

Unit 3(Trigonometric Functions)

1. Prove that $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

Sol. L.H.S. = $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$

$$= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{(\tan A - \sec A + 1)} \quad [\because \sec^2 A - \tan^2 A = 1]$$
$$= \frac{(\tan A + \sec A) - (\sec A + \tan A)(\sec A - \tan A)}{(1 - \sec A + \tan A)}$$
$$= \frac{(\sec A + \tan A)(1 - \sec A + \tan A)}{1 - \sec A + \tan A}$$
$$= \sec A + \tan A = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \frac{1 + \sin A}{\cos A} = \text{R.H.S.}$$

2. If $\frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$, then prove that $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha}$ is also equal to y .

Sol. We have, $\frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$

Now, $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} = \frac{(1 - \cos \alpha + \sin \alpha)}{(1 + \sin \alpha)} \cdot \frac{(1 + \cos \alpha + \sin \alpha)}{(1 + \cos \alpha + \sin \alpha)}$

$$= \frac{(1 + \sin \alpha)^2 - \cos^2 \alpha}{(1 + \sin \alpha)(1 + \sin \alpha + \cos \alpha)}$$
$$= \frac{1 + \sin^2 \alpha + 2 \sin \alpha - 1 + \sin^2 \alpha}{(1 + \sin \alpha)(1 + \sin \alpha + \cos \alpha)}$$

$$= \frac{2 \sin \alpha(1 + \sin \alpha)}{(1 + \sin \alpha)(1 + \sin \alpha + \cos \alpha)}$$
$$= \frac{2 \sin \alpha}{1 + \sin \alpha + \cos \alpha} = y$$

3. If $m \sin \theta = n \sin (\theta + 2\alpha)$, then prove that $\tan (\theta + \alpha) \cot \alpha = \frac{m+n}{m-n}$.

Sol. We have, $m \sin \theta = n \sin (\theta + 2\alpha)$

$$\therefore \frac{\sin(\theta + 2\alpha)}{\sin \theta} = \frac{m}{n}$$

Using componendo and dividendo, we get

$$\begin{aligned} \frac{\sin(\theta + 2\alpha) + \sin \theta}{\sin(\theta + 2\alpha) - \sin \theta} &= \frac{m+n}{m-n} \\ \Rightarrow \frac{2 \sin\left(\frac{\theta + 2\alpha + \theta}{2}\right) \cdot \cos\left(\frac{\theta + 2\alpha - \theta}{2}\right)}{2 \cos\left(\frac{\theta + 2\alpha + \theta}{2}\right) \cdot \sin\left(\frac{\theta + 2\alpha - \theta}{2}\right)} &= \frac{m+n}{m-n} \\ \Rightarrow \frac{\sin(\theta + \alpha) \cdot \cos \alpha}{\cos(\theta + \alpha) \cdot \sin \alpha} &= \frac{m+n}{m-n} \\ \Rightarrow \tan (\theta + \alpha) \cdot \cot \alpha &= \frac{m+n}{m-n} \end{aligned}$$

Q4. If $\cos (\alpha +) = 4/5$ and $\sin (\alpha -) = 5/13$, where α lie between 0 and $\pi/4$, then find the value of $\tan 2\alpha$.

Sol. We have $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$

$$\Rightarrow \tan(\alpha + \beta) = \pm \frac{3}{4}$$

$$\text{and } \tan(\alpha - \beta) = \pm \frac{5}{12}$$

Since $\alpha \in (0, \pi/2)$, $2\alpha \in (0, \pi)$, for which $\tan 2\alpha > 0$

Now, $\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{36 + 20}{48 - 15} = \frac{56}{33}$$

As other values of $\tan(\alpha + \beta)$ and $\tan(\alpha - \beta)$ gives negative value of $\tan 2\alpha$.

5. If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Sol. We have $\tan x = \frac{b}{a}$

$$\begin{aligned} \text{Now, } \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} &= \frac{(a+b) + (a-b)}{\sqrt{a^2 - b^2}} \\ &= \frac{2a}{a\sqrt{1 - \left(\frac{b}{a}\right)^2}} \\ &= \frac{2}{\sqrt{1 - \tan^2 x}} \\ &= \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}} \\ &= \frac{2 \cos x}{\sqrt{\cos 2x}} \end{aligned}$$

Q6. Prove that $\cos \theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2} = \sin \frac{7\theta}{2} \sin 4\theta$.

