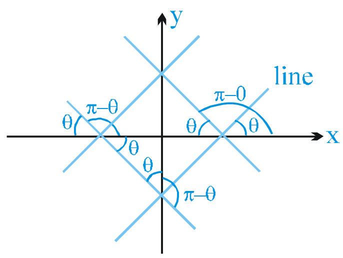
HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

2. Reflecting a graph over the x-axis results in the line M whose equation is ax – by = c, while a reflection through the y-axis results in the line N whose equation is –ax + by = c. Both clearly have slope equal to a/b (from, say, the slope-intercept form of the equation.)



6. $AP = \sqrt{x^2+(y-4)^2}$

$BP = \sqrt{x^2+(y+4)^2}$

$\because$ $|AP - BP| = 6$

$AP - BP = \pm 6$

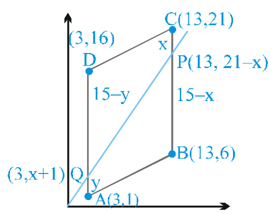
$\sqrt{x^2+(y-4)^2} - \sqrt{x^2+(y+4)^2} = \pm 6$

On squaring we get the locus of P is

$9x^2 - 7y^2 + 63 = 0$

9. as shown $= \dfrac{21-x}{13} = \dfrac{x+1}{3}$

$63 - 3x = 13x + 13$



$16x = 50$

$x = \dfrac{25}{8}$; Hence $m = \left(\dfrac{25}{8}+1\right)\cdot\dfrac{1}{3} = \dfrac{33}{24} = \dfrac{11}{8}$

11. Let (h, k) be the centroid of triangle

$3h = \cos\alpha + \sin\alpha + 1$

$\Rightarrow$ $(3h - 1) = \cos\alpha + \sin\alpha$ ......**(i)**

$3k = \sin\alpha - \cos\alpha + 2$

$\Rightarrow$ $(3k - 2) = \sin\alpha - \cos\alpha$ ......**(ii)**

square & add (i) & (ii)

$9(x^2 + y^2) + 6(x - 2y) = -3$

12. $D = 0$

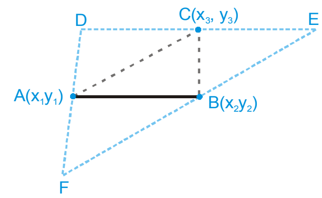
$x^2 = 4(x - y)^2$

$x = 2(x - y)$ or $x = -2(x - y)$

$x = 2y$ or $3x = 2y$

$\Rightarrow$ line pair with slope 3/2 and 1/2 $\Rightarrow$ D

13.



ABCD, ABEC, ACBF are three possible parallelograms.

14. $\Delta = \dfrac{1}{2}\begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} = 0$

$\Rightarrow$ $(2a - c)(2b - c) - (3a - c)(3b - c) = 0$

$\Rightarrow$ $4ab - 2ac - 2bc + c^2 - (9ab - 3ac - 3bc + c^2) = 0$

$\Rightarrow$ $ac + bc - 5ab = 0$

$\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{5}{c}$ $\Rightarrow$ $\dfrac{1}{a} + \dfrac{1}{b} = 2\left(\dfrac{5}{2c}\right)$

$\therefore$ a, $\dfrac{2c}{5}$, b are in H.P.

17. $(2y - x)(y - mx) = mx^2 - xy(2m + 1) + 2y^2 = 0$

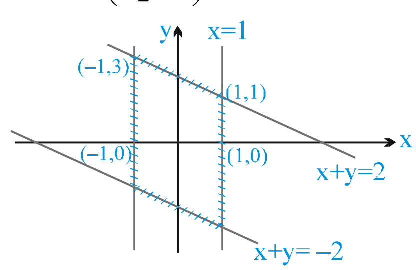
$\Rightarrow$ the equation to the pair of bisectors are :

$\dfrac{x^2 - y^2}{m - 2} = \dfrac{-2xy}{2m+1} \equiv 12x^2 - 7xy - 12y^2$

$\Rightarrow$ $\dfrac{2m+1}{12} = \dfrac{2(m-2)}{-7}$ or $38m = 41$ $\Rightarrow$ $m = \dfrac{41}{38}$

19. Figure is a parallelogram

Area $= 2\left(\dfrac{1+3}{2}\cdot 2\right) = 8$ Ans.



