Chapter 3. Trigonometric Functions

Question-1

Find the radian measure corresponding to the following degree measures:

a) 15^o
b) $-37^0\ 30'$
c) 240^0
d) 530^0

Solution:

a) $1^o = \frac{\pi}{180}$ radian

$15^o = \frac{\pi}{180} \times 15^o = \frac{\pi}{12}$ radian

b) $1^o = \frac{\pi}{180}$ radian

$-37^o30' = \frac{\pi}{180} \times -37.5^o = \frac{-5\pi}{24}$ radian

c) $1^o = \frac{\pi}{180}$ radian

$240^o = \frac{\pi}{180} \times 240 = \frac{4\pi}{3}$ radian

d) $1^o = \frac{\pi}{180}$ radian

$530^o = \frac{\pi}{180} \times 530 = \frac{53}{18}\pi$ radian

Question-2

Find the degree measure corresponding to the following radian measures:

a) $\frac{3}{4}$
b) -4
c) $\frac{5\pi}{3}$
d) $\frac{7\pi}{6}$

Solution:

a) 1 radian $= \frac{180^o}{\pi}$

$\frac{3}{4}$ radian $= \frac{180^o}{\pi} \times \frac{3}{4} = \frac{135^o}{\pi} = \frac{135^o}{22} \times 7 = 42.95 = 42^o57'$

b) 1 radian $= \frac{180^o}{\pi}$

-4 radian $= \frac{180^o}{\pi} \times -4 = \frac{-720}{22} \times 7 = -32.72 \times 7 = -229^o5'24''$

c) 1 radian $= \frac{180^o}{\pi}$

$\frac{5\pi}{3}$ radian $= \frac{180^o}{\pi} \times \frac{5\pi}{3} = 300^0$

d) 1 radian = $\frac{180^o}{\pi}$

$\frac{7\pi}{6}$ radian = $\frac{180}{\pi} \times \frac{7\pi}{6} = 210^o$

Question-3

A wheel makes 360 revolutions in one minute. Through how many radians does it turn is one second?

Solution:

One complete revolution = 2π360 revolutions = 360 × 2π= 720π radians / minute = 720π /60 radians/sec = 12π radians/sec.

Question-4

Find the degree measure of the angle subtended at the centre of a circle of diameter 200cm by an arc of length 22cm.

Solution:

Radius (r) = (200/2)cm = 100cm

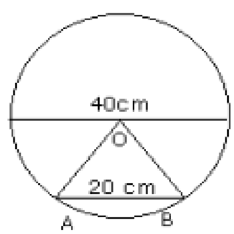
Length of an arc (l) = 22cm

θ = l/r = 22/100radians = $\frac{22}{100} \times \frac{180}{\pi} = 12^o 36'$

Question-5

In a circle of diameter 40cm. The length of a chord is 20cm. Find the length of minor arc corresponding to the chord.

Solution:



Question-6

If, in two circles, arcs of the same length subtend angles of 60^o and 75^o at the centre, find the ratio of their radii.

Solution:

Let the radii be r_1 and r_2. Let the angles subtend by the arcs in two circles be θ_1 and θ_2.

$l = \theta r$ where θ is the angle, l the length of an arc and r the radius of the circle.

$\theta_1 = 60^o = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$ radian

$\theta_2 = 75^o = 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$ radian

$\frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2$

$r_1 / r_2 = \frac{\frac{5}{12}}{\frac{1}{3}} = \frac{5}{4}$

Therefore the required ratio is 5:4.

Question-7

Find the angle in radian through which a pendulum swings if its length is 75cm and the tip described an arcs of length

(i) 10cm

(ii) 15cm

(iii) 21cm

Solution:

(i) Length of the pendulum (r) = 75cm

Length of an arc (l) = 10cm

$\theta = l/r$ = 10/75radians = 2/15radians

(ii) Length of the pendulum (r) = 75cm

Length of an arc (l) = 15cm

$\theta = l/r$ = 15/75 radians = 1/5radians

(iii) Length of the pendulum (r) = 75cm

Length of an arc (l) = 21cm

$\theta = l/r$ = 21/75 radians = 7/25radians

Question-8

Find the values of the other five trigonometric functions in the following problem: cosθ = -1/2, is quadrant III

Solution:

cosθ and secθ are negative; tan θ and cot θ are positive.

$\cos\theta = -\frac{1}{2}$

$\sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore \sin\theta = -\frac{\sqrt{3}}{2}$

$\therefore \tan\theta = \frac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}} = \sqrt{3}$

$\therefore \text{cosec}\theta = -\frac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$

$\sec\theta = \frac{1}{\frac{-1}{2}} = -2$

$\cot\theta = \frac{\frac{-1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Question-9

Find the values of the other five trigonometric functions in the following problem: sin $\theta - \frac{3}{5}, \theta$ is quadrant II

Solution:

sinθ and cosecθ positive; tanθ , cosθ , cotθ and secθ all are negative.

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{3}{5}\right)^2 = \frac{25-9}{25} = \frac{16}{25}$

$\cos\theta = -\frac{4}{5}$

$\therefore \tan\theta = \frac{\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}$

$\cot\theta = \frac{\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{3}$

$\text{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-4/5} = -5/4$

Question-10

Find the values of the other five trigonometric functions in the following problem: tanθ = $\frac{3}{4}$, θ is quadrant III

Solution:

sinθ , cosθ , cosecθ and secθ are negative in III rd quadrant tanθ and cotθ are positive in IIIrd quadrant.

$1+\tan^2\theta = \sec^2\theta 1+\frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1+\left(\frac{3}{4}\right)^2 = \sec^2\theta$

$\frac{16+9}{16} = \frac{25}{16} = \sec^2\theta$

$\sec\theta = \frac{-5}{4}$

$\cos\theta = \frac{-4}{5}$

$\sin^2\theta = 1-\left(\frac{4}{5}\right)^2 = \frac{9}{25}$

$\therefore \sin\theta = \frac{-3}{5}$

$\text{cosec}\theta = \frac{-5}{3}$

$\cot\theta = \frac{4}{3}$

Question-11

Find the values of the other five trigonometric functions in the following problem: sec θ = 13/5, θ lies in fourth quadrant.

Solution:

sin θ , tan θ , cosec θ and cot θ are negative in IV th quadrant sec θ and cos θ are positive in IVth quadrant.

sec θ = 13/5

cos θ = 1/sec θ = 5/13

$\sin^2\theta = 1-\left(\frac{5}{13}\right)^2 - \frac{144}{169}$

$\therefore \sin\theta = \frac{-12}{13}$

$\text{cosec}\,\theta = \frac{-13}{12}$

$\therefore \tan\theta = \frac{\frac{-12}{13}}{\frac{5}{13}} = -\frac{12}{5}$

$\cot\theta = \frac{\frac{5}{13}}{-\frac{12}{13}} = -\frac{5}{12}$

Question-12

Find the value of the following trigonometric function: sin 765°

Solution:

$\sin 765^{o} = \sin \frac{17\pi}{4} = \sin\left(4\pi + \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Question-13

Find the value of the following trigonometric function: cosec(-1410°)

Solution:

$\text{cosec}(-1410^{o}) = -\text{cosec}\ 1410^{o} = -\text{cosec}\ (8\pi - 30^{o}) = \text{cosec}\ 30^{o} = 2$

Question-14

Find the value of the following trigonometric function: tan $\frac{13\pi}{3}$

Solution:

$\tan \frac{13\pi}{3} = \tan\left(4\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3}$

Question-15

Find the value of the following trigonometric function: cot$\left(-\frac{15\pi}{4}\right)$

Solution:

$\cot\left(-\frac{15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right) = -\cot\left(4\pi - \frac{\pi}{4}\right) = \cot \frac{\pi}{4} = 1$

Question-16

Prove that: $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$

Solution:

$\text{L.H.S} = \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$

$= \frac{1}{4} + \frac{1}{4} - 1$

$= \frac{1}{2} - 1$

$= -\frac{1}{2} = \text{R.H.S}$

Question-17

Prove that: $2\sin^2\frac{\pi}{6} + \text{cosec}\frac{7\pi}{6}\cos^2\frac{\pi}{3} = 0$

Solution:

L.H.S = $2\sin^2\frac{\pi}{6} + \text{cosec}\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\times\frac{1}{4} + \text{cosec}\left(\pi + \frac{\pi}{6}\right)\times\frac{1}{4}$

$= \frac{1}{2} - \text{cosec}\frac{\pi}{6}\times\frac{1}{4}$

$= \frac{1}{2} - 2\times\frac{1}{4}$

$= 0$ = R.H.S

Question-18

Prove that: $3\cos^2\frac{\pi}{4} + \sec\frac{2\pi}{3} + 5\tan^2\frac{\pi}{3} = \frac{29}{2}$

Solution:

L.H.S = $3\cos^2\frac{\pi}{4} + \sec\left(\pi - \frac{\pi}{3}\right) + 5\tan^2\frac{\pi}{3}$

$= 3\times\frac{1}{2} - \sec\frac{\pi}{3} + 5\tan^2\frac{\pi}{3}$

$= \frac{3}{2} - 2 + 5\times(\sqrt{3})^2$

$= \frac{3}{2} - 2 + 15$

$= 13 + \frac{3}{2}$

$= \frac{29}{2}$

= R.H.S

Question-19

Prove that: $\cot^2\frac{\pi}{6} + \text{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Solution:

L.H.S = $\cot^2\frac{\pi}{6} + \text{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= \cot^2\frac{\pi}{6} + \text{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\tan^2\frac{\pi}{6}$

