = 27$ is the required equation of the straight line.

Question-5

Find the equation of the straight line passing through the point (2, 2) and having intercepts whose sum is 9.

Solution:

The intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ where 'a' and 'b' are x and y intercepts respectively.

$a + b = 9$ (Given) ....(i)

$\frac{x}{a} + \frac{y}{b} = 1$

Since (2, 2) lies on the equation, $\frac{2}{a} + \frac{2}{b} = 1$

$2(a + b) = ab$

$ab = 18$ (from i)

$a = 18/b$

Substituting in (i)

$18 + b^2 = 9b$

$b^2 - 9b + 18 = 0$

$b^2 - 6b - 3b + 18 = 0$

$b(b - 6) - 3(b - 6) = 0$

$(b - 3)(b - 6) = 0$

$b = 3$ or 6

Therefore $a = 6$ or 3.

Therefore the required equation of straight line is $\frac{x}{3} + \frac{y}{6} = 1$ or $\frac{x}{6} + \frac{y}{3} = 1$.

Question-6

Find the equation of the straight line whose intercept on the x-axis is 3 times its intercept on the y-axis and which passes through the point (-1, 3).

Solution:

The intercept form is $\frac{x}{a}+\frac{y}{b} = 1$ where 'a' and 'b' are x and y intercepts respectively.

a = 3b (Given) ………………..(i)

$\frac{x}{3b}+\frac{y}{b} = 1$

Since (2, 2) lies on the equation, $\frac{2}{3b}+\frac{2}{b} = 1$

b = $\frac{2}{3}+\frac{2}{1}=\frac{2+6}{3}=\frac{8}{3}$

∴ a = 8.

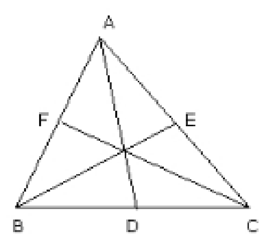
∴ The required equation of straight line is $\frac{x}{8}+\frac{3y}{8} = 1$ i.e., x + 3y = 8.

Question-7

Find the equations of the medians of the triangle formed by the point (2, 4), (4, 6) and (-6, -10).

Solution:

Let A(2, 4), B(4, 6) and C(-6, -10) be the given vertices of a Δ ABC. D, E, F re the mid-points of the sides BC, CA, AB respectively.



∴ D = $\left(\frac{4-6}{2},\frac{6-10}{2}\right) = \left(\frac{-2}{2},\frac{-4}{2}\right)$ = (-1, -2)

E = $\left(\frac{2-6}{2},\frac{4-10}{2}\right) = \left(\frac{-4}{2},\frac{-6}{2}\right)$ = (-2, -3)

F = $\left(\frac{2+4}{2},\frac{4+6}{2}\right) = \left(\frac{6}{2},\frac{10}{2}\right)$ = (3, 5)

∴ Equation of AD is

$\frac{y-2}{2+1} = \frac{x-4}{4+2}$

6(y - 2) = 3(x - 4)

2(y - 2) = (x - 4)

2y - 6 = x – 4

2y – x = 0

∴ Equation of BE is

$\frac{y-6}{6+3} = \frac{x-4}{4+2}$

6(y - 6) = 9(x - 4)

2(y - 6) = 3(x - 4)

2y - 12 = 3x - 12

3y - 3x = 0

∴ Equation of CF is

$\frac{y+10}{-10-5} = \frac{x+6}{-6-3}$

-9(y + 10) = -15(x + 6)

3(y + 10) = 5(x + 6)

3y + 30 = 5x + 30

3y – 5x = 0

Question-8

Find the length of the perpendicular from (3, 2) to the straight line 3x + 2y + 1 = 0.

Solution:

The perpendicular distance from (x_1, y_1) to the straight line ax + by + c = 0 is given by $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.∴ The length of the perpendicular from (3, 2) to the straight line 3x + 2y + 1 = 0 is $\left|\frac{3(3) + 2(2) + 1}{\sqrt{3^2 + 2^2}}\right| = \frac{14}{\sqrt{13}}$.--

Question-9

The portion of straight line between the axes is bisected at the point (-3, 2). Find its equation.

Solution:

The intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ where 'a' and 'b' are x and y intercepts respectively.

The straight line make the x intercept OA = a, and y intercept OB = b.

Then, A is (0, a) and B is (b, 0).

$\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

Mid-point of AB is $\left(\frac{b}{2}, \frac{a}{2}\right)$ = (-3, 2)

b = -6 and a = 4.

∴ The equation of straight line is $\frac{x}{4} + \frac{y}{-6} = 1$.

Question-10

Find the equation of the diagonals of quadrilateral whose vertices are (1, 2), (-2, -1), (3, 6) and (6, 8).

Solution:

Let A(1, 2), B(-2, -1), C(3, 6) and D(6, 8) be the vertices of quadrilateral ABCD.

Equation of the diagonal AC is $\frac{y-2}{2-6} = \frac{x-1}{1-3}$

$-2(y - 2) = -4(x - 1)$

$-2y + 4 = -4x + 1$

$4x - 2y + 3 = 0$

Equation of the diagonal BD is $\frac{y+1}{-1-8} = \frac{x+2}{-2-8}$

$-16(y + 1) = -9(x + 2)$

$-16y - 16 = -9x - 18$

$9x - 16y + 2 = 0$

Question-11

Find the equation of the straight line, which cut of intercepts on the axes whose sum and product are 1 and −6 respectively.

Solution:

The intercept form is $\frac{x}{a}+\frac{y}{b} = 1$ where 'a' and 'b' are x and y intercepts respectively.

$a + b = 1$ ....(i)

$ab = -6$ ....(ii)

$(a - b)^2 = 4ab - (a + b)^2 = 4(-6) - (1)^2 = -24 - 1 = 25$

$a - b = 5$ ....(iii)

Adding (i) and (iii)

$2a = 6$

$a = 3$

$\therefore b = -2$

$\therefore$ The equation of the straight line is $\frac{x}{3}+\frac{y}{-2} = 1$.

Question-12

Find the intercepts made by the line 7x + 3y − 6 = 0 on the coordinate axis.

Solution:

If y = 0 then x = 6/7

If x = 0 then y = 6/3 = 2

x - intercept is 6/7 and y − intercept is 2.

Question-13

What are the points on x-axis whose perpendicular distance from the straight line $\frac{x}{3}+\frac{y}{4}=1$ is 4?

Solution:

$\frac{x}{3}+\frac{y}{4}=1$

$4x + 3y = 12$

Any point on x – axis will have y coordinate as 0.

Let the point on x-axis be $P(x_1, 0)$.

The perpendicular distance from the point P to the given straight line is

$\left|\frac{4(x_1)+3(0)-12}{\sqrt{4^2+3^2}}\right| = 4$

$\left|\frac{4x_1-12}{5}\right| = 4$ or $\left|\frac{4x_1-12}{5}\right| = -4$

$4x_1 - 12 = 20$ or $4x_1 - 12 = -20$

$4x_1 = 32$ or $4x_1 = -8$

$x_1 = 8$ or $x_1 = -2$

Thus the required points are (8, 0) and (-2, 0).

Question-14

Find the distance of the line 4x – y = 0 from the point (4, 1) measured along the straight line making an angle of 135° with the positive direction of the x-axis.

Solution:

$m = \tan 135^\circ = -1$

Equation of a straight line having slope m = -1 and passing through (4, 1) is

$y - y_1 = m(x - x_1)$

$y - 1 = (-1)(x - 4)$

$y - 1 = -x + 4$

$x + y = 5$ ……………….(i)

$4x - y = 0$ ……………….(ii)

Solving (i) and (ii),

x = 1 and y = 4

∴ The distance between the points (1, 4) and (4, 1) is

$= \sqrt{(4-1)^2+(1-4)^2} = \sqrt{3^2+(-3)^2} = \sqrt{9+9} = 3\sqrt{2}$ units

Question-15

Find the angle between the straight lines 2x + y = 4 and x + 3y = 5

Solution:

Slope of the line 2x + y = 4 is m_1 = -2.

and slope of the line x + 3v = 5 is m_2 = -1/3

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \tan^{-1}\left|\frac{-2 + \frac{1}{3}}{1 + (-2)(-\frac{1}{3})}\right| = \tan^{-1}\left|\frac{\frac{-6+1}{3}}{1+\frac{2}{3}}\right| = \tan^{-1}\left|\frac{\frac{-5}{3}}{\frac{5}{3}}\right| = \tan^{-1}(-1) = 135^{o}$$

Question-16

Show that the straight lines 2x + y = 5 and x – 2y = 4 are at right angles.