Sol. L.H.S. = $\cos \theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2}$

$$\begin{aligned} &= \frac{1}{2} \left[2 \cos \theta \cdot \cos \frac{\theta}{2} - 2 \cos 3\theta \cdot \cos \frac{9\theta}{2} \right] \\ &= \frac{1}{2} \left(\cos \frac{3\theta}{2} + \cos \frac{\theta}{2} - \cos \frac{15\theta}{2} - \cos \frac{3\theta}{2} \right) \\ &= \frac{1}{2} \left[\cos \frac{\theta}{2} - \cos \frac{15\theta}{2} \right] = \frac{1}{2} \left[2 \sin \left(\frac{\theta + 15\theta}{4} \right) \cdot \sin \left(\frac{15\theta - \theta}{4} \right) \right] \\ &= \sin 4\theta \cdot \sin \frac{7\theta}{2} = \text{R.H.S.} \end{aligned}$$

Q7. If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then show that $a^2 + b^2 = m^2 + n^2$

Sol: We have, $a \cos \theta + b \sin \theta = m$ (i)

and $a \sin \theta - b \cos \theta = n$ (ii)

On squaring Eqs. (i) and (ii) and then adding the resulting equations, we get

$$\begin{aligned}m^2 + n^2 &= (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 \\&= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cdot \cos \theta + a^2 \sin^2 \theta \\&\quad + b^2 \cos^2 \theta - 2ab \sin \theta \cdot \cos \theta \\&= a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta) = a^2 + b^2\end{aligned}$$

Q8. Find the value of $\tan 22^\circ 30'$

Sol. We know that, $\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \frac{\sin \theta}{1 + \cos \theta}$

$$\therefore \tan 22^\circ 30' = \frac{\sin 45^\circ}{1 + \cos 45^\circ} = \frac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2} + 1}$$

Q9. Prove that $\sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$.

Sol: L.H.S. = $\sin 4A$

$$= 2 \sin 2A \cos 2A = 2(2 \sin A \cos A)(\cos^2 A - \sin^2 A)$$

$$= 4 \sin A \cdot \cos^3 A - 4 \cos A \sin^3 A = \text{R.H.S.}$$

Q10. If $\tan + \sin = m$ and $\tan - \sin = n$, then prove that $m^2 - n^2 = 4 \sin \tan$

Sol: We have, $\tan + \sin = m$ (i)

And $\tan - \sin = n$ (ii)

$$\text{Now, } m + n = 2 \tan$$

$$\text{And } m - n = 2 \sin.$$

$$(m + n)(m - n) = 4 \sin \tan$$

$$\tan m^2 - n^2 = 4 \sin \tan$$

Q11. If $\tan(A + B) = p$ and $\tan(A - B) = q$, then show that $\tan 2A = \frac{p+q}{1-pq}$

Sol: We have $\tan(A + B) = p$ and $\tan(A - B) = q$

$$\tan 2A = \tan [(A + B) + (A - B)]$$

$$= \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B)\tan(A - B)} = \frac{p + q}{1 - pq}$$

Q12. If $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$, then prove that $\cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta)$.

Sol. We have, $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$

$$\Rightarrow (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta - \sin^2 \alpha - \sin^2 \beta - 2 \sin \alpha \sin \beta = 0$$

$$\Rightarrow (\cos^2 \alpha - \sin^2 \alpha) + (\cos^2 \beta - \sin^2 \beta) = -2(\cos \alpha \cos \beta - \sin \alpha \sin \beta)$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta)$$

13. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then show that $\frac{\tan x}{\tan y} = \frac{a}{b}$.

Sol. Given that, $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

Using componendo and dividendo, we get

$$\Rightarrow \frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

$$\Rightarrow \frac{2 \sin\left(\frac{x+y+x-y}{2}\right) \cdot \cos\left(\frac{x+y-x+y}{2}\right)}{2 \cos\left(\frac{x+y+x-y}{2}\right) \cdot \sin\left(\frac{x+y-x+y}{2}\right)} = \frac{2a}{2b}$$

$$\Rightarrow \frac{\sin x \cdot \cos y}{\cos x \cdot \sin y} = \frac{a}{b} \Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$

14. If $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$, then show that $\sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$.