$= \cot^2\frac{\pi}{6} + \text{cosec}\frac{\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= (\sqrt{3})^2 + 2 + 3\left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + 2 + 1$

$= 6$

= R.H.S

Question-20

Prove that: $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Solution:

L.H.S = $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\sin^2\left(\pi - \frac{\pi}{4}\right) + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\sin^2\frac{\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\times\frac{1}{2} + 2\times\frac{1}{2} + 2\times 4$

$= 1 + 1 + 8$

$= 10$

= R.H.S

Question-21

Show that:

$\cos 70^o \cos 10^o + \sin 70^o \sin 10^o = ½$

Solution:

We know that

$\cos(\theta - \varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$ ∴ L.H.S = $\cos 70^o \cos 10^o + \sin 70^o \sin 10^o$

$= \cos(70^o - 10^o)$

$= \cos 60^o$

$= 1/2$

Question-22

Show that:

$\cos 130^o \cos 40^o + \sin 130^o \sin 40^o = 0$

Solution:

We know that

$\cos(\theta - \varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$ ∴ L.H.S = $\cos 130^o \cos 40^o + \sin 130^o \sin 40^o$

$= \cos(130^o - 40^o)$

$= \cos 90^o$

$= 0$

Question-23

Show that:

$$\sin(40^\circ + \theta)\cos(10^\circ + \theta) - \cos(40^\circ + \theta)\sin(10^\circ + \theta) = \tfrac{1}{2}$$

Solution:

We know that

$\sin(\theta - \varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$ L.H.S $= \sin(40^\circ + \theta)\cos(10^\circ + \theta) - \cos(40^\circ + \theta)\sin(10^\circ + \theta)$

$= \sin 30^\circ$

$= 1/2$

$=$ R.H.S

Question-24

Prove that:

$$\cos\left(\frac{\pi}{4} - \theta\right)\cos\left(\frac{\pi}{4} - \phi\right) - \sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} - \phi\right) = \sin(\theta + \varphi)$$

Solution:

We know that

$\cos(\theta + \varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi$ L.H.S $= \cos\left(\frac{\pi}{4} - \theta\right)\cos\left(\frac{\pi}{4} - \phi\right) - \sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} - \phi\right)$

$= \cos\left(\frac{\pi}{4} - \theta + \frac{\pi}{4} - \phi\right)$

$= \cos\left(\frac{\pi}{2} - (\theta + \phi)\right)$

$= \sin(\theta + \varphi)$

$=$ R.H.S

Question-25

Prove that:

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

Solution:

$$\text{L.H.S} = \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)}$$

$$= \frac{\dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}}$$

$$= \frac{\dfrac{1 + \tan x}{1 - \tan x}}{\dfrac{1 - \tan x}{1 + \tan x}}$$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

$=$ R.H.S

Question-26

Prove that :

$$\frac{\cos(\pi+\theta)\cos(-\theta)}{\sin(\pi-\theta)\cos\left(\frac{\pi}{2}+\theta\right)} = \cot^2\theta$$

Solution:

L.H.S = $\frac{\cos(\pi+\theta)\cos(-\theta)}{\sin(\pi-\theta)\cos\left(\frac{\pi}{2}+\theta\right)}$

= $\frac{(-\cos\theta)\cos\theta}{\sin\theta(-\sin\theta)}$

= $\cot^2\theta$ = R.H.S

Question-27

$\cos\theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta) = 0$.

Solution:

L.H.S = $\cos\theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta)$

= $\cos\theta - \cos\theta + \cos\theta - \cos\theta = 0$ = R.H.S

Question-28

$\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi + \theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right]=1$.

Solution:

L.H.S = $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi + \theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right]$

= $(-\sin\theta)(-\cos\theta)(\tan\theta + \cot\theta)$

= $\sin\theta\cos\theta(\tan\theta + \cot\theta)$

= $\sin^2\theta + \cos^2\theta = 1$ = R.H.S

Question-29

$\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x\cos(n + 2)x = \cos x$.

Solution:

L.H.S = $\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x\cos(n + 2)x$

= $\cos[(n + 1)x - (n + 2)x]$

= $\cos(-x)$

= $\cos x$

= RH.S

Question-30

Find the value of:

(i) cos 210º

(ii) sin 225º

(iii) tan 330º

(iv) cot (-315º)

Solution:

(i)cos 210º = cos (180º + 30º) = - cos 30º = - $\frac{\sqrt{3}}{2}$

(ii) sin 225º = sin (270º + 45º) = cos 45º = -$\frac{1}{\sqrt{2}}$

(iii) tan 330º = tan (360º - 30º) = - tan 30º = -$\frac{1}{\sqrt{3}}$

(iv) cot (-315º) = cot (315º) = cot (360º – 45º) = cot 45º = 1

Question-31

Find the value of tan(α + β), given that

cot α = ½, α ∈ $\left(\pi, \frac{3\pi}{2}\right)$ and sec β = -5/3, β ∈ $\left(\frac{\pi}{2}, \pi\right)$

Solution:

cot α = ½, α ∈ $\left(\pi, \frac{3\pi}{2}\right)$

and sec β = -5/3, β ∈ $\left(\frac{\pi}{2}, \pi\right)$

tanβ = -4/3

tan(α + β) = $\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$= \frac{2 - \frac{4}{3}}{1 - 2\times\frac{-4}{3}}$

$= \frac{\frac{2}{3}}{\frac{11}{3}}$

= 2/11

Question-32

Prove the following identity:

$\sin(150^o + x) + \sin(150^o - x) = \cos x$

Solution:

We know that

$2\sin\theta \cos\varphi = \sin(\theta + \varphi) + \sin(\theta - \varphi)$

L.H.S = $\sin(150^o + x) + \sin(150^o - x)$

$= 2\sin 150^o \cos x$

$= 2\sin(180^o - 30^o) \cos x$

$= 2\sin 30^o \cos x$

$= 2(1/2)\cos x$

$= \cos x$

= R.H.S

Question-33

Prove the following identity:

$\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\sin x$

Solution:

We know that

$-2\sin\theta \sin\varphi = \cos(\theta + \varphi) - \cos(\theta - \varphi)$

L.H.S = $-2\sin\frac{3\pi}{4}\sin x = -2\sin\left(\pi - \frac{\pi}{4}\right)\cos x = -2\sin\frac{\pi}{4}\sin x = -2\times\frac{1}{\sqrt{2}}\sin x = -\sqrt{2}\sin x$ =

R.H.S

Question-34

Prove the following identity:

$\cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right) = \sqrt{2}\cos x$

Solution:

We know that

$2\cos\theta \cos\varphi = \cos(\theta + \varphi) + \cos(\theta - \varphi)$

L.H.S = $2\cos\frac{\pi}{4}\cos x = 2\times\frac{1}{\sqrt{2}}\cos x = \sqrt{2}\cos x$ = R.H.S

Question-35

$\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Solution:

L.H.S = $\sin 2x + 2\sin 4x + \sin 6x$

$= 2\sin\frac{6x+2x}{2}\cos\frac{6x-2x}{2} + 2\sin 4x$

$= 2\sin 4x\cos 2x + 2\sin 4x$

$= 2\sin 4x(\cos 2x + 1)$

$= 4\sin 4x\cos^2 x$

= R.H.S

Question-36

$\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

L.H.S = $\sin^2 6x - \sin^2 4x$

[Using the formulae $2\sin^2 x = 1 - \cos 2x$]

$= \frac{1-\cos 12x}{2} - \frac{1-\cos 8x}{2}$

$= -\frac{1}{2}(\cos 12x - \cos 8x)$

$= -\frac{1}{2}[\cos(10x + 2x) - \cos(10x - 2x)]$

[Using the formulae $-2\sin\theta \sin\varphi = \cos(\theta + \varphi) - \cos(\theta - \varphi)$]

$= \frac{1}{2} \times 2 \sin 10x \sin 2x$

$= \sin 10x \sin 2x$

Question-37

$\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

L.H.S = $\cos^2 2x - \cos^2 6x$

[Using the formulae $2\cos^2 x = 1+ \cos 2x$]

$= \frac{1+\cos 4x}{2} - \frac{1+\cos 12x}{2}$

$= -\frac{1}{2}(\cos 12x - \cos 4x)$

$= -\frac{1}{2}[\cos(8x + 4x) - \cos(8x - 4x)]$

[Using the formulae $-2\sin\theta \sin\varphi = \cos(\theta + \varphi) - \cos(\theta - \varphi)$]

$= \frac{1}{2} \times 2 \sin 8x \sin 4x$

$= \sin 4x \sin 8x$

Question-38

cos7x + cos5x + cos3x + cosx = 4 cosx cos2x cos4x

Solution:

L.H.S = cos7x + cosx + cos5x + cos3x

= cos(4x + 3x) + cos(4x − 3x) + cos(4x + x) + cos(4x − x)

[Using the formula 2cosθ cosφ = cos(θ + φ) + cos(θ - φ)]

= 2cos4xcos3x + 2cos4xcosx

= 2cos4x(cos3x + cosx)

= 2cos4x(cos3x + cosx)

[Using the formula cosx + cosy = 2 cos $\frac{x+y}{2}$ cos $\frac{x-y}{2}$]

= 4cos4xcos $\frac{3x+x}{2}$ cos $\frac{3x-x}{2}$]

= 4cos4xcos2xcosx

= R.H.S

Question-39

cot4x(sin5x + sin3x) = cotx(sin5x − sin3x)

Solution:

L.H.S = cot4x(sin5x + sin3x)

[Using the formula sin5x + sin3x = 2 sin $\frac{x+y}{2}$ cos $\frac{x-y}{2}$]

= 2cot4x sin $\frac{5x+3x}{2}$ cos $\frac{5x-3x}{2}$

= 2cot4x sin $\frac{8x}{2}$ cos $\frac{2x}{2}$

= 2cot4x sin4x cosx

= 2cos4x cosx

R.H.S = cotx(sin5x − sin3x)

[Using the formula sin5x - sin3x = 2 cos $\frac{x+y}{2}$ sin $\frac{x-y}{2}$]

= 2cotx cos $\frac{5x+3x}{2}$ sin $\frac{5x-3x}{2}$

= 2cotx cos $\frac{8x}{2}$ sin $\frac{2x}{2}$

= 2cos4x cosx

∴ L.H.S = R.H.S

Question-40

tan3xtan2xtanx = tan3x – tan2x – tanx.