Solution:

Slope of the line 2x + y = 5 is m_1 = -2.

and slope of the line x - 2y = 4 is m_2 = ½

$m_1 m_2$ = -2(1/2) = -1

∴ The two straight lines are at right angles.

Question-17

Find the equation of the straight line passing through the point (1, -2) and parallel to the straight line 3x + 2y – 7 = 0.

Solution:

The straight line parallel to 3x + 2y – 7 = 0 is of the form 3x + 2y + k = 0 ....(i)

The point (1, -2) satisfies the equation (i)

Hence 3(1) + 2(-2) + k = 0

⇒ 3 - 4 + k = 0⇒ k = 1

∴ 3x + 2y + 1 = 0 is the equation of the required straight line.

Question-18

Find the equation of the straight line passing through the point (2, 1) and perpendicular to the straight line x + y = 9.

Solution:

The equation of the straight line perpendicular to the straight line x + y = 9 is of the form x - y + k = 0.

The point (2, 1) lies on the straight line x - y + k = 0.

∴ 2 - 1 + k = 0

k = -1

∴ The equation of the required straight line is x - y – 1 = 0.

Question-19

Find the point of intersection of the straight lines 5x + 4y – 13 = 0 and 3x + y – 5 = 0.

Solution:

Let (x_1, y_1) be the point of intersection. Then (x_1, y_1) lies on both the straight lines.

$\therefore 5x_1 + 4y_1 - 13 = 0$ ……………..(i)

$3x_1 + y_1 - 5 = 0$ ……………..(ii)

(ii) × 4 $12x_1 + 4y_1 - 20 = 0$ ……………(iii)

(i) – (iii) $-7x_1 + 7 = 0$

$x_1 = 1$

Substituting $x_1 = 1$ in (i) $5(1) + 4y_1 - 13 = 0$

$5 + 4y_1 - 13 = 0$

$4y_1 - 8 = 0$

$y_1 = 2$

∴ The point of intersection is (1, 2).

Question-20

If the two straight lines 2x – 3y + 9 = 0, 6x + ky + 4 = 0 are parallel, find k.

Solution:

The two given equations are parallel.

$2x - 3y + 9 = 0$ ……………..(i)

$6x + ky + 4 = 0$ ………………(ii)

∴ The coefficients of x and y are proportional $\frac{2}{6} = \frac{-3}{k}$.∴ k = -9.

Question-21

Find the distance between the parallel lines 2x + y – 9 = 0 and 4x + 2y + 7 = 0.

Solution:

The distance between the parallel lines is $\left|\frac{-9-\frac{7}{2}}{\sqrt{2^2+1^2}}\right| = \left|\frac{\frac{-25}{2}}{\sqrt{5}}\right| = \left|\frac{-25}{2\sqrt{5}}\right| = \frac{5\sqrt{5}}{2}$ units

Question-22

Find the values of p for which the straight lines 8px + (2 – 3p)y + 1 = 0 and px + 8y – 7 = 0 are perpendicular to each other.

Solution:

Slope of the line $8px + (2 - 3p)y + 1 = 0$ is $m_1 = 8p/(2 - 3p)$.

and slope of the line $px + 8y - 7 = 0$ is $m_2 = -p/8$

$m_1 m_2 = -1$

$\frac{8p}{2-3p} \times \frac{-p}{8} = -1$

$-p^2 = -(2 - 3p)$

$p^2 + 3p - 2 = 0$

$p^2 + 2p + p - 2 = 0$

$(p + 1)(p - 2) = 0$

∴ The two straight lines are at right angles.

Question-23

Find the equation of the straight line which passes through the intersection of the straight lines 2x + y = 8 and 3x – 2y + 7 = 0 and is parallel to the straight line 4x + y – 11 = 0.

Solution:

$2x + y - 8 = 0$ ....(i)

$3x - 2y + 7 = 0$ ....(ii)

(i) × 2

$4x + 2y - 16 = 0$ ....(iii)

(ii) + (iii)

$7x - 9 = 0$

$x = 9/7$

Substituting $x = 9/7$ in (i)

$2(9/7) + y - 8 = 0$

$18/7 + y - 8 = 0$

$y = 8 - 18/7 = 38/7$

Point of intersection of $2x + y = 8$ and $3x - 2y + 7 = 0$ is $(9/7, 38/7)$.

The equation of lines parallel the straight line $4x + y - 11 = 0$ is $4x + y + k = 0$.

$4(\frac{9}{7}) + \frac{38}{7} + k = 0$

$k = -\frac{74}{7}$ ∴ The required equation of straight line is $4x + y - \frac{74}{7} = 0$ i.e.,

$28x + 7y - 74 = 0$.

Question-24

Find the equation of the straight line passing through intersection of the straight lines 5x – 6y = 1 and 3x + 2y + 5 = 0 and perpendicular to the straight line 3x – 5y + 11 = 0.

Solution:

Equation of line through the intersection of straight lines 5x – 6y = 1 and 3x + 2y + 5 = 0 is

$$(5x - 6y - 1) + k(3x + 2y + 5) = 0$$

$(5 + 3k)x + (-6 + 2k)y + (-1 + 5k) = 0$

Slope of the above equation is $-(5 + 3k)/(-6 + 2k)$.

Above equation is perpendicular to 3x – 5y + 11 = 0.

Question-25

Find the equation of the straight line joining (4, -3) and the intersection of the straight lines 2x – y + 7 = 0 and x + y – 1 = 0.

Solution:

$2x - y + 7 = 0$ ………………(i)

$x + y - 1 = 0$ ………………(ii)

(i) + (ii)

$3x + 6 = 0$

$x = -2$

Substituting x = -2 in (i)

$2(-2) - y + 7 = 0$

$-4 - y + 7 = 0$

$y = 3$

The equation of the line joining (4, -3) and (-2, 3) is

$\frac{y+3}{-3-3} = \frac{x-4}{4+2}$

$6(y + 3) = -6(x - 4)$

$y + 3 + x - 4 = 0$

$x + y = 1$

Question-26

Find the equation of the straight line joining the point of the intersection of the straight lines 3x + 2y + 1 = 0 and x + y = 3 to the point of intersection of the straight lines y − x = 1 and 2x + y + 2 = 0.

Solution:

3x + 2y + 1 = 0 ....(i)

x + y = 3 ....(ii)

(ii) × -2

-2x - 2y + 6 = 0 ....(iii)

(i) + (iii)

x + 7 = 0

x = -7

Substituting x = -7 in (ii)

x + y − 3 = 0

-7 + y − 3 = 0

y = 10

∴ The point of intersection of lines 3x + 2y + 1 = 0 and x + y = 3 is (-7, 10).

y − x − 1 = 0 ....(i)

2x + y + 2 = 0 ....(ii)

(i) − (ii)

-3x - 3 = 0

x = -1

Substituting x = -1 in (ii)

y + 1 − 1 = 0

y = 0

∴ The point of intersection of lines y − x = 1 and 2x + y + 2 = 0 is (-1, 0).

The equation of the line joining (-7, 10) and (-1, 0) is

$\frac{y-10}{10-0} = \frac{x+7}{-7+1}$

-6(y - 10) = 10(x + 7)

-3(y - 10) = 5(x + 7)

-3y + 30 = 5x + 35

5x + 3y + 5 = 0.

∴ The required equation of straight line is 5x + 3y + 5 = 0.

Question-27

Show that the angle between 3x + 2y = 0 and 4x − y = 0 is equal to the angle between 2x + y = 0 and 9x + 32y = 41.

Solution:

Slope of the line 3x + 2y = 0 is m_1 = -3/2.

Slope of the line 4x − y = 0 is m_2 = 4.

Angle between the straight line 3x + 2y = 0 and 4x − y = 0 is tan θ_1

$$= \left|\frac{\frac{-3}{2}-4}{1+\left(\frac{-3}{2}\right)\times 4}\right| = \left|\frac{\frac{-3-8}{2}}{1-6}\right| = \left|\frac{\frac{-11}{2}}{-5}\right| = 11/10$$

Slope of the line 2x + y = 0 is m_1 = -2.