Sol. We have, $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$

$$\Rightarrow 1 + \tan^2 \theta = 1 + \frac{(\sin \alpha - \cos \alpha)^2}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow \sec^2 \theta = \frac{(\sin^2 \alpha + \cos^2 \alpha + 2 \sin \alpha \cos \alpha) + \sin^2 \alpha + \cos^2 \alpha - 2 \sin \alpha \cos \alpha}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow \sec^2 \theta = \frac{2}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow (\sin \alpha + \cos \alpha)^2 = 2 \cos^2 \theta \Rightarrow \sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$$

Q15. If $\sin \theta + \cos \theta = 1$, then find the general value of θ

Sol. $\sin \theta + \cos \theta = 1$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \sin \frac{\pi}{4} \Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}, n \in \mathbb{Z}$$

$$\therefore \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}, n \in \mathbb{Z}$$

Q16. Find the most general value of θ satisfying the equation $\tan \theta = -1$ and $\cos \theta = 1/\sqrt{2}$.

Sol: We have $\tan \theta = -1$ and $\cos \theta = 1/\sqrt{2}$.

So, θ lies in IV quadrant.

$$\theta = 7/4$$

So, general solution is $\theta = 7\pi/4 + 2n\pi, n \in \mathbb{Z}$

Q17. If $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$, then find the general value of θ

Sol: Given that, $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \frac{2}{\sin \theta}$$

$$\Rightarrow \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cdot \cos \theta} = \frac{2}{\sin \theta} \Rightarrow \frac{1}{\cos \theta} = 2$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \cos \theta = \cos \frac{\pi}{3}$$

$$\therefore \theta = 2n\pi \pm \frac{\pi}{3}, n \in Z$$

Q18. If $2 \sin^2 \theta = 3 \cos \theta$, where $0 \leq \theta \leq 2$, then find the value of θ

Sol. We have, $2 \sin^2 \theta = 3 \cos \theta$

$$\Rightarrow 2 - 2 \cos^2 \theta = 3 \cos \theta$$

$$\Rightarrow 2 \cos^2 \theta + 3 \cos \theta - 2 = 0$$

$$\Rightarrow (\cos \theta + 2)(2 \cos \theta - 1) = 0$$

$$\therefore \cos \theta = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3} \text{ or } 2\pi - \frac{\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

Q19. If $\sec x \cos 5x + 1 = 0$, where $0 < x < \pi/2$, then find the value of x .

Sol. We have, $\sec x \cos 5x + 1 = 0$

$$\Rightarrow \frac{\cos 5x}{\cos x} + 1 = 0$$

$$\Rightarrow \cos 5x + \cos x = 0 \Rightarrow 2 \cos 3x \cos 2x = 0$$

$$\Rightarrow \cos 3x = 0 \text{ or } \cos 2x = 0$$

$$\text{If } \cos 3x = 0, \text{ then } 3x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\text{If } \cos 2x = 0, \text{ then } 2x = \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{4}$$

Long Answer Type Questions

Q20. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$

Sol: We have $\sin(\theta + \alpha) = a$ ---(i)

$\sin(\theta + \beta) = b$ ---(ii)

$$\begin{aligned} \therefore \cos(\theta + \alpha) &= \sqrt{1 - a^2} \text{ and } \cos(\theta + \beta) = \sqrt{1 - b^2} \\ \therefore \cos(\alpha - \beta) &= \cos[(\theta + \alpha) - (\theta + \beta)] \\ &= \cos(\theta + \beta) \cos(\theta + \alpha) + \sin(\theta + \alpha) \sin(\theta + \beta) \\ &= \sqrt{1 - a^2} \sqrt{1 - b^2} + ab = ab + \sqrt{1 - a^2 - b^2 + a^2 b^2} \\ \Rightarrow \cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) \\ &= 2 \cos^2(\alpha - \beta) - 1 - 4ab \cos(\alpha - \beta) \\ &= 2 \cos(\alpha - \beta) [\cos(\alpha - \beta) - 2ab] - 1 \\ &= 2(ab + \sqrt{1 - a^2 - b^2 + a^2 b^2}) (ab + \sqrt{1 - a^2 - b^2 + a^2 b^2} - 2ab) - 1 \\ &= 2[(\sqrt{1 - a^2 - b^2 + a^2 b^2} + ab) (\sqrt{1 - a^2 - b^2 + a^2 b^2} - ab)] - 1 \\ &= 2[1 - a^2 - b^2 + a^2 b^2 - a^2 b^2] - 1 = 2 - 2a^2 - 2b^2 - 1 = 1 - 2a^2 - 2b^2 \end{aligned}$$