Solution:

tan3x = tan(2x + x)

= (tan2x + tanx)/(1 - tan2xtanx)

tan3x(1 - tan2xtanx) = tan2x + tanx

tan3x - tan3xtan2xtanx = tan2x + tanx

∴tan3xtan2xtanx = tan3x – tan2x – tanx.

Question-41

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \frac{-\sin 2x}{\cos 10x}$

Solution:

L.H.S = $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$= \frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$

$= \frac{-\sin\frac{14x}{2}\sin\frac{4x}{2}}{\cos\frac{20x}{2}\sin\frac{14x}{2}}$

$= \frac{-\sin 7x \sin 2x}{\cos 10x \sin 7x}$

$= \frac{-\sin 2x}{\cos 10x}$

= R.H.S

Question-42

Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$ **= tan4x**

Solution:

L.H.S = $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$= \frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$

$= \frac{\sin\frac{8x}{2}}{\cos\frac{8x}{2}}$

$= \frac{\sin 4x}{\cos 4x}$

= tan4x

= R.H.S

Question-43

Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\frac{x-y}{2}$

Solution:

L.H.S = $\frac{\sin x - \sin y}{\cos x + \cos y}$

$= \frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$

$= \frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}}$

$= \tan\frac{x-y}{2}$

= R.H.S

Question-44

Prove that (sinA + sin3A)/(cosA + cos3A) = tan2A

Solution:

L.H.S = (sinA + sin3A)/(cosA + cos3A) **=** 2sin2AcosA/2cos2AcosA = tan2A
= R.H.S

Question-45

Prove that $\frac{\sin x + \sin y}{\cos x + \cos y} = \tan\frac{x+y}{2}$

Solution:

L.H.S = $\frac{\sin x + \sin y}{\cos x + \cos y}$

$= \frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$

$= \frac{\sin\frac{x+y}{2}}{\cos\frac{x+y}{2}}$

$= \tan\frac{x+y}{2}$

= R.H.S

Question-46

Prove that (tan5θ+ tan3θ)/(tan5θ - tan3θ)= 4cos2θcos4θ

Solution:

L.H.S $= \dfrac{\dfrac{\sin 5\theta\cos 3\theta + \sin 3\theta\cos 5\theta}{\cos 5\theta\cos 3\theta}}{\dfrac{\sin 5\theta\cos 3\theta - \sin 3\theta\cos 5\theta}{\cos 5\theta\cos 3\theta}}$

$= \dfrac{\dfrac{\sin 8\theta}{\cos 5\theta\cos 3\theta}}{\dfrac{\sin 2\theta}{\cos 5\theta\cos 3\theta}}$

[sin(A+B) = sinAcosB + cosAsinB and sin (A-B) = sinAcosB - cosAsinB]

$= \dfrac{\sin 8\theta}{\sin 2\theta}$

$= \dfrac{2\sin 4\theta\cos 4\theta}{\sin 2\theta}$

$= \dfrac{2\times 2\sin 2\theta\cos 2\theta\cos 4\theta}{\sin 2\theta}$ (sin2θ = 2sinθ cosθ)

= 4 cos2θ cos4θ

= R.H.S

Hence proved

Question-47

Prove that $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$ =2sinx

Solution:

L.H.S $= \dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$= \dfrac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x}$

$= \dfrac{2\cos\dfrac{3x+x}{2}\sin\dfrac{3x-x}{2}}{\cos 2x}$

[Using the identity cos2x = $\cos^2 x - \sin^2 x$ and sinx – siny = 2cos $\frac{x+y}{2}$ sin $\frac{x-y}{2}$]

$= \dfrac{2\cos 2x \sin x}{\cos 2x}$

= 2sinx

= R.H.S

Question-48

Prove that $(3\sin A - \sin 3A)^{2/3} + (3\cos A + \cos 3A)^{2/3} = 4^{2/3}$

Solution:

L.H.S $= (3\sin A - \sin 3A)^{2/3} + (3\cos A + \cos 3A)^{2/3}$

$= [3\sin A - (3\sin A - 4\sin^3 A)]^{2/3} + [3\cos A + (4\cos^3 A - 3\cos A)]^{2/3}$

$= (4\sin^3 A)]^{2/3} + [4\cos^3 A]^{2/3}$

$= 4^{2/3}(\sin^2 A + \cos^2 A)^{2/3}$

$= 4^{2/3}\, 1^{2/3}$

$= 4^{2/3}$

= R.H.S

Question-49

Prove that $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

Solution:

L.H.S $= \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$

$= \frac{2\sin\frac{7x+5x}{2}\cos\frac{7x-5x}{2} + 2\sin\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2} + 2\cos\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}$

$= \frac{2\sin 6x\cos x + 2\sin 6x\cos 3x}{2\cos 6x\cos x + 2\cos 6x\cos 3x}$

$= \frac{2\sin 6x(\cos x + \cos 3x)}{2\cos 6x(\cos x + \cos 3x)}$

$= \tan 6x$

= R.H.S

Question-50

Prove that $\cos 4x = 1 - 8\sin^2 x\cos^2 x$.

Solution:

L.H.S $= \cos 4x = 1 - 2\sin^2 2x = 1 - 2(2\sin x\cos x)^2 = 1 - 8\sin^2 x\cos^2 x$ = R.H.S

Hence proved.

Trigonometric Functions

1. Prove that $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

2. If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$, then prove that $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is also equal to y.

$\left[\textbf{Hint}: \text{Express } \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} \cdot \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}\right]$

3. If $m \sin\theta = n \sin(\theta + 2\alpha)$, then prove that $\tan(\theta + \alpha)\cot\alpha = \frac{m+n}{m-n}$

[**Hint:** Express $\frac{\sin(\theta+2\alpha)}{\sin\theta} = \frac{m}{n}$ and apply componendo and dividendo]

4. If $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where α lie between 0 and $\frac{\pi}{4}$, find the value of $\tan 2\alpha$ [**Hint:** Express $\tan 2\alpha$ as $\tan(\alpha+\beta+\alpha-\beta)$]

5. If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$

6. Prove that $\cos\theta \cos\frac{\theta}{2} - \cos 3\theta \cos\frac{9\theta}{2} = \sin 7\theta \sin 8\theta$.

[**Hint:** Express L.H.S. $= \frac{1}{2}[2\cos\theta \cos\frac{\theta}{2} - 2\cos 3\theta \cos\frac{9\theta}{2}]$]

7. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then show that $a^2 + b^2 = m^2 + n^2$

8. Find the value of $\tan 22°30'$.

[**Hint:** Let $\theta = 45°$, use $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sin\theta}{1+\cos\theta}$]

9. Prove that $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$.

10. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sin\theta \tan\theta$
[**Hint:** $m + n = 2\tan\theta$, $m - n = 2\sin\theta$, then use $m^2 - n^2 = (m + n)(m - n)$]

11. If $\tan(A + B) = p$, $\tan(A - B) = q$, then show that $\tan 2A = \dfrac{p+q}{1-pq}$

[**Hint:** Use $2A = (A + B) + (A - B)$]

12. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then prove that $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$.
[**Hint:** $(\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$]

13. If $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$, then show that $\dfrac{\tan x}{\tan y} = \dfrac{a}{b}$ [**Hint:** Use Componendo and Dividendo].

14. If $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.

[**Hint:** Express $\tan\theta = \tan\left(\alpha - \frac{\pi}{4}\right) \Rightarrow \theta = \alpha - \frac{\pi}{4}$]

15. If $\sin\theta + \cos\theta = 1$, then find the general value of θ.

16. Find the most general value of θ satisfying the equation $\tan\theta = -1$ and $\cos\theta = \dfrac{1}{\sqrt{2}}$.

17. If $\cot\theta + \tan\theta = 2\,\mathrm{cosec}\,\theta$, then find the general value of θ.

18. If $2\sin^2\theta = 3\cos\theta$, where $0 \leq \theta \leq 2\pi$, then find the value of θ.

19. If $\sec x \cos 5x + 1 = 0$, where $0 < x \leq \frac{\pi}{2}$, then find the value of x.

20. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$
[**Hint:** Express $\cos(\alpha - \beta) = \cos((\theta + \alpha) - (\theta + \beta))$]

21. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$, then prove that $\tan\theta = \dfrac{1-m}{1+m}\cot\phi$.