24. $x^2+2\sqrt{2}\,xy+2y^2+4x+4\sqrt{2}\,y+1=0$

$\Rightarrow$ $(x+\sqrt{2}\,y+p)(x+\sqrt{2}\,y+q)=0$

$p+q=4$

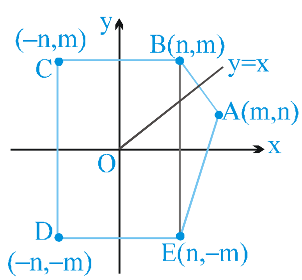
$\Rightarrow$ $pq=1$

Distance between parallel lines is $\left|\dfrac{p-q}{\sqrt{3}}\right|$ =

$$\frac{\sqrt{(p+q)^2-4pq}}{\sqrt{3}}=\frac{\sqrt{16-4}}{\sqrt{3}}=2$$

26. Area of rectangle $BCDE=4mn$

Area of $\Delta ABC=\dfrac{2m(m-n)}{2}$



$=m^2-mn$

$\therefore$ area of pentagon $=4mn+m^2-mn$

$=m^2+3mn$

30. Here, $x+2y-3=0$ and $3x+4y-7=0$ intersect $(1,1)$, which does not satisfy $2x+3y-4=0$ and $4x+5y-6=0$. Also, $3x+4y-7=0$ and $2x+3y-4=0$ intersect at $(5,-2)$ which does not satisfy $x+2y-3=0$ and $4x+5y-6=0$

Intersection point of $x+2y-3=0$ and $2x+3y-4=0$ is $(-1,2)$ which satisfy $4x+5y-6=0$.

Hence, only three lines are concurrent.

32. $m_1+m_2=-10$ $\Rightarrow$ $m_1m_2=\dfrac{a}{1}$

given $m_1=4m_2$ $\Rightarrow$ $m_2=-2,\ m_1=-8,\ a=16$

34. Homogenizing the curve with the help of the straight line.

$5x^2+12xy-6y^2+4x(x+ky)-2y(x+ky)+3(x+ky)^2=0$

$12x^2+(10+4k+6k)\,xy+(3k^2-2k-6)y^2=0$

Lines are equally inclined to the coordinate axes

$\therefore$ coefficient of $xy=0$

$\Rightarrow$ $10k+10=0$ $\Rightarrow$ $k=-1$

35. Curve passing through points of intersection of $S_1=0$ & $S_2=0$ is

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)+\lambda(x^2+y^2+2gx+2f+c)=0$$

above equation represents a pair of straight lines. They are parallel to the lines $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\lambda(x^2+y^2)=0$ which represents a pair of lines equally inclined to axis as the term containing xy is absent

36. Let the third PH reading is x

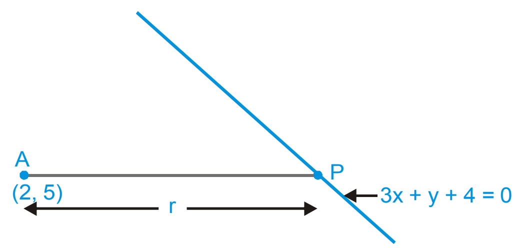
$$7.4<\frac{7.48+8.42+x}{3}<8.2$$

$22.2<15.90+x<24.6$

$6.3<x<8.7$

PH range should be in between 6.3 to 8.7

37. Let distance be 'r'.



Co-ordinates of 'P' are

$(2+r\cos\theta,\ 5+r\sin\theta)$ where $\tan\theta=\dfrac{3}{4}$

which lies on the line $3x+y+4=0$

$3(2+r\cos\theta)+5+r\sin\theta+4=0$

$$r\left(3.\frac{4}{5}+\frac{3}{5}\right)+15=0$$

$\Rightarrow$ $r=-\dfrac{15}{3}=-5$

but distance can not be negative

$\therefore$ $r=5$

EXERCISE - 2

Part # I : Multiple Choice

8. Use the condition of concurrency for three lines

13. The lines will pass through (4, 5) & parallel to the bisectors between them

$$\frac{3x-4y-7}{5} = \pm \frac{12x-5y+6}{13}$$

by taking + sign, we get $\quad 21x+27y+121=0$

Now by taking – sign, we get $\quad 99x-77y-61=0$

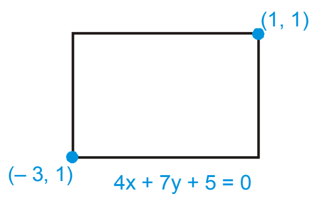
so slopes of bisectors are $-\frac{7}{9}, \frac{9}{7}$

Equation of lines are

$y-5=\frac{-7}{9}(x-4)$ and $y-5=\frac{9}{7}(x-4)$

$\Rightarrow \quad 7x+9y=73$ and $9x-7y=1$

18. Line $\perp$ to $4x+7y+5=0$ is



$7x-4y+\lambda=0$

It passes through (– 3, 1) and (1, 1)