21. If $\cos(\theta + \phi) = m \cos(\theta - \phi)$, then prove that $\tan \theta = \frac{1 - m}{1 + m} \cot \phi$.

Sol. Given that, $\cos(\theta + \phi) = m \cos(\theta - \phi)$

$$\Rightarrow \frac{\cos(\theta + \phi)}{\cos(\theta - \phi)} = \frac{m}{1}$$

Using componendo and dividendo rule, we get

$$\begin{aligned} \frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{\cos(\theta - \phi) + \cos(\theta + \phi)} &= \frac{1 - m}{1 + m} \\ \Rightarrow \frac{2 \sin\left(\frac{\theta + \phi - \theta + \phi}{2}\right) \cdot \sin\left(\frac{\theta - \phi + \theta + \phi}{2}\right)}{2 \cos\left(\frac{\theta - \phi + \theta + \phi}{2}\right) \cdot \cos\left(\frac{\theta - \phi - \theta - \phi}{2}\right)} &= \frac{1 - m}{1 + m} \\ \Rightarrow \frac{\sin \theta \cdot \sin \phi}{\cos \theta \cdot \cos \phi} &= \frac{1 - m}{1 + m} \\ \Rightarrow \tan \theta &= \left(\frac{1 - m}{1 + m}\right) \cot \phi \end{aligned}$$

Q22. Find the value of the expression

$$3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4(3\pi + \alpha) \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6(5\pi - \alpha) \right].$$

$$\begin{aligned} \text{Sol. } & 3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4(3\pi + \alpha) \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6(5\pi - \alpha) \right] \\ &= 3[\cos^4 \alpha + \sin^4 \alpha] - 2[\cos^6 \alpha + \sin^6 \alpha] \\ &= 3[(\cos^2 \alpha + \sin^2 \alpha)^2 - 2 \cos^2 \alpha \cdot \sin^2 \alpha] - 2[(\cos^2 \alpha + \sin^2 \alpha)^3 \\ &\quad - 3 \cos^2 \alpha \cdot \sin^2 \alpha (\cos^2 \alpha + \sin^2 \alpha)] \\ &= 3[1 - 2 \cos^2 \alpha \cdot \sin^2 \alpha] - 2[1 - 3 \cos^2 \alpha \cdot \sin^2 \alpha] = 3 - 2 = 1 \end{aligned}$$

Q23. If $a \cos 2\theta + b \sin 2\theta = c$ has α and β as its roots, then prove that $\tan \alpha + \tan \beta = 2b/a+c$

Sol. We have, $a \cos 2\theta + b \sin 2\theta = c$ (i)

$$\Rightarrow a \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + b \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = c$$

$$\Rightarrow a(1 - \tan^2 \theta) + 2b \tan \theta = c(1 + \tan^2 \theta)$$

$$\Rightarrow (c + a) \tan^2 \theta - 2b \tan \theta + c - a = 0 \quad \text{(ii)}$$

Equation (i) has roots α and β .

Thus, equation (ii) has roots $\tan \alpha$ and $\tan \beta$.

$$\therefore \text{Sum of roots, } \tan \alpha + \tan \beta = \frac{-(-2b)}{a+c} = \frac{2b}{a+c}$$

Q24. If $x = \sec \phi - \tan \phi$ and $y = \operatorname{cosec} \phi + \cot \phi$ then show that $xy + x - y + 1 = 0$.