[**Hint**: Express $\frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}=\frac{m}{1}$ and apply Componendo and Dividendo]

22. Find the value of the expression

$3\,[\sin^4(\frac{3\pi}{2}-\alpha)+\sin^4(3\pi+\alpha)]-2\,\{\sin^6(\frac{\pi}{2}+\alpha)+\sin^6(5\pi-\alpha)]$

23. If $a\cos 2\theta + b\sin 2\theta = c$ has α and β as its roots, then prove that

$\tan\alpha + \tan\beta = \frac{2b}{a+c}$.

[**Hint**: Use the identities $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ and $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$].

24. If $x = \sec\phi - \tan\phi$ and $y = \text{cosec}\,\phi + \cot\phi$ then show that $xy + x - y + 1 = 0$
[**Hint**: Find $xy + 1$ and then show that $x - y = -(xy + 1)$]

25. If θ lies in the first quadrant and $\cos\theta = \frac{8}{17}$, then find the value of

$\cos(30^\circ+\theta)+\cos(45^\circ-\theta)+\cos(120^\circ-\theta)$.

26. Find the value of the expression $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}$

[**Hint**: Simplify the expression to $2\,(\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8})$

$=2\left[\left(\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}\right)^2-2\cos^2\frac{\pi}{8}\cos^2\frac{3\pi}{8}\right]$

27. Find the general solution of the equation $5\cos^2\theta + 7\sin^2\theta - 6 = 0$
28. Find the general solution of the equation
$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

29. Find the general solution of the equation $(\sqrt{3}-1)\cos\theta + (\sqrt{3}+1)\sin\theta = 2$

[**Hint**: Put $\sqrt{3}-1 = r\sin\alpha$, $\sqrt{3}+1 = r\cos\alpha$ which gives $\tan\alpha = \tan(\frac{\pi}{4}-\frac{\pi}{6})$

$\Rightarrow \alpha = \frac{\pi}{12}$]

Choose the correct answer from the given four options in the Exercises 30 to 59 (M.C.Q.).

30. If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^2\theta + \operatorname{cosec}^2\theta$ is equal to

(A) 1
(B) 4
(C) 2
(D) None of these

31. If $f(x) = \cos^2 x + \sec^2 x$, then

(A) $f(x) < 1$
(B) $f(x) = 1$
(C) $2 < f(x) < 1$
(D) $f(x) \geq 2$

[Hint: A.M $\geq$ G.M.]

32. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then the value of $\theta + \phi$ is

(A) $\frac{\pi}{6}$
(B) π
(C) 0
(D) $\frac{\pi}{4}$

33. Which of the following is not correct?

(A) $\sin\theta = -\frac{1}{5}$
(B) $\cos\theta = 1$
(C) $\sec\theta = \frac{1}{2}$
(D) $\tan\theta = 20$

34. The value of $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$ is

(A) 0
(B) 1
(C) $\frac{1}{2}$
(D) Not defined

35. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is

(A) 1
(B) $\sqrt{3}$
(C) $\frac{\sqrt{3}}{2}$
(D) 2

36. The value of $\cos 1° \cos 2° \cos 3° \ldots \cos 179°$ is

(A) $\frac{1}{\sqrt{2}}$
(B) 0
(C) 1
(D) -1

37. If $\tan\theta = 3$ and θ lies in third quadrant, then the value of $\sin\theta$ is

(A) $\frac{1}{\sqrt{10}}$ (B) $-\frac{1}{\sqrt{10}}$ (C) $\frac{-3}{\sqrt{10}}$ (D) $\frac{3}{\sqrt{10}}$

38. The value of $\tan 75° - \cot 75°$ is equal to

(A) $2\sqrt{3}$ (B) $2+\sqrt{3}$ (C) $2-\sqrt{3}$ (D) 1

39. Which of the following is correct?

(A) $\sin 1° > \sin 1$ (B) $\sin 1° < \sin 1$

(C) $\sin 1° = \sin 1$ (D) $\sin 1° = \frac{\pi}{18°} \sin 1$

[Hint: 1 radian $= \frac{180°}{\pi} = 57° 30'$ approx]

40. If $\tan \alpha = \frac{m}{m+1}$, $\tan \beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

41. The minimum value of $3 \cos x + 4 \sin x + 8$ is

(A) 5 (B) 9 (C) 7 (D) 3

42. The value of $\tan 3A - \tan 2A - \tan A$ is equal to

(A) $\tan 3A \tan 2A \tan A$

(B) $-\tan 3A \tan 2A \tan A$

(C) $\tan A \tan 2A - \tan 2A \tan 3A - \tan 3A \tan A$

(D) None of these

43. The value of $\sin (45° + \theta) - \cos (45° - \theta)$ is

(A) $2 \cos\theta$ (B) $2 \sin\theta$ (C) 1 (D) 0

44. The value of $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$ is

(A) –1 (B) 0 (C) 1 (D) Not defined

45. $\cos 2\theta \cos 2\phi + \sin^2 (\theta - \phi) - \sin^2 (\theta + \phi)$ is equal to

(A) $\sin 2(\theta + \phi)$ (B) $\cos 2(\theta + \phi)$

(C) $\sin 2(\theta - \phi)$ (D) $\cos 2(\theta - \phi)$

[Hint: Use $\sin^2 A - \sin^2 B = \sin (A + B) \sin (A - B)$]

46. The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is

(A) $\frac{1}{2}$ (B) 1 (C) $-\frac{1}{2}$ (D) $\frac{1}{8}$

47. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then $\tan(2A + B)$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

48. The value of $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $-\frac{1}{4}$ (D) 1

[Hint: Use $\sin 18° = \frac{\sqrt{5}-1}{4}$ and $\cos 36° = \frac{\sqrt{5}+1}{4}$]

49. The value of $\sin 50° - \sin 70° + \sin 10°$ is equal to

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) 2

50. If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is equal to

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) -1

51. If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan\alpha)(1 + \tan\beta)$ is

(A) 1 (B) 2
(C) -2 (D) Not defined

52. If $\sin\theta = \frac{-4}{5}$ and θ lies in third quadrant then the value of $\cos\frac{\theta}{2}$ is

(A) $\frac{1}{5}$ (B) $-\frac{1}{\sqrt{10}}$ (C) $-\frac{1}{\sqrt{5}}$ (D) $\frac{1}{\sqrt{10}}$

53. Number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3

54. The value of $\sin\frac{\pi}{18}+\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\sin\frac{5\pi}{18}$ is given by

(A) $\sin\frac{7\pi}{18}+\sin\frac{4\pi}{9}$ (B) 1

(C) $\cos\frac{\pi}{6}+\cos\frac{3\pi}{7}$ (D) $\cos\frac{\pi}{9}+\sin\frac{\pi}{9}$

55. If A lies in the second quadrant and 3 tan A + 4 = 0, then the value of 2 cotA − 5 cos A + sin A is equal to

(A) $\frac{-53}{10}$ (B) $\frac{23}{10}$ (C) $\frac{37}{10}$ (D) $\frac{7}{10}$

56. The value of $\cos^2 48° - \sin^2 12°$ is

(A) $\frac{\sqrt{5}+1}{8}$ (B) $\frac{\sqrt{5}-1}{8}$

(C) $\frac{\sqrt{5}+1}{5}$ (D) $\frac{\sqrt{5}+1}{2\sqrt{2}}$

[**Hint**: Use $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$]

57. If $\tan\alpha = \frac{1}{7}$, $\tan\beta = \frac{1}{3}$, then cos 2α is equal to

(A) sin 2β (B) sin 4β (C) sin 3β (D) cos 2β

58. If $\tan\theta = \frac{a}{b}$, then $b\cos 2\theta + a\sin 2\theta$ is equal to

(A) a (B) b (C) $\frac{a}{b}$ (D) None

59. If for real values of x, $\cos\theta = x + \frac{1}{x}$, then

(A) θ is an acute angle (B) θ is right angle

(C) θ is an obtuse angle (D) No value of θ is possible

CBSE Class 11 Mathematics
Important Questions
Chapter 3
Trigonometric Functions

1 Marks Questions

1. Find the radian measure corresponding to 5° 37' 30''

Ans. $\left(\frac{\pi}{32}\right)^c$

2. Find the degree measure corresponding to $\left(\frac{11}{16}\right)^c$

Ans. 39°22'30''

3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

Ans. $\frac{5\pi}{12}\text{cm}$

4. Find the value of $\frac{19\pi}{3}$

Ans. $\sqrt{3}$

5. Find the value of sin(–1125°)

Ans. $\frac{-1}{\sqrt{2}}$

6. Find the value of tan 15°

Ans. $2-\sqrt{3}$

7. If sin A = $\frac{3}{5}$ and $\frac{\pi}{2}$ < A < , find cos A

Ans. $\frac{-4}{5}$

8. If tan A = $\frac{a}{a+1}$ and tan B = $\frac{1}{2a+1}$ then find the value of A + B.

Ans. $45°$

9. Express sin 12θ + sin 4θ as the product of sines and cosines.

Ans. 2 sin8θ cos4θ

10. Express 2 cos4x sin2x as an algebraic sum of sines or cosines.

Ans. sin 6x – sin2x

11. Write the range of cos θ

Ans. [–1,1]

12. What is domain of sec θ?

Ans. $R - \left\{(2n+1)\frac{\pi}{2};\ n \in z\right\}$

13.Find the principal solutions of cotx = 3

Ans. $\frac{5\pi}{6}, \frac{11\pi}{6}$

14. Write the general solution of cos θ = 0

Ans. $(2n+1)\frac{\pi}{2},\ n \in z$

15. If sinx = $\frac{\sqrt{5}}{3}$ **and 0 < x <** $\frac{\pi}{2}$ **find the value of cos 2x**

Ans. $-\frac{1}{9}$

16. If cosx = $\frac{-1}{3}$ **and x lies in quadrant III, find the value of sin** $\frac{x}{2}$

Ans. $\frac{\sqrt{6}}{3}$

17.Convert into radian measures. – 47^0 30′

Ans. - 47^0 30′ = - $\left(47+\frac{30}{60}\right)^0$

$= -\left(47\frac{1}{2}\right)^0$

$= -\left(\frac{95}{2}\times\frac{\pi}{180}\right)$ radians

$= -\ \frac{19\pi}{72}$ radians.