Slope of the line 9x + 32y = 41 is m_2 = -9/32.

Angle between the straight line 2x + y = 0 and 9x + 32y = 41 is tan θ_2 =

$$\left|\frac{-2+\frac{9}{32}}{1+(-2)\left(-\frac{9}{32}\right)}\right|$$

$$= \left|\frac{\frac{-64+9}{32}}{1+\frac{18}{32}}\right| = \left|\frac{\frac{-55}{32}}{\frac{50}{32}}\right| = 11/10 \therefore \tan\theta_1 = \tan\theta_2 \therefore \theta_1 = \theta_2$$

Question-28

Show that the triangle whose sides are y = 2x + 7, x − 3y − 6 = 0 and x + 2y = 8 is right angled. Find its other angles.

Solution:

Slope of the line y = 2x + 7 is m_1 = 2.

Slope of the line x − 3y − 6 = 0 is m_2 = 1/3.

Slope of the line x + 2y = 8 is m_3 = -1/2.

Angle between the straight line y = 2x + 7 and x − 3y − 6 = 0 is

$$\tan\theta_1 = \left|\frac{2-\frac{1}{3}}{1+(2)\left(\frac{1}{3}\right)}\right| = \left|\frac{\frac{6-1}{3}}{\frac{3+2}{3}}\right| = \left|\frac{\frac{5}{3}}{\frac{5}{3}}\right| = 1 \ \backslash\ \theta_1 = 45^o$$

Angle between the straight line x − 3y − 6 = 0 and x + 2y = 8 is

$$\tan\theta_2 = \left|\frac{\frac{1}{3}+\frac{1}{2}}{1+\left(\frac{1}{3}\right)\left(-\frac{1}{2}\right)}\right| = \left|\frac{\frac{2+3}{6}}{\frac{6-1}{6}}\right| = \left|\frac{\frac{5}{6}}{\frac{5}{6}}\right| = 1$$

$\therefore \theta_2 = 45^o$

Angle between the straight line x + 2y = 8 and y = 2x + 7 is

$$\tan\theta_3 = \left|\frac{2+\frac{1}{2}}{1+(2)\left(-\frac{1}{2}\right)}\right| = \left|\frac{\frac{4+1}{2}}{\frac{2-2}{2}}\right| = \left|\frac{\frac{5}{2}}{\frac{0}{2}}\right| \qquad \theta_3 = 90^o$$

∴ The triangle is right angled isosceles.

Question-29

Show that the straight lines 3x + y + 4 = 0, 3x + 4y - 15 = 0 and 24x − 7y − 3 = 0 form an isosceles triangle.

Solution:

Slope of the line 3x + y + 4 = 0 is m_1 = -3.

Slope of the line 3x + 4y - 15 = 0 is m_2 = -3/4.

Slope of the line 24x − 7y − 3 = 0 is m_3 = 24/7.

Angle between the straight line 3x + y + 4 = 0 and 3x + 4y - 15 = 0 is

$$\tan\theta_1 = \left|\frac{-3+\frac{3}{4}}{1+(-3)\left(-\frac{3}{4}\right)}\right| = \left|\frac{\frac{-12+3}{4}}{\frac{4+9}{4}}\right| = \left|\frac{9}{13}\right| = 9/13$$

Angle between the straight line 3x + 4y - 15 = 0 and 24x − 7y − 3 = 0 is

$$\tan\theta_2 = \left|\frac{-\frac{3}{4}-\frac{24}{7}}{1+\left(-\frac{3}{4}\right)\left(\frac{24}{7}\right)}\right| = \left|\frac{\frac{-21-96}{28}}{\frac{28-72}{28}}\right| = \left|\frac{\frac{-117}{28}}{\frac{-44}{28}}\right| = 117/44$$

Angle between the straight line 24x − 7y − 3 = 0 and 3x + y + 4 = 0 is

$$\tan\theta_3 = \left|\frac{\frac{24}{7}+3}{1+\left(\frac{24}{7}\right)(-3)}\right| = \left|\frac{\frac{24+21}{7}}{\frac{7-72}{7}}\right| = \left|\frac{\frac{45}{7}}{\frac{-65}{7}}\right| = 9/13$$

$\tan\theta_1 = \tan\theta_3$

∴ The triangle is isosceles.

Question-30

Show that the straight lines 3x + 4y = 13, 2x − 7y + 1 = 0 and 5x − y = 14 are concurrent.

Solution:

3x + 4y = 13 ....(i)

2x − 7y + 1 = 0 ....(ii)

5x − y = 14 ....(iii)

(iii) × 4

20x − 4y = 56 ....(iv)

(i) + (iv)

23x = 69

x = 3

Substitute x = 3 in (iii)

15 − y = 14

y = 1

∴ The point of intersection is (3, 1).

3(3) + 4(1) = 13

9 + 4 = 13

The point (3, 1) satisfies equation (i). Hence they are concurrent.

Question-31

Find 'a' so that the straight lines x − 6y + a = 0, 2x + 3y + 4 = 0 and x + 4y + 1 = 0 may be concurrent.

Solution:

$x - 6y + a = 0$ ....(i)

$2x + 3y + 4 = 0$ ....(ii)

$x + 4y + 1 = 0$ ....(iii)

2 × (iii)

$2x + 8y + 2 = 0$ ....(iv)

(ii) − (iv)

$-5y + 2 = 0$

$y = 2/5$

Substituting y = 2/5 in (iii)

$x + 4(2/5) + 1 = 0$

$x = -13/5$

Substituting (-13/5, 2/5) in (i),

$\frac{-13}{5} - 6\times \frac{2}{5} + a = 0$

$-25 + 5a = 0$

$a = 5$

Question-32

Find the values of 'a' for which the straight lines x + y − 4 = 0, 3x + 2 = 0 and x − y + 3a = 0 are concurrent.

Solution:

$x + y - 4 = 0$ ....(i)

$3x + 2 = 0$ ....(ii)

$x - y + 3a = 0$ ....(iii)

$x = -2/3$ ....(iv)

$(-2/3) + y - 4 = 0$

$y = 4 + 2/3 = 14/3$

Substituting (-2/3, 14/3) in (iii),

$(-2/3) - (14/3) + 3a = 0$

$(-16/3) + 3a = 0$

$a = 16/9$

Question-33

Find the coordinates of the orthocentre of the triangle whose vertices are the points (-2, -1), (6, -1) and (2, 5).

Solution:

Let A(-2, -1), B(6, -1) and C(2, 5) be the vertices of the triangle ABC.

Line AB is $\frac{y+1}{-1+1} = \frac{x+2}{-2-6}$

$x + 2 = 0$ …………………(i)

Line perpendicular to x + 2 = 0 is x + k = 0

If it passes through (2, 5), then 2 + k = 0, k = -2

∴ x − 2 = 0 is one altitude. …………………(ii)

Line BC is $\frac{y+1}{-1-5} = \frac{x-6}{6-2}$

$\frac{y+1}{-6} = \frac{x-6}{4}$

$2(y + 1) = -3(x - 6)$

$2y + 3x = 16$ …………………(iii)

Line perpendicular to 2y + 3x = 16 is 2x - 3y + k = 0.

If it passes through (-2, -1), then 2(-2) − 3(-1) + k = 0 i.e., -4 + 3 + k = 0 i.e., k = 1

∴ 2x - 3y + 1 = 0 is another altitude. …………………(iv)

Solving (ii) and (iv)

$x = 2$

$2(2) − 3y + 1 = 0$

$4 − 3y + 1 = 0$

$-3y = -5$

$y = 5/3$

∴ Orthocentre is (2, 5/3).

Question-34

If ax + by + c = 0, bx + cy + a = 0 and cx + ay + b = 0 are concurrent, show that $a^3 + b^3 + c^3 = 3abc$.

Solution:

The condition for three lines concurrency is $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$a(cb − a^2) − b(b^2 - ca) + c(ab − c^2) = 0$

$abc − a^3 − b^3 + abc + abc − c^3 = 0$

$abc − a^3 − b^3 + abc + abc − c^3 = 0$

\ $a^3 + b^3 + c^3 = 3abc$.