$$\text{Sol. } x = \sec \phi - \tan \phi \Rightarrow x = \frac{1 - \sin \phi}{\cos \phi}$$

$$y = \operatorname{cosec} \phi + \cot \phi \Rightarrow y = \frac{1 + \cos \phi}{\sin \phi}$$

$$\begin{aligned} \Rightarrow xy + x - y &= \frac{1 - \sin \phi}{\cos \phi} \frac{1 + \cos \phi}{\sin \phi} + \frac{1 - \sin \phi}{\cos \phi} - \frac{1 + \cos \phi}{\sin \phi} \\ &= \frac{(1 - \sin \phi)(1 + \cos \phi) + (1 - \sin \phi)\sin \phi - \cos \phi(1 + \cos \phi)}{\sin \phi \cos \phi} \\ &= \frac{1 - \sin \phi + \cos \phi - \sin \phi \cos \phi + \sin \phi - \sin^2 \phi - \cos \phi - \cos^2 \phi}{\sin \phi \cos \phi} \\ &= \frac{1 - \sin \phi \cos \phi - (\sin^2 \phi + \cos^2 \phi)}{\sin \phi \cos \phi} = -1 \end{aligned}$$

$$\therefore xy + x - y - 1 = 0$$

Q25. If θ lies in the first quadrant and $\cos \theta = 8/17$, then find the value of $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$.

Sol. Given that, $\cos \theta = \frac{8}{17}$

$$\Rightarrow \sin \theta = \frac{15}{17} \quad \text{[Since } \theta \text{ lies in the first quadrant]}$$

$$\text{Now, } \cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$$

$$= \cos 30^\circ \cos \theta - \sin 30^\circ \sin \theta + \cos 45^\circ \cos \theta + \sin 45^\circ \sin \theta + \cos 120^\circ \cos \theta + \sin 120^\circ \sin \theta$$

$$= \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta - \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2} \right) \cos \theta + \left(\frac{1}{\sqrt{2}} - \frac{1}{2} + \frac{\sqrt{3}}{2} \right) \sin \theta$$

$$= \left(\frac{\sqrt{3} + \sqrt{2} - 1}{2} \right) \frac{8}{17} + \left(\frac{\sqrt{2} - 1 + \sqrt{3}}{2} \right) \frac{15}{17}$$

$$= \left(\frac{8\sqrt{3} + 8\sqrt{2} - 8 + 15\sqrt{2} - 15 + 15\sqrt{3}}{34} \right) = \frac{23}{34} (\sqrt{3} + \sqrt{2} - 1)$$

Q26. Find the value of the expression $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

$$\begin{aligned}
\text{Sol. } & \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8} \\
&= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \left(\pi - \frac{3\pi}{8} \right) + \cos^4 \left(\pi - \frac{\pi}{8} \right) \\
&= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{\pi}{8} \\
&= 2 \left[\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} \right] \\
&= 2 \left[\cos^4 \frac{\pi}{8} + \cos^4 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \right] \\
&= 2 \left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} \right] \\
&= 2 \left[\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)^2 - 2 \cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8} \right] \\
&= 2 \left[1 - 2 \cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8} \right] \\
&= 2 - \left(2 \sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} \right)^2 \\
&= 2 - \left(\sin \frac{2\pi}{8} \right)^2 = 2 - \left(\frac{1}{\sqrt{2}} \right)^2 = 2 - \frac{1}{2} = \frac{3}{2}
\end{aligned}$$

Q27. Find the general solution of the equation $5 \cos^2 + 7 \sin^2 - 6 = 0$.

$$\begin{aligned}
\text{Sol. We have } & 5 \cos^2 \theta + 7 \sin^2 \theta - 6 = 0 \\
\Rightarrow & 5 \cos^2 \theta + 7(1 - \cos^2 \theta) - 6 = 0 \Rightarrow 5 \cos^2 \theta + 7 - 7 \cos^2 \theta - 6 = 0 \\
\Rightarrow & \cos^2 \theta = 1/2 \Rightarrow \cos^2 \theta = \cos^2 \frac{\pi}{4} \\
\therefore & \theta = n\pi \pm \frac{\pi}{4}, n \in \mathbb{Z}
\end{aligned}$$

Q28. Find the general solution of the equation $\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$.

$$\begin{aligned}
\text{Sol: We have, } & (\sin x + \sin 3x) - 3 \sin 2x = (\cos x + \cos 3x) - 3 \cos 2x \\
\Rightarrow & 2 \sin 2x \cos x - 3 \sin 2x = 2 \cos 2x \cos x - 3 \cos 2x \\
\Rightarrow & \sin 2x(2 \cos x - 3) = \cos 2x(2 \cos x - 3) \\
\Rightarrow & \sin 2x = \cos 2x \text{ (As } \cos x \neq 3/2) \\
\Rightarrow & \tan 2x = 1 \Rightarrow \tan 2x = \tan \pi/4 \\
\Rightarrow & 2x = n\pi + \pi/4, n \in \mathbb{Z} \\
x = & n\pi/2 + \pi/8, n \in \mathbb{Z}
\end{aligned}$$

Q29. Find the general solution of the equation $(\sqrt{3}-1)\cos + (\sqrt{3}+1)\sin = 2$.