18.Evaluate tan 75^0.

Ans. tan 75 = tan (45 + 30)

$$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

19.Prove that Sin (40 + θ). Cos (10 + θ) – Cos (40 + θ). Sin (10 + θ) = $\frac{1}{2}$

Ans. L. H. S = Sin (40 + θ). Cos (10 + θ) – Cos (40 + θ). Sin (10 +θ)

= Sin $\left[40 + \not\theta - 10 - \not\theta\right] = \text{Sin } 30 = \frac{1}{2}$

20.Find the principal solution of the eq. Sin x = $\frac{\sqrt{3}}{2}$

Ans. Sin x = $\frac{\sqrt{3}}{2}$

$$\text{Sin x} = \text{Sin } \frac{\pi}{3}$$

$$x = \frac{\pi}{3}$$

$$\mathit{Sin}\ \text{x} = \text{Sin}\left(\pi - \frac{\pi}{3}\right)$$

$$x = \frac{2\pi}{3}$$

21.Prove that $\text{Cos}\left(\frac{\pi}{4}+x\right)+\text{Cos}\left(\frac{\pi}{4}-x\right)=\sqrt{2}\ \text{Cos x}$

Ans. L. H. S = Cos $\left(\frac{\pi}{4}+x\right)+\text{Cos}\left(\frac{\pi}{4}-x\right)$

$2\ \text{Cos}\ \frac{\pi}{4}.\ \text{Cos x}\ [\because\ \text{Cos (A + B) + Cos (A - B) = 2 Cos A. Cos B}$

$= 2\ \frac{1}{\sqrt{2}}.\ \text{Cos x} \qquad = \sqrt{2}\ \text{Cos x}$

22.Convert into radian measures. -37^0 30'

Ans. - 37^0 30' = - $\left(37+\frac{\not{3}\not{0}\ ^1}{\not{6}\not{0}\ ^2}\right)^0$

$=-\left(\frac{75}{2}\right)^0$

$=\ -\ \frac{75}{2}\times\frac{\pi}{180}\ radien$

$=-\frac{5\pi}{24}$

23.Prove Sin (n+1) x Sin (n+2) x + Cos (n+1) x. Cos (n+2) x = Cos x

Ans.L.H.S = Cos (n+1) x Cos (n+2) x + Sin (n+1) x Sin (n+2) x

=Cos $\left[(n+1)x-(n+2)x\right]$

=Cos x

24.Find the value of Sin $\frac{31\pi}{3}$

Ans. Sin $\frac{31\pi}{3}$ = Sin $\left(10\pi+\frac{\pi}{3}\right)$

= sin $\left(2\pi\times 5+\frac{\pi}{3}\right)$

= Sin $\frac{\pi}{3}$

= $\frac{\sqrt{3}}{2}$

25.Find the principal solution of the eq. tan x = $\frac{-1}{\sqrt{3}}$

Ans. tan x = $-\frac{1}{\sqrt{3}}$

$\tan x = \tan\left(\pi-\frac{\pi}{6}\right)$

$x = \frac{5\pi}{6}$

$\tan x = -\frac{1}{\sqrt{3}}$

$\tan x = \tan\left(2\pi-\frac{\pi}{6}\right)$

$x = \frac{11\pi}{6}$

26.Convert into radian measures. $5^{0}37'\ 30''$

Ans. $5^0\ 37^1\ 30^{11}$ = 5^0 + $\left(37+\frac{30}{60}\right)^1$

$$=5^0+\left(\frac{75}{2}\right)^1$$

$$=5^0+\left(\frac{75}{2}\times\frac{1}{60}\right)^0$$

$$=5^0+\left(\frac{5}{8}\right)^0$$

$$=\left(5\frac{5}{8}\right)^0$$

$$=\left(\frac{45}{8}\right)^0$$

$$=\frac{45}{8}\times\frac{\pi}{180}\ \text{radian}$$

$$=\frac{\pi}{32}\ \text{radian.}$$

27.Prove Cos 70^0. Cos 10^0 + **Sin 70^0. Sin 10^0** = $\frac{1}{2}$

Ans. L. H. S. = Cos (70 – 10) = Cos 60 = $\frac{1}{2}$

28.Evaluate 2 Sin $\frac{\pi}{12}$.

Ans.2 Sin $\frac{\pi}{12}=2\ \text{Sin}\left[\frac{\pi}{4}-\frac{\pi}{6}\right]$

$$= 2\left[\sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}\right]$$

$$= 2\left[\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2}\right]$$

$$= \frac{\sqrt{3}-1}{\sqrt{2}}$$

29.Find the solution of Sin x = $-\frac{\sqrt{3}}{2}$

Ans. Sin x = $-\frac{\sqrt{3}}{2}$

$$\text{Sin x} = \text{Sin}\left(\pi + \frac{\pi}{3}\right)$$

$$\text{Sin x} = \text{Sin}\ \frac{4\pi}{3}$$

$$\text{if Sin}\ \theta = \text{Sin}\ \alpha$$

$$\theta = \text{n}\ \pi + (-1)^{n}.\ \alpha$$

$$x = \text{n}\,\pi + (-1)^{n}.\frac{4\pi}{3}$$

30.Prove that $\frac{\text{Cos } 9^0 - \text{Sin } 9^0}{\text{Cos } 9^0 + \text{Sin } 9^0} = \tan 36^0$

Ans.L. H. S = tan 36^0

$$= \tan(45^0 - 9^0)$$

$$= \frac{1 - \tan 9^0}{1 + \tan 9^0}$$

$$= \frac{\cos 9^0 - \sin 9^0}{\cos 9^0 + \sin 9^0}$$

31.Find the value of tan $\frac{19\pi}{3}$.

Ans. $\tan \frac{19\pi}{3} = \tan\left(6\pi + \frac{\pi}{3}\right)$

$$= \tan\left(3 \times 2\pi + \frac{\pi}{3}\right)$$

$$= \tan \frac{\pi}{3} = \sqrt{3}$$

32.Prove Cos 4x = 1 – 8 Sin^2 x. Cos^{2}x

Ans. L. H. S = Cos 4x

$$= 1 - 2 \sin^2 2x \quad \left[\because \cos 2\theta = 1 - 2\sin^2 \theta\right.$$

$$= 1 - 2 (\sin 2x)^2$$

$$= 1 - 2 (2 \sin x. \cos x)^2$$

$$= 1 - 2 (4 \sin^2 x . \cos^2 x)$$

$$= 1 - 8 \sin^2 x. \cos^2 x.$$

33.Prove

$$\frac{\text{Cos}(\pi+x)\,.\,\text{Cos}(-x)}{\text{Sin}(\pi-x).\ \text{Cos}\left(\frac{\pi}{2}+x\right)} = \text{Cot}^2 x$$

Ans.L. H. S = $\frac{-\text{Cos x. Cos x}}{\text{-Sin x. Sin x}} = \text{Cot}^2\ x.$

34.Prove that tan 56^0 = $\frac{\text{Cos } 11^0 + \text{Sin } 11^0}{\text{Cos } 11^0 - \text{Sin } 11^0}$

Ans.L. H. S = tan 56^0

= tan (45^0 + 11^0)

$$= \frac{\tan 45^0 + \tan 11^0}{1 - \tan 45^0\ .\ \tan 11^0}$$

$$= \frac{1 + \tan 11^0}{1 - \tan 11^0}$$

$$= \frac{\text{Cos } 11^0 + \text{Sin } 11^0}{\text{Cos } 11^0 - \text{Sin } 11^0} = \text{R H S}$$

35.Prove that Cos 105^0 + **Cos 15^0** = **Sin 75^0 – Sin 15^0**

Ans.L. H. S = Cos 105^0 + Cos 15^0

$= \text{Cos}(90^0 + 15^0) + \text{Cos}(90^0 - 75^0)$

$= -\ \text{Sin } 15^0 + \text{Sin } 75^0$

$= \text{Sin } 75^0 - \text{Sin } 15^0$

36.Find the value of Cos (- 1710^0).

Ans. Cos (-1710^0) = Cos (1800-90)[Cos (-θ) = Cos θ

= Cos [5 × 360 +90]

= Cos $\frac{\pi}{2}$ = 0

37.A wheel makes 360 revolutions in 1 minute. Through how many radians does it turn in 1 second.

Ans.N. of revolutions made in 60 sec. = 360

N. of revolutions made in 1 sec = $\frac{360}{60} = 6$

Angle moved in 6 revolutions = 2 π × 6 = 12 π

38.Prove Sin^2 6x – Sin^2 4x = Sin2 x. Sin 10 x.