Question-35

Find the coordinates of the orthocentre of the triangle formed by the straight lines x + y − 1 = 0, x + 2y − 4 = 0 and x + 3y − 9 = 0.

Solution:

Let the equation of sides AB, BC and CA of a Δ BC be represented by

$x + y - 1 = 0$ ....(i)

$x + 2y - 4 = 0$ ....(ii)

$x + 3y - 9 = 0$ ....(iii)

(i) × 2

$2x + 2y - 2 = 0$ ....(iv)

(ii) - (iv)

$-x - 2 = 0$

$x = -2$

Substituting x = -2 in (ii)

$-2 + 2y - 4 = 0$

$2y = 6$

$y = 3$

The vertex A is (-2, 3).

The equation of the straight line CA is x + 3y − 9 = 0. The straight line perpendicular to its is of the form 3x - y + k = 0 ....(v)

A(-2, 3) satisfies the equation (v)

$\therefore 3x - y + k = 0$

$3(-2) - 3 + k = 0$

$k = 9$

The equation of AD is 3x - y + 9 = 0. ....(vi)

(ii) − (iii)

$-y + 5 = 0$

$y = 5$

Substituting y = 5 in (ii)

$x + 2(5) - 4 = 0$

$x + 10 - 4 = 0$

$x = -6$

The vertex C is (-6, 5).

The equation of the straight line AB is x + y − 1 = 0. The straight line perpendicular to its is of the form x - y + k = 0 ....(vii)

C(-6, 5) satisfies the equation (vii)

$\therefore$ x - y + k = 0

-6 - 5 + k = 0

k = 11

The equation of CE is x − y + 11 = 0. ....(viii)

(vi) − (viii)

2x − 2 = 0

x = 1

Substituting x = 1 in (viii)

1 − y + 11 = 0

y = 12

$\therefore$ The orthocentre O is (1, 12).

Question-36

The equation of the sides of a triangle are x + 2y = 0, 4x + 3y = 5 and 3x + y = 0. Find the coordinates of the orthocentre of the triangle.

Solution:

Let the equation of sides AB, BC and CA of a Δ BC be represented by

x + 2y = 0 ....(i)

4x + 3y = 5 ....(ii)

3x + y = 0 ....(iii)

Substituting x = -2y (ii)

4(-2y) + 3y = 5

-8y + 3y = 5

y = -1

$\therefore$ x = 2

The vertex B is (2, -1).

The equation of the straight line AC is 3x + y = 0. The straight line perpendicular to its is of the form x - 3y + k = 0 ....(iv)

B(2, -1) satisfies the equation (iv)

$\therefore 2 - 3(-1) + k = 0$

$2 + 3 + k = 0$

$k = -5$

The equation of BD is $x - 3y - 5 = 0$. ………………(v)

Substituting $x = -2y$ (iii)

$3(-2y) + y = 0$

$-6y + y = 0$

$-5y = 0$

$y = 0$

$\therefore x = 0$

The vertex A is (0, 0).The equation of the straight line BC is $4x + 3y = 5$. The straight line perpendicular to its is of the form $3x - 4y + k = 0$ ……………….. (vi)

A(0, 0) satisfies the equation (vi)

$\therefore 3(0) - 4(0) + k = 0$

$k = 0$

The equation of AE is $3x - 4y = 0$ ………………(vii)

3 × (v) – (vii)

$3x - 9y - 15 = 0$

$\therefore -5y - 15 = 0$

$y = -15/5 = -3$

Substitute $y = -3$ in (vii)

$3x - 4(-3) = 0$

$3x + 12 = 0$

$x = -4$∴ The orthocentre O is (-4, -3).

CBSE Class11 Mathematics

Important Questions

Chapter 10

Straight Lines

1 Marks Questions

1. Find the slope of the lines passing through the point (3,-2) and (-1,4)

Ans. Slope of line through (3,-2) and (-1, 4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-(-2)}{-1-3}$$

$$= \frac{6}{-4} = \frac{-3}{2}$$

2. Three points $P(h, k)$, $Q(x_1, y_1)$ and $R(x_2, y_2)$ lie on a line. Show that $(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$

Ans. Since P, Q, R are collinear

Slope of PQ= slope of QR

$$\frac{y_1 - k}{x_1 - h} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{\cancel{-}(k - y_1)}{\cancel{-}(h - x_1)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$$

3. Write the equation of the line through the points $(1, -1)$ and $(3, 5)$

Ans. Req. eq. $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$y + 1 = \frac{5+1}{2}(x - 1)$$

$$-3x + y + 4 = 0$$

4. Find the measure of the angle between the lines $x + y + 7 = 0$ and $x - y + 1 = 0$

Ans. $x + y + 7 = 0$

$$m_1 = \frac{-1}{1}$$

$$x - y + 1 = 0$$

$$m_2 = \frac{-1}{-1} = 1$$

Slopes of the two lines are 1 and -1 as product of these two slopes is -1, the lines are at right angles.

5. Find the equation of the line that has y-intercept 4 and is $\perp$ to the line $y = 3x - 2$

Ans. $y = 3x - 2$

Slope (m) = $\frac{-3}{-1} = 3$, slope of any line $\perp$ it is $-\frac{1}{3}$

$$C = 4$$

Req. eq. is $y = mx + c$

$$y = \frac{-1}{3}x + 4$$

6. Find the equation of the line, which makes intercepts -3 and 2 on the x and y-axis respectively.

Ans. Req. eq. $\frac{x}{a}+\frac{y}{b}=1$

$a=-3, b=2$

$\therefore \frac{x}{-3}+\frac{y}{2}=1$

2x - 3y + 6 = 0

7. Equation of a line is $3x-4y+10=0$ find its slope.

Ans. $m=\frac{-\text{coff. of } x}{\text{coff. of } y}$

$=\frac{-3}{-4}=\frac{3}{4}$

8. Find the distance between the parallel lines $3x-4y+7=0$ and $3x-4y+5=0$

Ans. $A=3, B=-4, C_1=7$ and $C_2=5$

$d=\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}$

$=\frac{|7-5|}{\sqrt{(3)^2+(-4)^2}}$

$=\frac{2}{5}$

9. Find the equation of a straight line parallel to y-axis and passing through the point

(4,-2)

Ans. Equation of line parallel to y -axis is $x = a.....(i)$

Eq. (i) passing through (-4,2)

$a = -4$

So $x = -4$

$x + 4 = 0$

10. If $3x - by + 2 = 0$ and $9x + 3y + a = 0$ represent the same straight line, find the values of a and b.

Ans. ATQ

$$\frac{3}{9} = \frac{-b}{3} = \frac{2}{a}$$

$b = -1$

$\Rightarrow a = 6$

11. Find the distance between $P(x_1 y_1)$ and $Q(x_2, y_2)$ when PQ is parallel to the y -axis.

Ans. When PQ is parallel to the y -axis,

Then $x_1 = x_2$

$$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}$$

$$= | y_2 - y_1 |$$

12. Find the slope of the line, which makes an angle of 30^0 with the positive direction of

y-axis measured anticlockwise.

