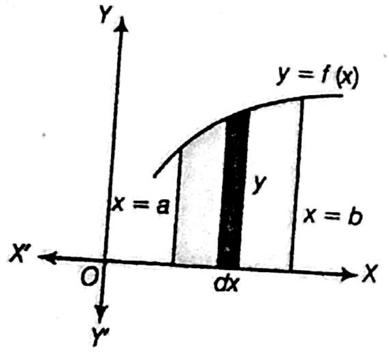
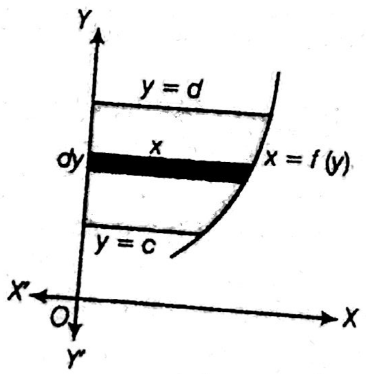
Application of Integrals

Concept Related to Bounded Area

1. The area bounded by the curve $y = f(x)$, the X-axis and the ordinates $x = a$ and $x = b$ is calculated as Area $= \int_a^b y\,dx = \int_a^b f(x)\,dx$.



2. The area bounded by the curve $x = f(y)$, the Y-axis and the lines $y = c$ and $y = d$ is calculated as Area $= \int_c^d x\,dy = \int_c^d f(y)\,dy$.

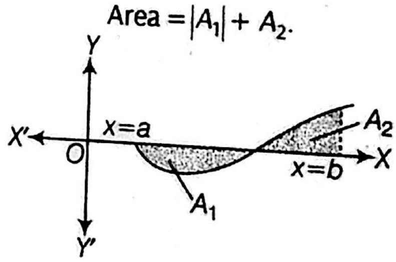


NOTE (i) If area lies below X-axis or left of Y-axis, then it is negative and in such a case, we take its absolute value because the area of region is always positive,

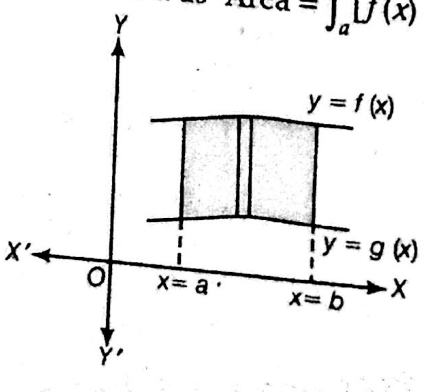
i.e. $A = \left|\int_a^b f(x)dx\right|$ or $A = \left|\int_c^d f(y)dy\right|$.

(ii) It may happen that some portion of the curve is above X-axis and some is below the X-axis, then the area bounded by the curve $y = f(x)$, X-axis and the ordinates $x = a$ and $x = b$ is given by

Area $= |A_1| + A_2$.

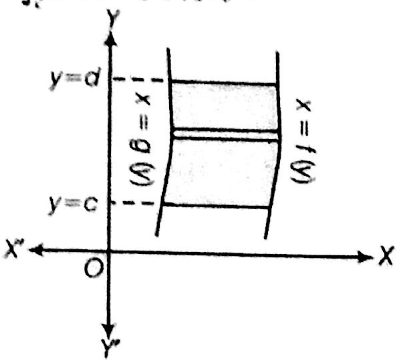


3. The area bounded by the two curves $y = f(x)$ and $y = g(x)$ such that $0 \le g(x) \le f(x)$, $\forall\ x \in [a, b]$, the X-axis and the lines $x = a$ and $x = b$ is calculated as Area $= \int_a^b [f(x) - g(x)]\,dx$.



4. The area bounded by the two curves $x = f(y)$ and $x = g(y)$ such that $0 \le g(y) \le f(y)$, $\forall\, y \in [c, d]$, the Y-axis and the lines $y = c$ and $y = d$ is calculated as

Area $= \int_c^d [f(y) - g(y)]\, dy$.



5. The area bounded by the two curves $f(x)$ and $g(x)$ such that $f(x) \ge g(x)$ in $[a, c]$ and $f(x) \le g(x)$ in $[c, b]$, $a < c < b$, is calculated as

Area $= \int_a^c [f(x) - g(x)]\, dx + \int_c^b [g(x) - f(x)]\, dx$.