$-11-4+l=0 \quad \Rightarrow \quad l=25$

$7-4+l=0 \quad \Rightarrow \quad l=-3$

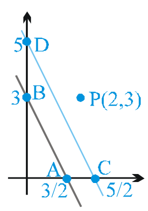
Hence lines are $7x-4y+25=0$, $7x-4y-3=0$

line || to $4x+7y+5=0$ passing through (1, 1) is $4x+7y+l=0$

$\Rightarrow \quad l=-11$

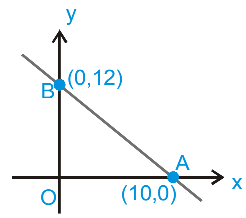
$\Rightarrow \quad 4x+7y-11=0$

Part # II : Assertion & Reason

3. 

; P lies outside the quadrilateral

4. S_1 : equation of such line is $\frac{x}{5}+\frac{y}{6}=2$



$\Rightarrow \quad$ Area of $\Delta OAB = \frac{1}{2} \times 10 \times 12 = 60$

S_2 : In this situation area obtained is least infact.

6. $AB = \sqrt{(8)^2+(19)^2} = \sqrt{425}$; $AC = \sqrt{(16)^2+(13)^2}$

$\therefore \quad \Delta$ is isosceles

8. $ax^3+bx^2y+cxy^2+dy^3=0$

since this is homogeneous pair represent there straight lines passing through origin

$ax^3+bx^2y+cxy^2+dy^3=(y-m_1x)(y-m_2x)(y-m_3x)$

or $\quad$ put $y=mx$ in given equation we get

$m^3d+cm^2+bm+a=0$

$$m_1+m_2+m_3=\frac{-c}{d}$$

$$m_1m_2+m_2m_3+m_3m_1=\frac{+b}{d}$$

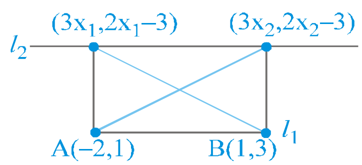
$$m_1m_2m_3=\frac{+a}{d}$$

given two lines + hence $m_1m_2=-1 \Rightarrow m_3=a/d$

eliminate m_3 from remaining equation

10. $m_{l_1}=\frac{2}{3}$

$$m_{l_2}=\frac{2(x_2-x_1)}{3(x_2-x_1)}=\frac{2}{3}$$



$$A=\frac{1}{2}\begin{vmatrix}-2 & 1 & 1\\ 1 & 3 & 1\\ 3x & (2x-3) & 1\end{vmatrix}=8$$

EXERCISE - 3

Part # I : Matrix Match Type

1. **(A)** Let the lines $4x + 5y = 0$ and $7x + 2y = 0$ represents the sides AB & AD of the parallelogram ABCD, then the vertices of

A, B, D are (0,0), $\left(\frac{5}{3}, -\frac{4}{3}\right)$ and $\left(-\frac{2}{3}, \frac{7}{3}\right)$ respectively

the mid point of BD is $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\therefore$ the equation of the line passing through $\left(\frac{1}{2}, \frac{1}{2}\right)$ and (0, 0) will be $x - y = 0$ which is the required equation of the other diagonal

So $a = 1, b = -1, c = 0$

$\therefore$ $a + b + c = 0$

(B) Joint equation of lines OA & OB, O being the origin will be

$2x^2 - by^2 + (2b - 1)\,xy - (x + by)(-2x+by) = 0$

$\Rightarrow$ $4x^2 - (b + b^2)y^2 + (3b - 1)xy = 0$

If these lines are perpendicular then

$4 - b - b^2 = 0 \Rightarrow b + b^2 = 4$

(C) Equation of line passing through intersection of $4x + 3y = 12$ and $3x + 4y = 12$ will be

$(4x + 3y - 12) + \lambda(3x + 4y - 12) = 0$

If passes through (3, 4) $\Rightarrow$ $(12 + \lambda(13)) = 0$

$\Rightarrow$ $\lambda = -\frac{12}{13}$

$\therefore$ Equation of the required line

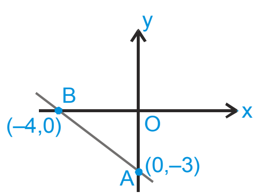
$16x - 9y - 12 = 0$

length of intercepts on x and y axes are $\frac{3}{4}$ and $\frac{4}{3}$

So $ab = 1$

2. **(A)** Slope of such line is ± 1

(B) area of $\Delta OAB = \frac{1}{2} \times 3 \times 4 = 6$ sq. units



(C) To represent pair of straight lines $\begin{vmatrix} 2 & -1 & -3 \\ -1 & -1 & 3 \\ -3 & 3 & c \end{vmatrix} = 0$