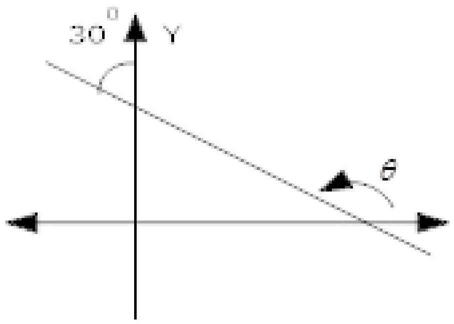
Ans. Let θ be the inclination of the line

$\theta = 120^0$

$m = \tan 120^0$

$= \tan(90+30)$

$= -\sqrt{3}$



13. Determine x so that the inclination of the line containing the points $(x, -3)$ and $(2, 5)$ is 135.

Ans.

$$\frac{5-(-3)}{2-x} = \tan 135$$

$$\left[\begin{array}{l} \because m = \tan\theta \\ m = \dfrac{y_2 - y_1}{x_2 - x_1} \end{array}\right]$$

$$\frac{5+3}{2-x} = -1$$

$x = 10$

14. Find the distance of the point $(4,1)$ from the line $3x-4y-9=0$

Ans. Let d be the req. distance

$$d=\frac{|3.(4)-4(1)-9|}{\sqrt{(3)^2+(-4)^2}}$$

$$=\frac{|-1|}{5}=\frac{1}{5}$$

15. Find the value of x for which the points $(x,-1),(2,1)$ and $(4,5)$ are collinear.

Ans. Let $A(x,-1),B(2,1),C(4,5)$

Slope of AB= Slope of BC

$$\frac{1+1}{2-x}=\frac{5-1}{4-2}$$

$$\frac{2}{2-x}=\frac{4}{2}$$

$2-x=1$

$-x=-1$

$x=1$

16. Find the angle between the x -axis and the line joining the points $(3,-1)$ and $(4,-2)$

Ans. $m_1=0$ [Slope of x -axis]

$m_2=$slope of line joining points $(3,-1)$ and $(4,-2)$

$$= \frac{-2-(-1)}{4-3} = -1$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1+m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{0+1}{1+0\times(-1)}\right|$$

$$\tan\theta = 1$$

$$\theta = 45^0$$

17. Using slopes, find the value of x for which the points $(x,-1),(2,1)$ and $(4,5)$ are collinear.

Ans. Since the given points are collinear slope of the line joining points $(x,-1)$ and $(2,1)$ =slope of the line joining points $(2,1)$ and $(4,5)$

$$\Rightarrow \frac{2}{2-x} = \frac{2}{1}$$

$$x = 1$$

18. Find the value of K so that the line $2x+ky-9=0$ may be parallel to $3x-4y+7=0$

Ans. ATQ

Slope of 1st line = slope of 2nd line

$$\frac{-2}{k} = \frac{-3}{-4}$$

$$\Rightarrow k = \frac{-8}{3}$$

19. Find the value of K, given that the distance of the point $(4,1)$ from the line $3x-4y+K=0$ is 4 units.

Ans. We are given that distance of (4,0) from the line $3x-4y+k=0$ is 4

$$\frac{|3(4)-4(1)+k|}{\sqrt{(3)^2+(-4)^2}}=4$$

$$|k+8|=4\times 5$$
$$k=12,-28$$

20. Find the equation of the line through the intersection of $3x-4y+1=0$ $5x+y-1=0$ which cuts off equal intercepts on the axes.

Ans. Slope of a line which makes equal intercept on the axes is -1any line through the intersection of given lines is

$$(3x-4y+1)+K(5x-y-1)=0$$
$$(3+5K)x+y(K-4)+1-K=0$$

$$m=-\frac{(3+5K)}{K-4}=-1$$

$$K=\frac{-7}{4}$$

21. Find the distance of the point $(2,3)$ from the line $12x-5y=2$

Ans. $d=\frac{|12x-5y-2|}{\sqrt{(12)^2+(-5)^2}}$

$$d=\frac{|12\times 2-5\times 3-2|}{\sqrt{169}}$$

$= \frac{|-41|}{13} = \frac{41}{13}$

22. Find the equation of a line whose perpendicular distance from the origin is 5 units and angle between the positive direction of the x -axis and the perpendicular is 30^0.

Ans. $p = 5, \alpha = 30^0$

Req. eq. $x\cos\alpha + y\sin\alpha = p$

$x\cos 30^0 + y\sin 30^0 = 5$

$\sqrt{3}x + y - 10 = 0$

23. Write the equation of the lines for which $\tan\theta = \frac{1}{2}$, where Q is the inclination of the line and x intercept is 4.

Ans. $m = \tan\theta = \frac{1}{2}$ and $d = 4$

$y = \frac{1}{2}(x-4)$ $[\because y = m(x-d)]$

$2y - x + 4 = 0$

24. Find the Angle between the x -axis and the line joining the points $(3,-1)$ and $(4,-2)$

Ans. Let $A(3,-1)$ $B(4,-2)$

Slope of $AB = \frac{-2-(-1)}{4-2}$

$= -1$

$\tan\theta = -1$

$\theta = 135 \left[\begin{array}{l}\text{where } \theta \text{ is the angle which AB makes} \\ \text{with positive direction of } x-axis\end{array}\right]$

25. Find the equation of the line intersecting the x -axis at a distance of 3 unit to the left of origin with slope -2.

Ans. The line passing through (-3,0) and has slope = -2

Req. eq. is

$y - 0 = -2(x+3)$

$2x + y + 6 = 0$

CBSE Class 12 Mathematics

Important Questions

Chapter 10

Straight Lines

4 Marks Questions

1. If p is the length of the $\perp$ from the origin on the line whose intercepts on the axes are a and b. show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Ans. Equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}-1=0$$

The distance of this line from the origin is P

$$\therefore P=\frac{\left|\frac{0}{a}+\frac{0}{b}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}} \qquad \left[d=\frac{|ax+by+c|}{\sqrt{a^2+b^2}}\right]$$

$$\frac{P}{1}=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\frac{1}{P}=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

Sq. both side

$$\frac{1}{P^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

2.Find the value of p so that the three lines $3x+y-2=0$, $px+2y-3=0$ and $2x-y-3=0$ may intersect at one point.

Ans. $3x+y-2=0........(i)$

$px+2y-3=0.......(ii)$

$2x-y+3=0.......(iii)$

On solving eq. (i) and (iii)

$x=1$, And $y=-1$

Put x, y in eq. (ii)

$P(1)+2(-1)-3=0$

$p-2-3=0$
$p=5$

3.Find the equation to the straight line which passes through the point (3,4) and has intercept on the axes equal in magnitude but opposite in sign.

Ans. Let intercept be a and –a the equation of the line is

$$\frac{x}{a}+\frac{y}{-a}=1$$

$\Rightarrow x-y=a.......(i)$

Since it passes through the point (3, 4)

$3-4=a$
$a=-1$

Put the value of a in eq. (i)

$x-y=-1$
$x-y+1=0$

4.By using area of Δ. Show that the points $(a,b+c),(b,c+a)$ and $(c,a+b)$ are collinear.

Ans. Area of $\Delta=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$=\frac{1}{2}|a(c+a)-b(b+c)+b(a+b)-c(c+a)+c(b+c)-a(a+b)|$

$=\frac{1}{2}.0=0$

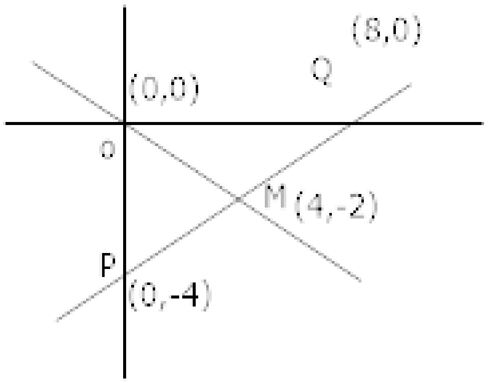
5. Find the slope of a line, which passes through the origin, and the midpoint of the line segment joining the point $p(0,-4)$ and $Q(8,0)$

Ans. Let m be the midpoint of segment PQ then $M=\left(\frac{0+8}{2},\frac{-4+0}{2}\right)$

$=(4,-2)$

Slope of $OM=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-2-0}{4-0}=\frac{-1}{2}$



6.Find equation of the line passing through the point $(2,2)$ and cutting off intercepts on the axes whose sum is 9

Ans. Req. eq. be $\frac{x}{a}+\frac{y}{b}=1......(i)$

$$a+b=9$$
$$b=9-a$$

$$\Rightarrow \frac{x}{a}+\frac{y}{9-a}=1$$

This line passes through $(2,2)$

$$\therefore \frac{2}{a}+\frac{2}{9-a}=1$$

$$a^2-9a+18=0$$

$$a^2-6a-3a+18=0$$

$$a(a-6)-3(a-6)=0$$

$$(a-6)(a-3)=0$$
$$a=6,3$$

$$a=6 \quad a=3$$
$$b=3 \quad b=6$$

$\frac{x}{6}+\frac{y}{3}=1 \quad \frac{x}{3}+\frac{y}{6}=1$

7.Reduce the equation $\sqrt{3}x+y-8=0$ into normal form. Find the values p and ω .