Ans.L. H. S = Sin^2 6x – Sin^2 4x

= Sin (6x + 4x). Sin (6x – 4x)

= Sin 10x . Sin 2x

39.Prove that $\frac{\tan 69 + \tan 66}{1 - \tan 69. \tan 66} = -1$

Ans.L. H. S = tan (69 + 66)

= tan (135)

= tan (90 + 45)

= - tan 45

= -1

40.Prove that $\frac{\text{Sin x}}{1 + \text{Cos x}} = \tan \frac{x}{2}$

Ans.L. H. S

$$\frac{2Sin\frac{x}{2}.Cos\frac{x}{2}}{2Cos^{x}\frac{x}{2}}$$

$= \tan \frac{x}{2}$

CBSE Class 12 Mathematics

Important Questions

Chapter 3

Trigonometric Functions

4 Marks Questions

Prove the following Identities

1.The minute hand of a watch is 1.5 cm long. How far does it tip move in 40 minute?

Ans. r = 1.5 cm

Angle made in 60 mint = 360^0

Angle made in 1 min = $\frac{360}{60}$ = 60^0

Angle made in 40 mint = 6×40

= 240^0

$\Theta = \frac{l}{r}$

$$\cancel{240}^{\,4} \times \frac{\pi}{\cancel{180}\,3} = \frac{l}{1.5}$$

$$\frac{4 \times 3.14}{\cancel{3}} = \frac{l}{\frac{\cancel{15}\,\cancel{5}}{\cancel{10}\,2}}$$

$2 \times 3.14 = l$

$6.28 = l$

$l = 6.28$ cm

2. Show that tan 3x. tan 2x. tan x = tan 3x – tan 2x – tan x

Ans. Let 3x = 2x + x

tan 3x = tan (2x + x)

$$\frac{\tan\ 3x}{1} = \frac{\tan 2x + \tan x}{1 - \tan 2x . \tan x}$$

$$\tan 3x\ (1 - \tan 2x . \tan x) = \tan 2x + \tan x$$

$$\tan 3x - \tan 3x . \tan 2x . \tan x = \tan 2x + \tan x$$

$$\tan 3x . \tan 2x . \tan x = \tan 3x - \tan 2x - \tan x$$

3.Find the value of tan $\frac{\pi}{8}$.

Ans. Let x = $\frac{\pi}{8}$

$$\tan\ 2x = \frac{2\ \tan x}{1 - \tan^2 x} t$$

$$\tan\left(\not{2}\frac{\pi}{\not{8}\ 4}\right) = \frac{2\ \tan\ \pi/8}{1 - \tan^2\ \pi/8}$$

$$1 = \frac{2\ \tan\ \pi/8}{1 - \tan^2\ \pi/8}$$

put tan $\pi/8 =$

$$\frac{1}{1} = \frac{2t}{1-t^2}$$

$$2t = 1 - t^2$$

$t^2 + 2t - 1 = 0$

$$t = \frac{-2 \pm 2\sqrt{2}}{2\times 1}$$

$= -1 \pm \sqrt{2}$

$= \pm\sqrt{2} - 1$

$= \sqrt{2} - 1$ or $-\sqrt{2} - 1$

$\tan \pi/8 = \sqrt{2} - 1$

4.Prove that $\dfrac{Sin(x+y)}{Sin(x-y)} = \dfrac{\tan x + \tan y}{\tan x - \tan y}$

Ans.L.H.S = $\dfrac{Sin(x+y)}{Sin(x-y)}$

$$= \frac{Sinx.Cosy + Cosx.Siny}{Sinx.Cosy - Cosx.Siny}$$

$DividingNandDbyCosx.Cosy$

$$= \frac{\tan x + \tan y}{\tan x - \tan y}$$

5.If in two circles, arcs of the same length subtend angles 60^0 and 75^0 at the centre find the ratio of their radii.

Ans. $\theta = \dfrac{l}{r_1}$

$$60 \times \frac{\pi}{18} = \frac{l}{r_1}$$

$$r_1 = \frac{3l}{\pi} \quad (1)$$

$$\theta = \frac{l}{r_2}$$

$$75 \times \frac{\pi}{18} = \frac{l}{r_2}$$

$$r_2 = \frac{12l}{5\pi} \quad (2)$$

(1) ÷ (2)

$$\frac{r_1}{r_2} = \frac{\frac{3l}{\pi}}{\frac{12l}{5\pi}}$$

$$= \frac{3l}{\pi} \times \frac{5\pi}{12l}$$

$$= 5 : 4$$

6.Prove that Cos 6x= 32 Cos^{6}x – 48 Cos^4 x + 18 Cos^2 x-1

Ans.L.H.S. = Cos 6x

= $\cos 2(3x) = 2\cos^2 3x - 1 = \cos 2(3x)$

$$= 2(4\cos^3 x - 3\cos x)^2 - 1$$

$$= 2\left[16\cos^6 x + 9\cos^2 x - 24\cos^4 x\right] - 1$$

$$= 32\cos^6 x + 18\cos^2 x - 48\cos^4 x - 1$$

$$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$$

7.Solve Sin2x-Sin4x+Sin6x=o

Ans. $Sin\ 6x+\sin 2x-\sin 4x=0$

$$2\sin\left(\frac{6x+2x}{2}\right).\cos\left(\frac{6x-2x}{2}\right)-\sin 4x=0$$

$$2\sin 4x.\cos 2x-\sin 4x=0$$

$$\sin 4x\left[2\cos 2x-1\right]=0$$

$$\sin 4x=0$$

$$4x=n\pi$$

$$x=\frac{n\pi}{4}$$

$$2\cos 2x-1=0$$

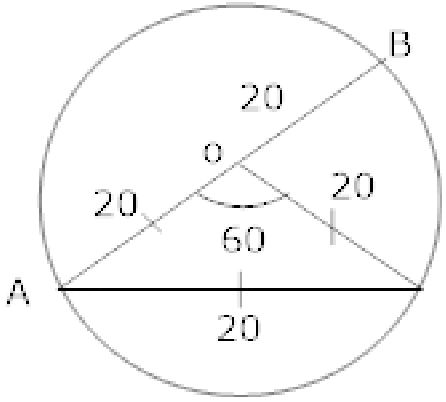
$$\cos 2x=\cos\frac{\pi}{3}$$

$$2x=2n\pi\pm\frac{\pi}{3}$$

$$x=n\pi\pm\frac{\pi}{6}$$

8.In a circle of diameter 40cm, the length of a chord is 20cm. Find the length of minor are of the chord.

Ans.



$\Theta = 60^0$

$$\theta = \frac{l}{r}$$

$$60 \times \frac{\pi}{180\ 3} = \frac{l}{20}$$

$$l = \frac{20\pi}{3} \text{ cm.s}$$

9.Prove that tan 4x = $\dfrac{4 \tan x\,(1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Ans. L. H. S = tan 4x

$$= \frac{2 \tan 2x}{1 - \tan^2 2x}$$

$$= \frac{2.\dfrac{2\tan x}{1-\tan^2 x}}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\frac{4\tan x}{1-\tan^2 x}}{\frac{(1-\tan^2 x)^2 - 4\tan^2 x}{(1-\tan^2 x)^2}}$$

$$= \frac{4\tan x}{(1-\tan^2 x)} \times \frac{(1-\tan^2 x)}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

10.Prove that (Cos x + Cos y)^2 + (Sin x – Sin y)^2 = 4 Cos^2 $\left(\frac{x+y}{2}\right)$

Ans. L. H. S = (Cos x + Cos y)^2 + (Sin x – Sin y)^2

$$=\left(2\ \text{Cos}\ \frac{x+y}{2}.\ \text{Cos}\ \frac{x-y}{2}\right)^2 + \left(2\ \text{Cos}\ \left(\frac{x+y}{2}\right).\ \text{Sin}\ \left(\frac{x-y}{2}\right)\right)^2$$

$$= 4\ \text{Cos}^2\ \frac{x+y}{2}.\ \text{Cos}^2\left(\frac{x-y}{2}\right) + 4\ \text{Cos}^2\ \frac{x+y}{2}.\ \text{Sin}^2\ \frac{x-y}{2}$$

$$=\ 4\ \text{Cos}^2\ \left(\frac{x+y}{2}\right)\left[\text{Cos}^2\ \frac{x-y}{2} + Sin^2\ \frac{x-y}{2}\right]$$

$$=\ 4\ \text{Cos}^2\ \left(\frac{x+y}{2}\right)$$

11.If Cot x = - $\frac{5}{12}$, x lies in second quadrant find the values of other five trigonometric functions.

Ans. Cot x = $-\frac{5}{12}$

$\tan x = -\frac{12}{5}$

$\text{Sec}^2 x = 1 + \tan^2 x$

$\text{Sec } x = \pm\frac{13}{5}$

$\text{Sec } x = -\frac{13}{5}$ [$\because$ x lies in IInd quad.