Sol. $(\sqrt{3}-1)\cos\theta + (\sqrt{3}+1)\sin\theta = 2$ (i)

Put $\sqrt{3}-1 = r \sin\alpha$ and $\sqrt{3}+1 = r \cos\alpha$

$\therefore r^2 = (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2 \Rightarrow r^2 = 8 \Rightarrow r = 2\sqrt{2}$

Also, $\tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} \Rightarrow \alpha = \frac{\pi}{12}$

From eq. (i), we have

$$r \sin\alpha \cos\theta + r \cos\alpha \sin\theta = 2 \Rightarrow r \sin(\theta + \alpha) = 2$$

$$\Rightarrow \sin(\theta + \alpha) = \frac{1}{\sqrt{2}} \Rightarrow \sin(\theta + \alpha) = \sin\frac{\pi}{4}$$

$$\Rightarrow \theta + \alpha = n\pi + (-1)^n \frac{\pi}{4}, n \in Z$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}, n \in Z$$

Objective Type Questions

Q30. If $\sin + \operatorname{cosec} = 2$, then $\sin^2 + \operatorname{cosec}^2$ is equal to

- (a) 1
- (b) 4
- (c) 2
- (d) None of these

Sol. (c)

$$\sin\theta + \operatorname{cosec}\theta = 2$$

$$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$$

$$\Rightarrow \sin^2\theta + 1 = 2\sin\theta \Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$$

$$\therefore \sin^2\theta + \operatorname{cosec}^2\theta = 1 + 1 = 2$$

Q31. If $f(x) = \cos^2 x + \sec^2 x$, then

- (a) $f(x) < 1$
- (b) $f(x) = 1$
- (c) $2 < f(x) < 1$
- (d) $f(x) \geq 2$

Q32. If $\tan\theta = 1/2$ and $\tan\phi = 1/3$, then the value of $\theta + \phi$ is

- (a) $\frac{\pi}{6}$
- (b) π
- (c) 0
- (d) $\frac{\pi}{4}$

Sol. (d) We have, $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{5}{6}}{1 - \frac{1}{6}} = 1$$

$$\therefore \theta + \phi = \frac{\pi}{4}$$

Q33. Which of the following is not correct?

- (a) $\sin\theta = -1/5$
- (b) $\cos\theta = 1$
- (c) $\sec\theta = -1/2$
- (d) $\tan\theta = 20$

Sol: (c)

We know that, the range of $\sec \theta$ is $\mathbb{R} - (-1, 1)$.

Hence, $\sec \theta$ cannot be equal to $-1/2$

Q34. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$ is

- (a) 0
- (b) 1
- (c) $1/2$
- (d) Not defined

Sol: (b)

$\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$

$= [\tan 1^\circ \tan 2^\circ \dots \tan 44^\circ] \tan 45^\circ [\tan (90^\circ - 44^\circ) \tan (90^\circ - 43^\circ) \dots \tan (90^\circ - 1^\circ)]$

$= [\tan 1^\circ \tan 2^\circ \dots \tan 44^\circ] [\cot 44^\circ \cot 43^\circ \dots \cot 1^\circ]$

$= 1 \cdot 1 \dots 1 \cdot 1 = 1$

35. The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ is

- (a) 1
- (b) $\sqrt{3}$
- (c) $\frac{\sqrt{3}}{2}$
- (d) 2

Sol. (c)

$$\text{We know that } \cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\therefore \frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Q36. The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$ is

- (a) $1/\sqrt{2}$
- (b) 0
- (c) 1
- (d) -1

Sol: (b)

Since $\cos 90^\circ = 0$, we have

$\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 90^\circ \dots \cos 179^\circ = 0$