$\Rightarrow$ $c = 3$

(D) Lines represented by given equation are $x + y + a = 0$ and $x + y - 9a = 0$

$\therefore$ distance between these parallel lines is

$= \frac{10a}{\sqrt{2}} = 5\sqrt{2}a$

Part # II : Comprehension

Comprehension # 5

1. $d(OR) = d(AR)$

$|x - 0| + |y - 0| = |x - 1| + |y - 2|$

$x + y = |x - 1| + |y - 2|$ $(\because x > 0, y > 0)$

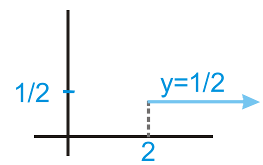
$x + y = -x + 1 - y + 2$

$2x + 2y = 3.$ $(\because 0 \le x < 1 \ \&\ 0 \le y < 2)$

2. $d(OS) = d(BS)$

$|x - 0| + |y - 0| = |x - 2| + |y - 3|$

$x + y = x - 2 + 3 - y$ $(\because x \ge 2 \ \&\ 0 \le y < 3).$



$y = 1/2.$

which is an infinite ray

3. $d(TO) = d(TC)$

$|x - 0| + |y - 0| = |x - 4| + |y - 3|$

$x + y = |x - 4| + |y - 3|$

Case : I $0 \le x < 4$ & $0 \le y < 3.$

$x + y = -x + 4 - y + 3$

$x + y = 7/2.$

Case : II $0 \le x < 4$ & $y \ge 3.$

$x + y = -x + 4 + y - 3$

$x = 1/2.$

Case : III $x \ge 4$ & $0 \le y < 3.$

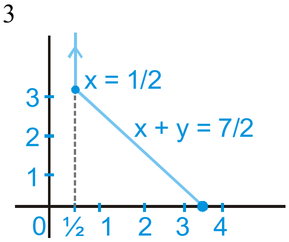
$x + y = x - 4 - y + 3$

$y = -1/2.$

Case:IV $x \ge 4$ & $y \ge 3.$

$x + y = x - 4 + y - 3$

$0 = -7$ (so rejected)



Comprehension # 6

Slopes of the lines

$3x + 4y = 5$ is $m_1 = -\frac{3}{4}$

and $4x - 3y = 15$ is $m_2 = \frac{4}{3}$

$\because \quad m_1 m_2 = -1$

$\therefore$ given lines are perpendicular and $\angle A = \frac{\pi}{2}$

Now required equation of BC is

$(y - 2) = \frac{m \pm \tan(\pi/4)}{1 \mp m\tan(\pi/4)}(x - 1)$ ......(i)

where m = slope of AB $= -\frac{3}{4}$

$\therefore$ equation of BC is (on solving (1))

$x - 7y + 13 = 0$ and $7x + y - 9 = 0$

$L_1 \equiv x - 7y + 13 = 0$

$L_2 \equiv 7x + y - 9 = 0$

1. $c + f = 4$

2. Equation of a straight line through (2, 3) and inclined at an angle of $(\pi/3)$ with y-axis $((\pi/6)$ with x-axis) is

$\frac{x-2}{\cos(\pi/6)} = \frac{y-3}{\sin(\pi/6)} \Rightarrow x - \sqrt{3}y = 2 - 3\sqrt{3}$

Points at a distance $c + f = 4$ units from point P are

$(2 + 4\cos(\pi/6), 3 + 4\sin(\pi/6)) \equiv (2 + 2\sqrt{3}, 5)$

and $(2 - 4\cos(\pi/6), 3 - 4\sin(\pi/6)) \equiv (2 - 2\sqrt{3}, 1)$

only **(A)** is true out of given options

3. Let required line be $x + y = a$

which is at $|b - 2a - 1| = |5 - 4 - 4\sqrt{3} - 1| = 4\sqrt{3}$ units from origin

$\therefore$ required line is $x + y - 4\sqrt{6} = 0$ (since intercepts are on positive axes only)

EXERCISE - 4
Subjective Type

3. $ax^2 + 2hxy + by^2 = (y - m_1x)(y - m_2x)$

given that $m_2 = m_1^n$

Hence $m_1 + m_2 = -\frac{2h}{b} \Rightarrow m_1 + m_1^n = -\frac{2h}{b}$

$\Rightarrow m_1.m_1^n = \frac{a}{b} \qquad \Rightarrow m_1 = \left(\frac{a}{b}\right)^{\frac{1}{1+n}}$

Eliminate m_1 from both.