$\text{Cos } x = -\frac{5}{13}$

$\text{Sin } x = \tan x . \text{ Cos } x$

$= \frac{-12}{5} \times \left(\frac{-5}{13}\right) \qquad = \frac{12}{13}$

$\text{Cosec } x = \frac{13}{12}$

12.Prove that $\frac{\text{Sin } 5x - 2\text{Sin } 3x + \text{Sin } x}{\text{Cos } 5x - \text{Cos } x} = \tan x$

Ans.L. H. S = $\frac{\text{Sin } 5x + \text{Sin} x - 2 \text{ Sin } 3x}{\text{Cos } 5x - \text{Cos } x}$

$= \frac{2\text{Sin } 3x . \text{ Cos } 2x - 2 \text{ Sin } 3x}{-2 \text{ Sin } 3x . \text{ Sin } 2x}$

$= \frac{\cancel{2 \text{ Sin } 3x} (\text{Cos } 2x - 1)}{-\cancel{2 \text{ Sin } 3x} . \text{ Sin } 2x}$

$= \frac{\cancel{-}(1 - \text{Cos } 2x)}{\cancel{-}\text{Sin } 2x}$

$$= \frac{\cancel{2}\ \cancel{\text{Sin}}^{\cancel{2}}\ x}{\cancel{2}\ \cancel{\text{Sin}}\ x.\ \text{Cos}\ x}$$

$$= \frac{\text{Sin x}}{\text{Cos x}} = \tan x$$

13.Prove that Sin x + Sin 3x + Sin 5x + Sin 7x = 4 Cos x. Cos 2x. Sin 4x

Ans.L. H. S. = Sin x + Sin 3x + Sin 5x + Sin 7x

= Sin x + Sin 7x + Sin 3x + Sin 5x

$$= 2\ \text{Sin}\left(\frac{x+7x}{2}\right).\ \text{Cos}\left(\frac{x-7x}{2}\right) + 2\ \text{Sin}\left(\frac{3x+5x}{2}\right)\ \text{Cos}\left(\frac{3x-5x}{2}\right)$$

$$= 2\ \text{Sin}\ 4x.\ \text{Cos}\ 3x + 2\ \text{Sin}\ 4x.\ \text{Cos}\ x$$

$$= 2\ \text{Sin}\ 4x\ [\text{Cos}\ 3x + \text{Cos}\ x]$$

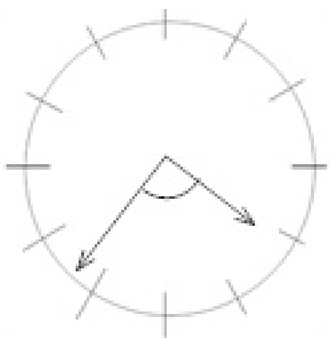
$$= 2\ \text{Sin}\ 4x\left[2\ \text{Cos}\left(\frac{3x+x}{2}\right).\ \text{Cos}\left(\frac{3x-x}{2}\right)\right]$$

$$= 2\ \text{Sin}\ 4x\ [\ 2\ \text{Cos}\ 2x.\ \text{Cos}\ x]$$

$$= 4\ \text{Cos}\ x.\ \text{Cos}\ 2x.\ \text{Sin}\ 4x$$

14.Find the angle between the minute hand and hour hand of a clock when the time is 7. 20.

Ans. Angle made by mint hand in 15 mint= 15 $\times$ 6 = 90^0



Angle made by hour hand in 1 hr = 30^0

in 60 minute = $\frac{30}{60} = \frac{1}{2}$

$[\because$ Angle Traled by hr hand in 12 hr = 360^0

in 20 minute = $\frac{1}{2} \times 20 = 10^0$

Angle made = 90 + 10 = 100^0

15.Show that $\sqrt{2+\sqrt{2+2\ \text{Cos}\ 4\theta}} = 2\ \text{Cos}\ \theta$

Ans.L H. S = $\sqrt{2+\sqrt{2+2\text{Cos}\ 4\theta}}$

$$= \sqrt{2+\sqrt{2(1+\text{Cos}\ 4\theta)}}$$

$$= \sqrt{2+\sqrt{2.2\ \text{Cos}^2\ 2\theta}}$$

$$= \sqrt{2+2\ \text{Cos}\ 2\theta}$$

$$= \sqrt{2(1+\text{Cos}\ 2\theta)}$$

$$= \sqrt{2.2\ \text{Cos}^2\theta}$$

$$= 2\ \text{Cos}\ \theta$$

16.Prove that Cot 4x (Sin 5x + Sin 3x) = Cot x (Sin 5x – Sin 3x)

Ans.L. H. S = Cot 4x (Sin 5x + Sin 3x)

$$= \frac{\text{Cos}\ 4x}{\text{Sin}\ 4x}\left[2\ \text{Sin}\ \frac{5x+3x}{2}.\ \text{Cos}\ \frac{5x-3x}{2}\right]$$

$$= \frac{\text{Cos } 4x}{\cancel{\text{Sin } 4x}} \; 2 \, \cancel{\text{Sin } 4x} . \text{ Cos } x$$

$$= 2 \text{ Cos } 4x. \text{ Cos } x$$

R. H. S = Cot x (Sin 5x – Sin 3x)

$$= \frac{\text{Cos x}}{\text{Sin x}} \left[2 \text{ Cos } \frac{5x+3x}{2} . \text{ Sin } \frac{5x-3x}{2} \right]$$

$$= \frac{\text{Cos x}}{\cancel{\text{Sin x}}} \left[2 \text{ Cos } 4x . \cancel{\text{Sin x}} \right]$$

$$= 2 \text{ Cos } 4x. \text{ Cos } x$$

L. H. S = R. H. S

CBSE Class 12 Mathematics
Important Questions
Chapter 3
Trigonometric Functions

6 Marks Questions

1. Find the general solution of sin2x + sin4x + sin6x = 0

Ans. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}, n \in z$

2. Find the general solution of cos θ cos2 θ cos3θ = $\frac{1}{4}$

Ans. $(2n+1)\frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in z$

3.If Sin α + Sin β = a and Cos α + Cos β = b show that Cos (α + β) = $\frac{b^2 - a^2}{b^2 + a^2}$

Ans. $b^2 + a^2 = (\text{Cos } \alpha + \text{Cos } \beta)^2 + (\text{Sin } \alpha + \text{Sin } \beta)^2$

$= \text{Cos}^2 \alpha + \text{Cos}^2 \beta + 2 \text{ Cos } \alpha . \text{ Cos } \beta + \text{Sin}^2 \alpha + \text{Sin}^2 \beta + 2 Sin \alpha . \text{ Sin } \beta$

$= 1 + 1 + 2 (\text{Cos } \alpha . \text{ Cos } \beta + \text{Sin } \alpha . \text{ Sin } \beta)$

$= 2 + 2 \text{ Cos } (\alpha - \beta) \quad (1)$

$b^2 - a^2 = (\text{Cos } \alpha + \text{Cos } \beta)^2 - (\text{Sin } \alpha + \text{Sin } \beta)^2$

$= (\text{Cos}^2 \alpha - \text{Sin}^2 \beta) + (\text{Cos}^2 \beta - \text{Sin}^2 \alpha) + 2 \text{ Cos } (\alpha + \beta)$

$= \text{Cos } (\alpha + \beta) \text{ Cos } (\alpha - \beta) + \text{Cos } (\beta + \alpha) \text{Cos } (\alpha - \beta) + 2 \text{ Cos } (\alpha + \beta)$

$= 2 \text{ Cos } (\alpha + \beta) . \text{ Cos } (\alpha - \beta) + 2 \text{ Cos } (\alpha + \beta)$

$$= Cos\ (\alpha + \beta)\ \left[2\ \text{Cos}\ (\alpha - \beta) + 2\right]$$

$$= \text{Cos}\ (\alpha + \beta).\ (b^2 + a^2)\ [\text{from}\ (1)$$

$$\frac{b^2 - a^2}{b^2 + a^2} = \text{Cos}\ (\alpha + \beta)$$

4. Prove that Cos α + Cos β + Cos γ + Cos (α + β + γ)

$$= 4\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right).\ \text{Cos}\left(\frac{\gamma+\alpha}{2}\right)$$

Ans.L. H. S.

$$= \text{Cos}\ \alpha + \text{Cos}\ \beta + \text{Cos}\ \gamma + \text{Cos}\ (\alpha + \beta + \gamma)$$

$$= \text{Cos}\ \alpha + \text{Cos}\ \beta) + \left[\text{Cos}\ \gamma + \text{Cos}\ (\alpha + \beta + \gamma)\right]$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\ \text{Cos}\left(\frac{\alpha+\beta+\gamma+\gamma}{2}\right).\ \text{Cos}\left(\frac{\alpha+\beta+\gamma-\gamma}{2}\right)$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[\text{Cos}\left(\frac{\alpha-\beta}{2}\right) + \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\ \text{Cos}\left(\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right).\ \text{Cos}\left(\frac{\frac{\alpha+\beta+2\gamma}{2}-\frac{\alpha-\beta}{2}}{2}\right)\right]$$

$$= 2\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\ \text{Cos}\left(\frac{\alpha+\gamma}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right)\right]$$

$$= 4\ \text{Cos}\left(\frac{\alpha+\beta}{2}\right).\ \text{Cos}\left(\frac{\beta+\gamma}{2}\right).\ \text{Cos}\left(\frac{\gamma+\alpha}{2}\right)$$

5.Prove that Sin3x +Sin2x-Sin2x=4Sinx.Cos $\frac{x}{2}$. Cos $\frac{3x}{2}$

Ans.

$$(\sin 3x - \sin x) + \sin 2x$$

$$= 2\cos\left(\frac{3x+x}{2}\right).\sin\left(\frac{3x-x}{2}\right) + \sin 2x$$

$$= 2\cos 2x.\sin x + \sin 2x$$

$$= 2\cos 2x.\sin x + 2\sin x.\cos x$$

$$= 2\sin x[\cos 2x + \cos x]$$

$$= 2\sin x\left[2\cos x\frac{3x}{2}.\cos\frac{x}{2}\right]$$

$$= 4\sin x\cos\frac{3x}{2}\cos\frac{x}{2}$$

6.Prove that 2Cos $\frac{\pi}{13}$.Cos $\frac{9\pi}{13}$ + Cos $\frac{3\pi}{13}$ + Cos $\frac{5\pi}{13}$ =0

Ans.L.H.S.

$$= 2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\left(\frac{\pi}{13} + \frac{9\pi}{13}\right) + \cos\left(\frac{\pi}{13} - \frac{9\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= -\cos\frac{3\pi}{13} - \cos\frac{5\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= 0$$

7. Find the value of tan (α + β) Given that

$$\text{Cot } \alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right) \text{ and Sec } \beta = -\frac{5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$$

Ans.