Q37. If $\tan \theta = 3$ and θ lies in the third quadrant, then the value of $\sin \theta$ is

- (a) $\frac{1}{\sqrt{10}}$
- (b) $-\frac{1}{\sqrt{10}}$
- (c) $\frac{-3}{\sqrt{10}}$
- (d) $\frac{3}{\sqrt{10}}$

Sol. (c)

$$\tan \theta = 3 \Rightarrow \cot \theta = \frac{1}{3}$$

$$\text{Now, } \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta = 1 + \frac{1}{9} = \frac{10}{9}$$

$$\Rightarrow \sin^2 \theta = \frac{9}{10} \Rightarrow \sin \theta = -\frac{3}{\sqrt{10}} \text{ (as } \theta \text{ lies in third quadrant)}$$

Q38. The value of $\tan 75^\circ - \cot 75^\circ$ is equal to

- (a) $2\sqrt{3}$ (b) $2 + \sqrt{3}$ (c) $2 - \sqrt{3}$ (d) 1

Sol. (a)

$$\begin{aligned}\tan 75^\circ - \cot 75^\circ &= \frac{\sin 75^\circ}{\cos 75^\circ} - \frac{\cos 75^\circ}{\sin 75^\circ} = \frac{2(\sin^2 75^\circ - \cos^2 75^\circ)}{2 \sin 75^\circ \cos 75^\circ} = \frac{-2 \cos 150^\circ}{\sin 150^\circ} \\ &= -2 \cot 150^\circ = -2 \cot (180^\circ - 30^\circ) = 2 \cot 30^\circ = 2\sqrt{3}\end{aligned}$$

Q39. Which of the following is correct?

- (a) $\sin 1^\circ > \sin 1$
 (b) $\sin 1^\circ < \sin 1$
 (c) $\sin 1^\circ = \sin 1$
 (d) $\sin 1^\circ = \pi/18^\circ \sin 1$

Sol: We know that, in first quadrant if θ is increasing, then $\sin \theta$ is also increasing.
 $\therefore \sin 1^\circ < \sin 1$ [$\because 1 \text{ radian} = 57^\circ 30'$]

40. If $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

Sol. (d) Given that, $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$

$$\begin{aligned}\text{Now, } \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)} \\ &= \frac{m(2m+1) + m+1}{(m+1)(2m+1) - m} = \frac{2m^2 + 2m + 1}{2m^2 + 3m + 1 - m} = 1\end{aligned}$$

$$\therefore \alpha + \beta = \frac{\pi}{4}$$

Q41. The minimum value of $3 \cos x + 4 \sin x + 8$ is

- (a) 5
 (b) 9
 (c) 7
 (d) 3

Sol: (d)

$$\begin{aligned}3 \cos x + 4 \sin x + 8 &= 5 \left(\frac{3}{5} \cos x + \frac{4}{5} \sin x \right) + 8 \\ &= 5(\sin \alpha \cos x + \cos \alpha \sin x) + 8 \\ &= 5 \sin(\alpha + x) + 8, \text{ where } \tan \alpha = 3/4\end{aligned}$$

Q42. The value of $\tan 3A - \tan 2A - \tan A$ is

- (a) $\tan 3A \cdot \tan 2A \cdot \tan A$
 (b) $-\tan 3A \cdot \tan 2A \cdot \tan A$
 (c) $\tan A \cdot \tan 2A - \tan 2A \cdot \tan 3A - \tan 3A \cdot \tan A$
 (d) None of these

Sol: (a)

$$3A = A + 2A$$

$$\Rightarrow \tan 3A = \tan (A + 2A)$$

$$\begin{aligned} \Rightarrow \tan 3A &= \tan A + \tan 2A / 1 - \tan A \cdot \tan 2A \\ \Rightarrow \tan A + \tan 2A &= \tan 3A - \tan 3A \cdot \tan 2A \cdot \tan A \\ \Rightarrow \tan 3A - \tan 2A - \tan A &= \tan 3A \cdot \tan 2A \cdot \tan A \end{aligned}$$