4. The combined equation of AB and AD is

$S_1 \equiv ax^2 + 2hxy + by^2 = 0$

Now equation of lines through (p,q) and parallel to $S_1 = 0$ is

$S_2 \equiv a(x-p)^2 + 2h(x-p)(y-q) + b(y-q)^2 = 0$

Hence equation of diagonal BD is $S_1 - S_2 = 0$

$\Rightarrow (2x - p)(ap + hq) + (2y - q)(hp + bq) = 0$

5. Consider a line $\ell x + my + n = 0$

point $\left(\frac{r^3}{r-1}, \frac{r^2-3}{r-1}\right)$ lies on the above line

$\therefore \ell\left(\frac{r^3}{r-1}\right) + m\left(\frac{r^2-3}{r-1}\right) + n = 0$

$\ell r^3 + mr^2 + nr - (3m + n) = 0$

a, b, c are the roots of the equation.

$a+b+c = \frac{-m}{\ell}$, $ab+bc+ca = \frac{n}{\ell}$, $abc = \frac{3m+n}{\ell}$

Now taking LHS

$3(a + b + c) = \frac{-3m}{\ell}$

RHS

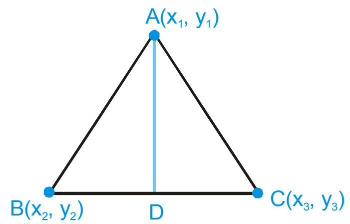
$ab + bc + ca - abc = \frac{n}{\ell} - \left(\frac{3m+n}{\ell}\right) = -\frac{3m}{\ell}$

10. (i) D is mid point of BC Hence co-ordinates of D are

$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Therefore, equation of the median AD is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{x_2+x_3}{2} & \frac{y_2+y_3}{2} & 1 \end{vmatrix} = 0$$



Applying $R_3 \to 2R_3$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2+x_3 & y_2+y_3 & 2 \end{vmatrix} = 0$$

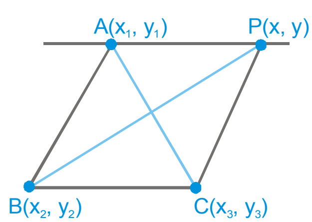
$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

(using the addition property of determinants)

(ii) Let P(x, y) be any point on the line parallel to BC

Area of ΔABP = Area of ΔACP

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

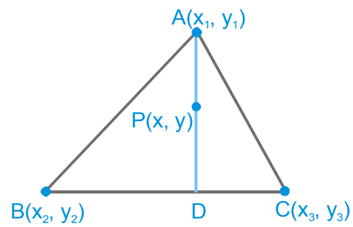
This gives the equation of line AP.

(iii) Let AD be the internal bisector of angle A,

$$\therefore \frac{BD}{DC} = \frac{BA}{CA} = \frac{c}{b}$$

$$\therefore D \equiv \left(\frac{cx_3+bx_2}{c+b}, \frac{cy_3+by_2}{c+b}\right)$$

Let P(x,y) be any point on AD then P,A,D are collinear



$$\therefore \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{cx_3+bx_2}{b+c} & \frac{cy_3+by_2}{b+c} & 1 \end{vmatrix} = 0$$

$R_3 \to (b+c)\, R_3$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ cx_3+bx_2 & cy_3+by_2 & b+c \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ cx_3 & cy_3 & c \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ bx_2 & by_2 & b \end{vmatrix} = 0$$

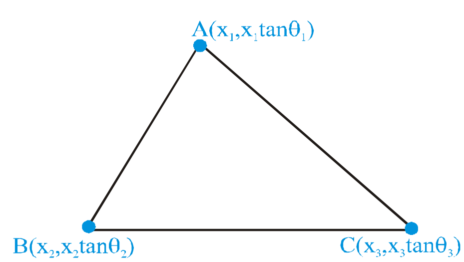
(Addition property)

$$\Rightarrow c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

This is the equation of AD.

11. Circumcentre is origin

$\therefore$ $OA^2 = OB^2 = OC^2$



$x_1^2 + x_1^2 \tan^2\theta_1 = x_2^2 + x_2^2 \tan^2\theta_2$

$= x_3^2 + x_3^2 \tan^2\theta_3 = r^2$

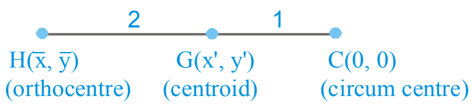
$x_1 = r\cos\theta_1, x_2 = r\cos\theta_2, x_3 = r\cos\theta_3$

$\therefore$ co-ordinate of vertices of the triangle become -

$A(r\cos\theta_1, r\sin\theta_1), B(r\cos\theta_2, r\sin\theta_2),$

$C(r\cos\theta_3, r\sin\theta_3)$

$$x' = \frac{\Sigma r\cos\theta_1}{3}, \quad y' = \frac{\Sigma r\sin\theta_1}{3}$$