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha . \tan\beta} \quad (1)$$

$$\text{Cot } \alpha = \frac{1}{2},$$

$$\Rightarrow \tan\alpha = 2$$

$$1 + \tan^2\beta = \text{Sec}^2\beta$$

$$1 + \tan^2\beta = \left(\frac{-5}{3}\right)^2 \quad \left[\because \text{Sec } \beta = \frac{-5}{3} 1\right)$$

$$\tan\beta = \pm \frac{4}{3}$$

$$\tan\beta = -\frac{4}{3} \left[\because \beta \in \left(\frac{\pi}{2} / x\right)\right.$$

put $\tan\alpha$, and $\tan\beta$ in eq. (

$$\tan(\alpha+\beta) = \frac{2-\frac{4}{3}}{1-2\left(-\frac{4}{3}\right)}$$

$$= \frac{2}{11}$$

8.Prove that $\frac{\text{Sec } 8A - 1}{\text{Sec } 4A - 1} = \frac{\tan 8A}{\tan 2A}$

Ans.L. H. S=

$$\frac{\frac{1}{Cos\ 8A}-1}{\frac{1}{\text{Cos } 4A}-1}$$

$$= \frac{1 - \text{Cos } 8A}{1 - \text{Cos } 4A} \times \frac{\text{Cos } 4A}{\text{Cos } 8A}$$

$$= \frac{2 \text{ Sin}^2 4A}{2 \text{ Sin}^2 2A} . \frac{\text{Cos } 4A}{\text{Cos } 8A}$$

$$= \frac{(2 \text{ Sin } 4A. \text{ Cos } 4A). \text{ Sin } 4A}{2 \text{ Sin}^2 2A. \text{ Cos } 8A}$$

$$= \frac{\text{Sin } 8A. \left(\cancel{2}\ \cancel{\text{Sin}}\ \cancel{2}A\ .\ \text{Cos } 2A\right)}{\cancel{2}\ \cancel{\text{Sin}}^{\cancel{2}}\ 2A.\ \text{Cos } 8A}$$

$$= \frac{\text{Sin } 8A \text{ Cos } 2A}{\text{Sin } 2A. \text{ Cos } 8A}$$

$$= \frac{\tan 8A}{\tan 2A}$$

9.Prove that $\text{Cos}^2 x + \text{Cos}^2\left(x+\frac{\pi}{3}\right)+Cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$

Ans.L. H. S = $\frac{1+\text{Cos }2x}{2}+\frac{1+\text{Cos}\left(2x+\frac{2\pi}{3}\right)}{2}+\frac{1+\text{Cos}\left(2x-\frac{2\pi}{3}\right)}{2}$

$=\frac{1}{2}\left[1+1+1+\text{Cos }2x+\text{Cos}\left(2x+\frac{2\pi}{3}\right)+\text{Cos}\left(2x-\frac{2\pi}{3}\right)\right]$

$=\frac{1}{2}\left[3+Cos2x+Cos\left(2x+\frac{2\pi}{3}\right)+Cos\left(2x-\frac{2\pi}{3}\right)\right]$

$=\frac{1}{2}\left[3+Cos2x+2Cos\left(\frac{2x+\frac{2\pi}{3}+2x-\frac{2\pi}{3}}{2}\right).\text{Cos}\left(\frac{2x+\frac{2\pi}{3}-2x+\frac{2\pi}{3}}{3}\right)\right]$

$=\frac{1}{2}\left[3+\text{Cos }2x+2\text{ Cos }2x.\text{ Cos }\frac{4\pi}{6}\right]$

$=\frac{1}{2}\left[3+\text{Cos }2x+2\text{ Cos }2x.\text{ Cos }\frac{2\pi}{3}\right]$

$=\frac{1}{2}\left[3+\text{Cos }2x+2\text{ Cos }2x.\text{ Cos}\left(\pi-\frac{\pi}{3}\right)\right]$

$=\frac{1}{2}\left[3+\text{Cos }2x+2\text{ Cos }2x.\left(\frac{-1}{2}\right)\right]$

$=\frac{3}{2}$.

10..Prove that Cos 2x. Cos $\frac{x}{2}$ - $\text{Cos }3x.\text{ Cos }\frac{9x}{2}=\text{Sin }5x\text{ Sin }\frac{5x}{2}$

Ans.L. H. S = $\frac{1}{2}\left[2\text{ Cos }2x\text{ Cos }\frac{x}{2}-2\text{Cos }3x.\text{ Cos }\frac{9x}{2}\right]$

$=\frac{1}{2}\left[\text{Cos}\left(2x+\frac{x}{2}\right)+Cos\left(2x-\frac{x}{2}\right)-Cos\left(\frac{9x}{2}+3x\right)-Cos\left(\frac{9x}{2}-3x\right)\right]$

$$= \frac{1}{2}\left[Cos\frac{5x}{2} + \not{C}\not{o}\not{x}\frac{\not{3}x}{\not{2}} - Cos\frac{15x}{2} - \not{C}\not{o}\not{x}\frac{\not{3}x}{\not{2}} \right]$$

$$= \frac{1}{2}\left[Cos\frac{5x}{2} - Cos\frac{15x}{2} \right]$$

$$= \frac{1}{2}\left[-2Sin\left(\frac{\frac{5x}{2}+\frac{15x}{2}}{2} \right).Sin\left(\frac{\frac{5x}{2}-\frac{15x}{2}}{2} \right) \right]$$

$$= \text{ - Sin 5x. Sin } \left(\frac{-5x}{2} \right)$$

$$= \text{ Sin 5x. Sin } \frac{5x}{2}.$$

11.Prove that Cos 20^0. Cos 40^0. Cos 60^0 Cos 80^0 = $\frac{1}{16}$

Ans.L. H. S = Cos 20^0. Cos 40^0. Cos 60^0. Cos 80^0.

= Cos 60. Cos 20^0. Cos 40^0. Cos 80.

$$= \frac{1}{2}.\frac{1}{2} \text{ Cos}40^0 \text{ (2 Cos20. Cos80)}^0$$

$$= \frac{1}{4}\text{Cos}40^0 \text{ [Cos (80 + 20) + Cos (80 - 20)]}$$

$$= \frac{1}{4}\text{Cos}40^0 \left[\text{Cos } 100^0 + \text{Cos } 60 \right]$$

$$= \frac{1}{4}\text{Cos}40^0 \left[\text{Cos } 100 + \frac{1}{2} \right]$$

$= \frac{1}{8}\left(2\text{ Cos }100.\text{ Cos }40^0\right) + \frac{1}{8}\text{ Cos }40$

$= \frac{1}{8}\left[\text{Cos }(100 + 40^0) + \text{Cos }(100 - 40^0)\right] + \frac{1}{8}\,Cos\,40^0$

$= \frac{1}{8}\left[\text{Cos }140^0 + \text{Cos }60\right] + \frac{1}{8}\text{ Cos }40^0$

$= \frac{1}{8}\left[\text{Cos }140 + \frac{1}{2}\right] + \frac{1}{8}\text{ Cos }40^0$

$= \frac{1}{8}\text{ Cos }140^0 + \frac{1}{16} + \frac{1}{8}\text{ Cos }40^0$

$= \frac{1}{8}\text{ Cos }(180 - 40) + \frac{1}{16} + \frac{1}{8}\text{ Cos }40^0$

$= -\frac{1}{8}\,\not{C}\not{o}\not{s}\,\not{4}\not{0}^{\not{0}} + \frac{1}{16} + \frac{1}{8}\,\not{C}\not{o}\not{s}\,\not{4}\not{0}^{\not{0}}$

$= \frac{1}{16}$

12.If tan x = $\frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, Find the value of Sin $\frac{x}{2}$, Cos $\frac{x}{2}$ and tan $\frac{x}{2}$.

Ans.$\pi < x < \frac{3\pi}{2}$ [Given

Cos x is – tive

$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Sin $\frac{x}{2}$ is + tive and Cos $\frac{x}{2}$ is - tive.

$1 + \tan^2 x = \text{Sec}^2 x \;\; \frac{5}{4}$

$$1 + \left(\frac{3}{4}\right)^2 = \mathrm{Sec}^2 x$$

$$\mathrm{Sec}\ x = \pm$$

$$\mathrm{Cos}\ x = \pm\ \frac{5}{4}$$

$$\mathrm{Cos}\ x = -\ \frac{5}{4} \quad \left[\because\ \pi < x < \frac{3\pi}{2}\right.$$

$$\mathrm{Sin}\ \frac{x}{2} = \sqrt{\frac{1 - Cosx}{2}}$$

$$= \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

$$\mathrm{Cos}\ \frac{x}{2} = -\sqrt{\frac{1 + Cosx}{2}}$$

$$= -\sqrt{\frac{1 - \frac{4}{5}}{2}} = -\sqrt{\frac{1}{10}} = \frac{-1}{\sqrt{10}}$$

$$\tan\ \frac{x}{2} = \frac{\frac{3}{\sqrt{10}}}{\frac{-1}{\sqrt{10}}} = -3$$