Ans. $\sqrt{3}x+y-8=0$

$\sqrt{3}x+y=8......(i)$

$\sqrt{\left(\sqrt{3}\right)^2+(1)^2}=2$

Dividing (i) by 2

$\frac{\sqrt{3}}{2}x+\frac{y}{2}=4$

$x\cos 30^0+y.\sin 30=4.......(ii)$

Comparing (ii) with

$x\cos\omega+y\sin\omega=p$

$p=4$

$\omega=30^0$

8. Without using the Pythagoras theorem show that the points $(4,4)$,$(3,5)$ and $(-1,-1)$ are the vertices of a right angled Δ.

Ans. The given points are $A(4,4)$,$B(3,5)$ and $C(-1,-1)$

Slope of $AB=\frac{5-4}{3-4}=-1$

Slope of $BC = \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of $AC = \frac{-1-4}{-1-4} = +1$

Slope of $AB \times$ slope of $AC = -1$

$\Rightarrow AB \perp AC$

Hence ΔABC is right angled at A.

9.The owner of a milk store finds that, he can sell 980 liters of milk each week at 14 liter and 1220 liter of milk each week at Rs 16 liter. Assuming a linear relationship between selling price and demand how many liters could he sell weekly at Rs 17 liter?

Ans. Assuming sell along x -axis and cost per litre along y -axis, we have two points $(980,14)$ and $(1220,16)$ in $x\ y$ plane

$$y-14=\frac{16-14}{1220-980}(x-980)$$

$$y-14=\frac{\cancel{2}}{{}_{120}\cancel{240}}(x-980)$$

$$120y-14\times 120=x-980$$

$$120y-1680=x-980$$

$$x-120y=-700$$

When $y=17$

$x-120\times 17=-700 \Rightarrow x=1340$ litres.

10.The line through the points (h,3) and (4,1) intersects the line $7x-9y-19=0$ at right angle. Find the value of h.

Ans. Slope of line joining (h,3) and (4,1)

$$= \frac{1-3}{4-h} = \frac{-2}{4-h}$$

Given line is $7x - 9y - 19 = 0$

Slope of this line = $\frac{-7}{-9}$

ATQ

$$\left(\frac{-2}{4-h}\right) \times \left(\frac{7}{9}\right) = -1$$

$$\Rightarrow h = \frac{22}{9}$$

11.Find the equations of the lines, which cut off intercepts on the axes whose sum and product are 1 and -6 respectively.

Ans. ATQ $a + b = 1 ....(i)$

$ab = -6 ....(ii)$

$b = 1 - a \quad [from(i)]$

Put b in eq. (ii)

$$a(1-a) = -6$$
$$a - a^2 = -6$$

$$a^2 - a - 6 = 0$$

$$(a-3)(a+2) = 0$$

$$a = 3, -2$$

When $a = 3$

$b = -2$

Eq. of the line is

$$\frac{x}{3} + \frac{y}{-2} = 1$$

$$2x - 3y - 6 = 0$$

When $a = -2$

$b = 3$

Eq. of the line is

$$\frac{x}{-2} + \frac{y}{3} = 1$$

$$3x - 2y + 6 = 0$$

12. The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines.

Ans. Let the slope of one line is m and other line is $2m$

$$\frac{1}{3} = \left|\frac{2m - m}{1 + (2m)(m)}\right|$$

$$\frac{1}{3} = \left|\frac{m}{1 + 2m^2}\right|$$

$$\pm\frac{1}{3} = \frac{m}{1 + 2m^2}$$

$$\frac{1}{3}=\frac{m}{1+2m^2}$$

$$2m^2-3m+1=0$$
$$2m^2-2m-m+1=0$$

$$2m(m-1)-1(m-1)=0$$

$$(m-1)(2m-1)=0$$

$$m=1, m=\frac{1}{2}$$

$$\frac{-1}{3}=\frac{m}{1+2m^2}$$

$$-1-2m^2=3m$$
$$2m^2+3m+1=0$$

$$2m^2+2m+m+1=0$$
$$2m(m+1)+1(m+1)=0$$

$$(m+1)(2m+1)=0$$

$$m=-1$$

$$m=\frac{-1}{2}$$

13.Point $R(h,k)$ divides a line segment between the axes in the ratio 1:2. Find equation of the line.

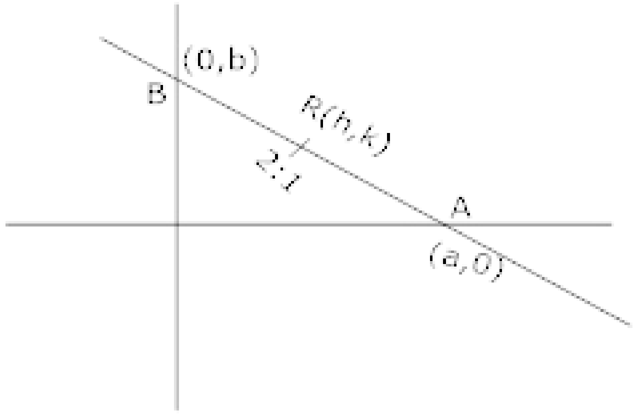
Ans. Let eq. be $\frac{x}{a}+\frac{y}{b}=1$.......$(i)$

It is given that $R(h,k)$ divides AB in the ratio 1:2

$\therefore (h,k) = \left(\frac{2a}{3}, \frac{b}{3}\right)$

$\frac{2a}{3} = h$

$a = \frac{3h}{2}$



$k = \frac{b}{3}$

$b = 3k$

Put a and b in eq.......... (i)

$\frac{x}{\frac{3h}{2}} + \frac{y}{3k} = 1$

$\frac{2x}{h} + \frac{y}{k} = 3$

14.The Fahrenheit temperature F and absolute temperature K satisfy a linear equation. Given that K=273 when F=32 and that K= 373 when F=212 Express K in terms of F and find the value of F when K=0

Ans. Let F along x -axis and K along y-axis

$$K-273=\frac{373-273}{212-32}(F-32)\left[\because y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$$

$$K-273=\frac{100}{180}(F-32)$$

$$K=\frac{5}{9}(F-32)+273$$

15. If three points $(h,0)(a,b)$ and $(0,k)$ lie on a line, show that $\frac{a}{h}+\frac{b}{k}=1$

Ans. Let $A(h,0)B(a,b)$ and $C(0,k)$

Slope of AB = slope of BC

$$\frac{b-0}{a-h}=\frac{k-b}{0-a}$$

$$\frac{b}{a-h}=\frac{h-b}{-a}$$

$$(a-h)(k-b)=-ab$$

$$ak-ab-hk+hb=-ab$$

$$ak+hb=hk$$

$$\frac{ak}{hk}+\frac{hb}{hk}=1$$

$$\frac{a}{h}+\frac{b}{k}=1$$

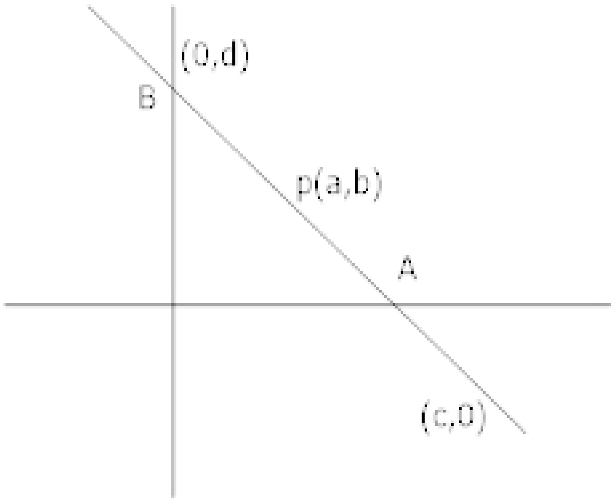
16. $p(a,b)$ is the mid point of a line segment between axes. Show that equation of the

line is $\frac{x}{a}+\frac{y}{b}=2$

Ans. Req. eq. be

$$\frac{x}{c}+\frac{y}{d}=1......(i)$$

P is the mid point



Coordinate of $p=\left(\frac{c}{2},\frac{d}{2}\right)$

$$(a,b)=\left(\frac{c}{2},\frac{d}{2}\right)$$

$$\frac{a}{1}=\frac{c}{2}$$

$$c=2a$$

$$\frac{b}{1}=\frac{d}{2}$$

$$d=2b$$

Put the value of C and D in eq. (i)

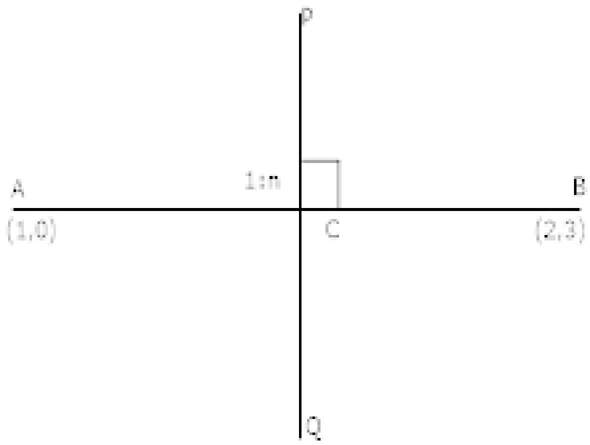
$$\frac{x}{2a}+\frac{y}{2b}=1$$

$$\frac{x}{a}+\frac{y}{b}=2$$

17.The line $\perp$ to the line segment joining the points $(1, 0)$ and $(2, 3)$ divides it in the ratio $1:n$ find the equation of the line.