Now, $x' = \dfrac{0+\bar{x}}{3}$

$$\bar{x} = r(\cos\theta_1 + \cos\theta_2 + \cos\theta_3)$$

$$\bar{y} = r(\sin\theta_1 + \sin\theta_2 + \sin\theta_3)$$

$$\therefore \frac{\bar{x}}{\bar{y}} = \frac{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}$$

13. Let point of intersection of lines is (x, y) using parametric form of line

$$\frac{x-2}{\cos\theta} = \frac{y-1}{\sin\theta} = 3$$

$x = 3\cos\theta + 2,\ y = 3\sin\theta + 1$

This point satisfy equation of line

$$4y - 4x + 4 + 3\sqrt{2} + 3\sqrt{10} = 0$$

$$12(\sin\theta - \cos\theta) = -3\sqrt{2}(1+\sqrt{5})$$

$$\sin\theta - \cos\theta = -\frac{(1+\sqrt{5})}{2\sqrt{2}}$$

$$\Rightarrow \cos(\theta + 45°) = -\frac{(1+\sqrt{5})}{4} \qquad \text{...... (i)}$$

$$\Rightarrow \cos(\theta + 45°) = \cos(180° - 36°)$$

$$\Rightarrow \cos(\theta + 45°) = \cos 144° \quad \Rightarrow \quad \theta = 99°$$

Now from (i)

$$\cos(\theta + 45°) = \cos(180° + 36°) \quad \Rightarrow \quad \theta = 171°$$

14. $y + 2at = tx - at^3$

slope = t.

Let is passes through P(h, k)

$\therefore\ k + 2at = th - at^3$

$at^3 + t(2a - h) + k = 0$ ... **(1)**

$$t_1t_2t_3 = -\frac{k}{a} \qquad \{t_1t_2 = -1\}$$

$$t_3 = \frac{k}{a}$$

Substituting t_3 in (1) we can get the desired locus

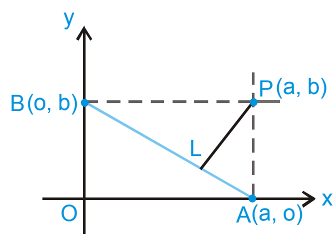
16. $a^2 + b^2 = c^2$ .... **(i)**

Let L is (x_1, y_1)

L is foot of perpendicular from point P(a, b) on line AB

equation of AB is $bx + ay - ab = 0$

$$\Rightarrow \frac{x_1 - a}{b} = \frac{y_1 - b}{a} = \frac{-(ab + ab - ab)}{a^2 + b^2}$$



$$\frac{x_1 - a}{b} = \frac{y_1 - b}{a} = \frac{-ab}{c^2}$$

$\Rightarrow x_1 = a - \dfrac{ab^2}{c^2} = \dfrac{a(c^2 - b^2)}{c^2} = a^3/c^2 \Rightarrow a^3 = c^2x_1$ ..... **(ii)**

similarly $b^3 = c^2y_1$ ..... **(iii)**

using these relations **(ii)** & **(iii)** in equation **(i)**, we get required locus.

20. Since A (4, 2) and B (2,4) both lies same side of

$3x + 2y + 10 = 0$

(i) $PA + PB \geq AB$

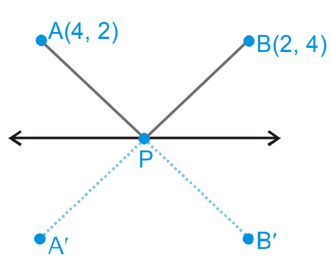
$PA + PB' \geq AB \quad \Rightarrow \quad PA + PB = PA + PB'(\text{min.}) = AB$

Hence A, P, B′ are collinear.

Image of B(2,4) in $3x + 2y + 10 = 0$....is ...**(i)**

$$\frac{x-2}{3} = \frac{y-4}{2} = -2\left(\frac{6+8+10}{3^2+2^2}\right)$$

$$\Rightarrow \quad B'(x, y)\left(-\frac{118}{13}, \frac{-44}{13}\right)$$

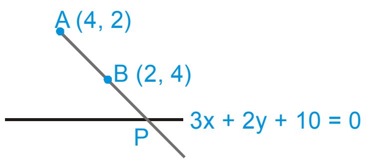


now equation of AB′ is $y - 2 = \dfrac{2+\frac{49}{13}}{4+\frac{118}{13}}(x-4)$

$\Rightarrow 7x - 17y + 6 = 0$ ....**(ii)**

solving **(i)** and **(ii)** we get $\left(-\dfrac{14}{5}, -\dfrac{4}{5}\right)$

(ii) in any triangle.