Q43. The value of $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$ is

- (a) $2 \cos$
- (b) $2 \sin$
- (c) 1
- (d) 0

Sol: (d)

$$\begin{aligned} \sin(45^\circ + \theta) - \cos(45^\circ - \theta) &= \sin(45^\circ + \theta) - \sin(90^\circ - (45^\circ - \theta)) \\ &= \sin(45^\circ + \theta) - \sin(45^\circ + \theta) = 0 \end{aligned}$$

Q44. The value of $(\pi/4 + \theta) \cot(\pi/4 - \theta)$ is

- (a) -1
- (b) 0
- (c) 1
- (d) Not defined

Sol. (c)

$$\begin{aligned} \cot\left(\frac{\pi}{4} + \theta\right) \cot\left(\frac{\pi}{4} - \theta\right) &= \cot\left(\frac{\pi}{4} + \theta\right) \cot\left(\frac{\pi}{2} - \left(\frac{\pi}{4} - \theta\right)\right) \\ &= \cot\left(\frac{\pi}{4} + \theta\right) \tan\left(\frac{\pi}{4} + \theta\right) = 1 \end{aligned}$$

45. $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$ is equal to

- (a) $\sin 2(\theta + \phi)$
- (b) $\cos 2(\theta + \phi)$
- (c) $\sin 2(\theta - \phi)$
- (d) $\cos 2(\theta - \phi)$

Sol. (b)

$$\begin{aligned} \cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi) &= \cos 2\theta \cos 2\phi + \sin(\theta - \phi + \theta + \phi) \sin(\theta - \phi - \theta - \phi) \\ &= \cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi = \cos(2\theta + 2\phi) = \cos 2(\theta + \phi) \end{aligned}$$

Q46. The value of $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ$ is

- (a) $1/2$
- (b) 1
- (c) $-1/2$
- (d) $1/8$

Sol. (c)

$$\begin{aligned} \cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ &= (\cos 12^\circ + \cos 132^\circ) + (\cos 84^\circ + \cos 156^\circ) \\ &= 2 \cos 72^\circ \cos 60^\circ + 2 \cos 120^\circ \cos 36^\circ \\ &= \cos 72^\circ - \cos 36^\circ = \sin 18^\circ - \cos 36^\circ \\ &= \left(\frac{\sqrt{5}-1}{4}\right) - \left(\frac{\sqrt{5}+1}{4}\right) = \frac{-1}{2} \end{aligned}$$

Q47. If $\tan A = 1/2$ and $\tan B = 1/3$ then $\tan(2A + B)$ is equal to

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Sol. (c)

$$\text{We have, } \tan A = \frac{1}{2} \text{ and } \tan B = \frac{1}{3}$$

$$\text{Now, } \tan(2A + B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \cdot \tan B}$$

$$\text{Also, } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$$

$$\therefore \tan(2A + B) = \frac{\frac{4}{3} + \frac{1}{3}}{1 - \frac{4}{3} \cdot \frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3$$

48. The value of $\sin \frac{\pi}{10} \sin \frac{13\pi}{10}$ is

- (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{4}$ (d) 1

Sol. (c)

$$\begin{aligned} \sin \frac{\pi}{10} \sin \frac{13\pi}{10} &= \sin \frac{\pi}{10} \sin \left(\pi + \frac{3\pi}{10} \right) = -\sin \frac{\pi}{10} \sin \frac{3\pi}{10} \\ &= -\sin 18^\circ \sin 54^\circ = -\sin 18^\circ \cos 36^\circ \\ &= -\left(\frac{\sqrt{5}-1}{4} \right) \left(\frac{\sqrt{5}+1}{4} \right) = \frac{1-5}{16} = -\frac{1}{4} \end{aligned}$$

Q49. The value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is equal to

- (a) 1
(b) 0
(c) 1
(d) 2

Sol. (b)

$$\begin{aligned} &\sin 50^\circ - \sin 70^\circ + \sin 10^\circ \\ &= 2 \cos \left(\frac{50^\circ + 70^\circ}{2} \right) \sin \left(\frac{50^\circ - 70^\circ}{2} \right) + \sin 10^\circ \\ &= -2 \cos 60^\circ \sin 10^\circ + \sin 10^\circ = -2 \cdot \frac{1}{2} \sin 10^\circ + \sin 10^\circ = 0 \end{aligned}$$

Q50. If $\sin + \cos = 1$, then the value of $\sin 2$ is

- (a) 1
(b) 1
(c) 0
(d) -1