Ans. Coordinate of $c\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$

$$m_{AB}=3$$
$$m_{PQ}=-\frac{1}{3}$$



Eq. of PQ is

$$\frac{y}{1}-\frac{3}{1+n}=-\frac{1}{3}\left(\frac{x}{1}-\frac{2+n}{1+n}\right)$$

$$(n+1)x+3(n+1)y-(n+11)=0$$

CBSE Class 12 Mathematics
Important Questions
Chapter 10
Straight Lines

6 Marks Questions

1. Find the values of k for the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$

(a). Parallel to the x-axis

(b). Parallel to y-axis

(c). Passing through the origin

Ans.

(a) The line parallel to x-axis if coeff. Of x =0

$k-3=0$

$k=3$

(b) The line parallel to y-axis if coeff. Of y =0

$4-k^2=0$

$k=\pm 2$

(c) Given line passes through the origin if (0, 0) lies on given eq.

$(k-3).(0)-(4-k^2)(0)+k^2-7k+6=0$

$(k-6)(k-1)=0$

$k=6,1$

2. If p and q are the lengths of $\perp$ from the origin to the lines.

$x\cos\theta - y\sin\theta = k\cos 2\theta$, and $x\sec\theta + y\text{cosec}\theta = k$ respectively, prove that $p^2 + 4q^2 = k^2$

Ans.

$$P = \frac{|0.\cos\theta - 0\sin\theta - k\cos 2\theta|}{\sqrt{(\cos\theta)^2 + (-\sin\theta)^2}} \quad \begin{bmatrix} \perp \textit{ from origin} \\ \because (0,0) \end{bmatrix}$$

$$P = K\cos 2\theta.....(i)$$

$$q = \frac{|0.\sec\theta + 0\,\text{cosec}\theta - k|}{\sqrt{\sec^2\theta + \text{cosec}^2\theta}}$$

$$= \frac{K}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}}$$

$$= \frac{k\cos\theta.\sin\theta}{\sqrt{\sin^2\theta + \cos^2\theta}} = \frac{1}{2}k.\sin\theta.\cos\theta$$

$$2q = k.\sin 2\theta......(ii)$$

Squaring (i) and (ii) and adding

$$P^2 + (2q)^2 = K^2\cos^2 2\theta + K^2\sin^2 2\theta$$

$$P^2 + 4q^2 = K^2\left(\cos^2 2\theta + \sin^2 2\theta\right)$$

$$p^2 + 4q^2 = k^2$$

3.Prove that the product of the $\perp$ drawn from the points $\left(\sqrt{a^2 - b^2}, 0\right)$ and $\left(-\sqrt{a^2 - b^2}, 0\right)$ to the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2.

Ans. Let

$$p_1 = \frac{\left|\frac{\sqrt{a^2-b^2}}{a}.\cos\theta - 1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2 + \left(\frac{\sin\theta}{b}\right)^2}} \left[\because \perp \text{ from the points} \sqrt{a^2-b^2}, 0\right]$$

Similarly p_2 be the distance from $\left(-\sqrt{a^2-b^2}, 0\right)$ to given line

$$p_2 = \frac{\left|-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2 + \left(\frac{\sin\theta}{b}\right)^2}}$$

$$p_1 p_2 = \frac{\left|\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\left(-\frac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\right|}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}$$

$$= \frac{\left|\left(\frac{a^2-b^2}{a^2}\right).\cos^2\theta - 1\right|}{\frac{b^2\cos^2\theta + a^2\sin^2\theta}{a^2b^2}}$$

$$= \frac{\left|a^2\cos^2\theta - b^2\cos^2\theta - a^2\right| a^2b^2}{a^2(a^2\sin^2\theta + b^2\cos^2\theta)}$$

$$= \frac{\left|-(a^2\sin^2\theta + b^2\cos^2\theta)\right| b^2}{a^2\sin^2\theta + b^2\cos^2\theta} \qquad \left[\because a^2\cos^2\theta - a^2 = a^2(\cos^2\theta - 1)\right.$$

$$= \frac{(a^2\sin^2\theta + b^2\cos^2\theta)\ b^2}{a^2\sin^2\theta + b^2\cos^2\theta}$$

$= b^2$

4. Find equation of the line mid way between the parallel lines $9x+6y-7=0$ and $3x+2y+6=0$

Ans. The equations are

$$9x+6y-7=0$$

$$3\left(3x+2y-\frac{7}{3}\right)=0$$

$$3x+2y-\frac{7}{3}=0.......(i)$$

$$3x+2y+6=0.....(ii)$$

Let the eq. of the line mid way between the parallel lines (i) and (ii) be

$$3x+2y+k=0.......(iii)$$

ATQ

Distance between (i) and (iii) = distance between (ii) and (iii)

$$\left|\frac{K+\frac{7}{3}}{\sqrt{9+4}}\right|=\left|\frac{K-6}{\sqrt{9+4}}\right| \quad \left[\because d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}\right]$$

$$K+\frac{7}{3}=K-6$$

$$K=\frac{11}{6}$$

Req. eq. is

$3x+2y+\frac{11}{6}=0$

5. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1,2) in the line $x-3y+4=0$

Ans. Let $Q(h,k)$ is the image of the point $p(1,2)$ in the line.

$x-3y+4=0$......(i)

Coordinate of midpoint of $PQ=\left(\frac{h+1}{2},\frac{k+2}{2}\right)$

This point will satisfy the eq. ……(i)

$\left(\frac{h+1}{2}\right)-3\left(\frac{k+2}{2}\right)+4=0$

$h-3k=-3$.......(i)

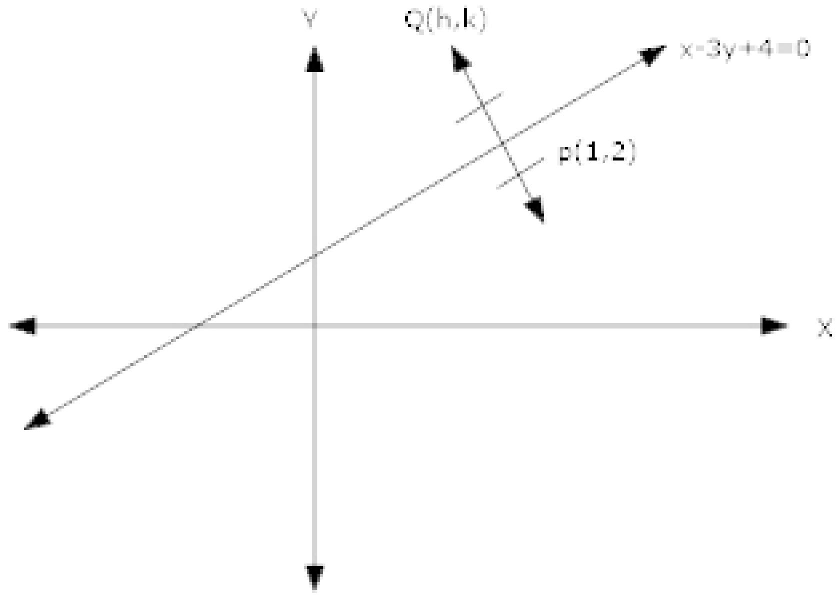
(Slope of line PQ) $\times$ (slope of line $x-3y+4=0)=-1$

$\left(\frac{k-2}{h-1}\right)\left(\frac{-1}{-3}\right)=-1$

$3h+k=5$......(ii)

On solving (i) and (ii)

$h=\frac{6}{5}$ and $k=\frac{7}{5}$



6.A person standing at the junction (crossing) of two straight paths represented by the equations $2x-3y+4=0$ and $3x+4y-5=0$ wants to reach the path whose equation is $6x-7y+8=0$ in the least time. Find equation of the path that he should follow.