$|PA - PB| \le AB$

Hence $|PA - PB| = AB$ when P, A, B are collinear

Hence equation of AB is

$y - 2 = -1\,(x - 4)$

$x + y - 6 = 0$ ...... **(i)**

solving **(i)** with $3x + 2y + 10 = 0$

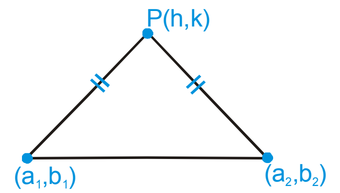
we get $(-22, 28)$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. $(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$

$2h(a_1 - a_2) + 2k(b_1 - b_2) + \left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right) = 0$



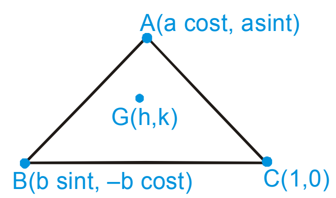
compare with $(a_1 - a_2)x + (b_1 - b_2)\,y + c = 0$

$c = \dfrac{\left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right)}{2}$.

2. $3h - 1 = a\cos t + b\sin t$

$3k = a\sin t - b\cos t$

squaring and add. (Locus)



$(3x - 1)^2 + 9y^2 = a^2 + b^2$

3. $x^2 - 2pxy - y^2 = 0$

pair of angle bisector of this pair $\dfrac{x^2 - y^2}{1-(-1)} = \dfrac{xy}{-p}$

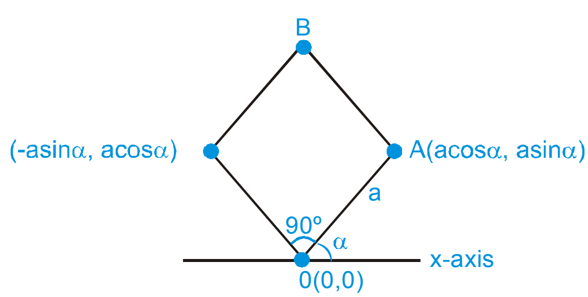
$\Rightarrow \quad x^2 - y^2 + \dfrac{2}{p}\,xy = 0$

compare this bisector pair with $x^2 - 2qxy - y^2 = 0$

$\dfrac{2}{p} = -2q \quad \Rightarrow \quad pq = -1.$

4. Equation of AC

$y - a\sin\alpha = \dfrac{\sin\alpha - \cos\alpha}{\cos\alpha + \sin\alpha}(x - a\cos\alpha)$



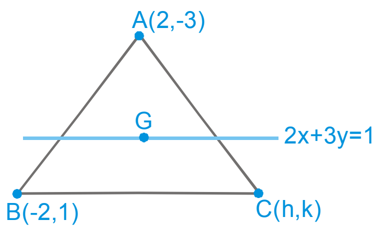
$y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha)$

$= a(\sin\alpha\cos\alpha + \sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha)$

$y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a.$

5. $G\left(\frac{h}{3}, \frac{k-2}{3}\right)$

$\Rightarrow \frac{2h}{3} + (k-2) = 1 \Rightarrow 2h + 3k = 9$



Locus $2x + 3y = 9$.

6. Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

it passes through (4, 3) $\frac{4}{a} + \frac{3}{b} = 1$

sum of intercepts is –1

$\Rightarrow a + b = -1 \qquad \Rightarrow a = -1 - b$

$\Rightarrow \frac{4}{-1-b} + \frac{3}{b} = 1 \Rightarrow 4b - 3 - 3b = -b - b^2$

$\Rightarrow b^2 + 2b - 3 = 0 \qquad \Rightarrow b = -3, 1$

$b = 1, \quad a = -2 \qquad \frac{x}{-2} + \frac{y}{1} = 1$

$b = -3, \quad a = 2 \qquad \frac{x}{2} + \frac{y}{-3} = 1.$

7. $x^2 - 2cxy - 7y^2 = 0$

sum of the slopes $m_1 + m_2 = \frac{2c}{-7}$

Product of slopes $m_1m_2 = \frac{-1}{7}$

given $m_1 + m_2 = 4m_1m_2 \quad \Rightarrow \frac{2c}{-7} = \frac{-4}{7} \quad \Rightarrow c = 2.$

8. Pair $6x^2 - xy + 4cy^2 = 0$ has its one line $3x + 4y = 0$

$\Rightarrow y = \frac{-3x}{4}$

$6x^2 + \frac{3x^2}{4} + 4c\,\frac{9x^2}{16} = 0 \Rightarrow 24x^2 + 3x^2 + 9cx^2 = 0$

$\Rightarrow c = -3.$

9. $ax + 2by + 3b = 0$

$bx - 2ay - 3a = 0$

$$\frac{x}{-6ab + 6ab} = \frac{y}{3b^2 + 3a^2} = \frac{1}{-2a^2 - 2b^2}$$

Hence point of intersection (0, –3/2)

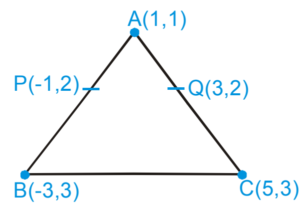
Line parallel to x-axis y = –3/2.