Ans. $2x-3y-4=0......(i)$

$3x+4y-5=0......(ii)$

$6x-7y+8=0.......(iii)$

On solving eq. (i) and (ii)

We get $\left(\frac{31}{17},\frac{-2}{17}\right)$

To reach the line (iii) in least time the man must move along the $\perp$ from crossing point $\left(\frac{31}{17},\frac{-2}{17}\right)$ to (iii) line

Slope of (iii) line is $\frac{6}{7}$

Slope of required path $= \frac{-7}{6}$ $[\because m_1 \times m_2 = -1]$

$$y - \left(-\frac{2}{17}\right) = \frac{-7}{6}\left(x - \frac{31}{17}\right)$$

$$119x + 102y = 205$$

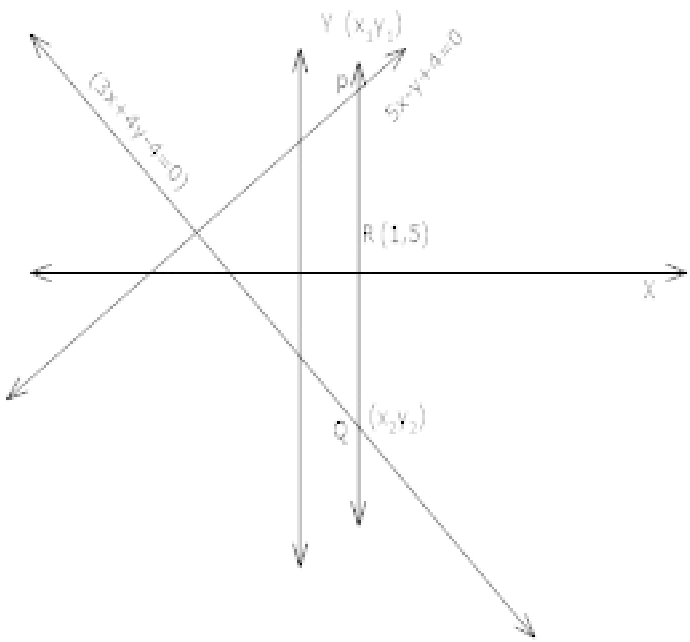
7. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$ obtain its equation.

Ans. $p(x_1 y_1)$ lies on $5x - y + 4 = 0$

$$\Rightarrow 5x_1 - y_1 + 4 = 0$$

And $Q(x_2 y_2)$ lies on $3x + 4y - 4 = 0$

$$3x_2 + 4y_2 - 4 = 0$$



On solving

$$y_1 = 5x_1 + 4$$

$$y_2 = \frac{4-3x_2}{4}$$

Since R is the mid point of PQ

$$\frac{x_1+x_2}{2} = 1, \frac{y_1+y_2}{2} = 5$$

$$x_1 + x_2 = 2, \ y_1 + y_2 = 10$$

On solving

$$x_1 = \frac{26}{23}, \ x_2 = \frac{20}{23}$$

And $y_1 = \frac{222}{23}, \ y_2 = \frac{8}{23}$

Eq. of PQ

$$y - \frac{222}{23} = \frac{\frac{8}{23} - \frac{222}{23}}{\frac{20}{23} - \frac{26}{23}}\left(x - \frac{26}{23}\right)$$

$$107x - 3y - 92 = 0$$

8.Find the equations of the lines which pass through the point $(4, 5)$ and make equal angles with the lines $5x - 12y + 6 = 0$ and $3x - 4y - 7 = 0$

Ans. The slopes of the given lines are $\frac{5}{12}$ and $\frac{3}{4}$

Let m be the slope of a required line

ATQ

$$\left|\frac{m-\frac{5}{12}}{1+m.\frac{5}{12}}\right|=\left|\frac{m-\frac{3}{4}}{1+m.\frac{3}{4}}\right|$$

$$\Rightarrow\left|\frac{12m-5}{12+5m}\right|=\left|\frac{4m-3}{4+3m}\right|$$

$$\frac{12m-5}{12+5m}=\frac{4m-3}{4+3m}$$

$$16m^2=-16$$

$$m^2=-1$$

Neglect

$$\frac{12m-5}{12+5m}=-\frac{4m-3}{4+3m}$$

$$m=\frac{4}{7},\frac{-7}{4}$$

Req. eq. are

$$y-5=\frac{4}{7}(x-4)$$

$$4x-7y+19=0$$

$$y-5=\frac{-7}{4}(x-4)$$

$$7x+4y-48=0$$

Straight lines

1. Find the equation of the straight line which passes through the point $(1, -2)$ and cuts off equal intercepts from axes.
2. Find the equation of the line passing through the point $(5, 2)$ and perpendicular to the line joining the points $(2, 3)$ and $(3, -1)$.
3. Find the angle between the lines $y = (2 - \sqrt{3})(x + 5)$ and $y = (2 + \sqrt{3})(x - 7)$.
4. Find the equation of the lines which passes through the point $(3, 4)$ and cuts off intercepts from the coordinate axes such that their sum is 14.
5. Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.
6. Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.
7. Find the equation of lines passing through $(1, 2)$ and making angle 30° with y-axis.
8. Find the equation of the line passing through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ and parallel to the line $3x + 4y = 7$.
9. For what values of a and b the intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ are equal in length but opposite in signs to those cut off by the line $2x - 3y + 6 = 0$ on the axes.
10. If the intercept of a line between the coordinate axes is divided by the point $(-5, 4)$ in the ratio $1 : 2$, then find the equation of the line.
11. Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of x-axis.

    [**Hint**: Use normal form, here $\omega = 30^\circ$.]
12. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by $3x + 4y = 4$ and the opposite vertex of the hypotenuse is $(2, 2)$.

13. If the equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$, then find the length of the side of the triangle.

[**Hint**: Find length of perpendicular (p) from $(2, -1)$ to the line and use $p = l \sin 60°$, where l is the length of side of the triangle].

14. A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P.

[**Hint**: Let the slope of the line be m. Then the equation of the line passing through the fixed point $P(x_1, y_1)$ is $y - y_1 = m(x - x_1)$. Taking the algebraic sum of perpendicular distances equal to zero, we get $y - 1 = m(x - 1)$. Thus (x_1, y_1) is (1, 1).]

15. In what direction should a line be drawn through the point (1, 2) so that its point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point.

16. A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point.

[**Hint**: $\frac{x}{a} + \frac{y}{b} = 1$ where $\frac{1}{a} + \frac{1}{b} =$ constant $= \frac{1}{k}$ (say). This implies that $\frac{k}{a} + \frac{k}{b} = 1 \Rightarrow$ line passes through the fixed point (k, k).]

17. Find the equation of the line which passes through the point $(-4, 3)$ and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point.

18. Find the equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point (3, 2) is $\frac{7}{5}$.

19. If the sum of the distances of a moving point in a plane from the axes is 1, then find the locus of the point. [**Hint**: Given that $|x| + |y| = 1$, which gives four sides of a square.]

20. P_1, P_2 are points on either of the two lines $y - \sqrt{3}\,|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P_1, P_2 on the bisector of the angle between the given lines.

[**Hint**: Lines are $y = \sqrt{3}x + 2$ and $y = -\sqrt{3}\ x + 2$ according as $x \geq 0$ or $x < 0$. y-axis is the bisector of the angles between the lines. P_1, P_2 are the points on these lines at a distance of 5 units from the point of intersection of these lines which have a point on y-axis as common foot of perpendiculars from these points. The y-coordinate of the foot of the perpendicular is given by $2 + 5\cos 30°$.]

21. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P, then show that $a^4 + b^4 = 0$.