10. $\because$ a, b, c are in H.P. $\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$

given line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$

Clearly line passes through (1, –2).

11. Centroid is $\left(1, \frac{7}{3}\right)$



12. Pair of lines $ax^2 + 2(a+b)xy + by^2 = 0$

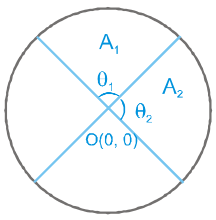
Area of sector $A_1 = \frac{1}{2}r^2\theta_1$

$A_2 = \frac{1}{2}r^2\theta_2$

$\theta_1 + \theta_2 = 180^\circ$

given $A_1 = 3A_2 \quad \Rightarrow \theta_1 = 3\theta_2$

$\Rightarrow \theta_2 = 45^\circ, \theta_1 = 135^\circ$



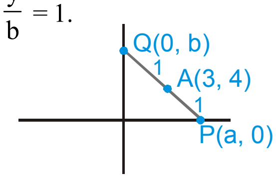
Angle between lines is $= \left|\frac{2\sqrt{(a+b)^2 - ab}}{a+b}\right| = 1$

$\Rightarrow 4\left(a^2 + b^2 + ab\right) = a^2 + b^2 + 2ab$

$\Rightarrow 3a^2 + 3b^2 + 2ab = 0.$

13. Let equation of line is $\frac{x}{a}+\frac{y}{b}=1$.

By section formula



$\frac{a}{2}=3 \quad \Rightarrow \quad a=6$

$\frac{b}{2}=4 \quad \Rightarrow \quad b=8$

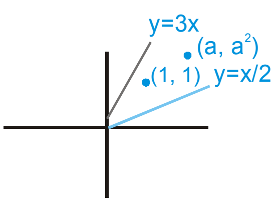
$\frac{x}{6}+\frac{y}{8}=1 \quad \Rightarrow \quad 4x+3y=24.$

14. Since (1, 1) and (a, a^2) Both lies same side with respect to both lines

$a-2a^2<0 \quad \Rightarrow \quad 2a^2-a>0$

$\Rightarrow \quad a(2a-1)>0$

$a \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$



$3a-a^2>0 \quad \Rightarrow \quad a^2-3a<0 \quad \Rightarrow \quad a \in (0, 3)$

Hence after taking intersection $a \in \left(\frac{1}{2}, 3\right)$.

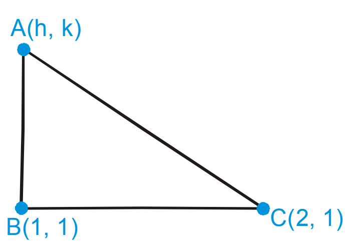
15. $AB=\sqrt{(h-1)^2+(k-1)^2}$

$BC=1$

$AC=\sqrt{(h-2)^2+(k-1)^2}$

$AB^2+BC^2=AC^2$

$\Rightarrow \quad (h-1)^2+(k-1)^2+1=(h-2)^2+(k-1)^2$



$\Rightarrow \quad 2h=2 \Rightarrow h=1$

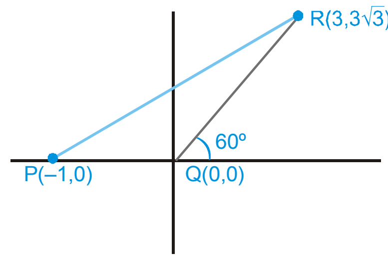
Area of $\Delta ABC=\frac{1}{2}\sqrt{(h-1)^2+(k-1)^2}\times 1=1$

$(K-1)^2=4 \quad \Rightarrow \quad k-1=\pm 2 \quad \Rightarrow \quad k=3,-1.$

16. The line segment QR makes an angle 60º with the positive direction of x-axis.
hence bisector of angle PQR will make 120º with +ve direction of x-axis.

$\therefore$ Its equation

$y-0=\tan 120^\circ (x-0)$



$y=-\sqrt{3}\,x$

$x\sqrt{3}+y=0$

17. Bisector of $x=0$ and $y=0$ is either $y=x$ or $y=-x$

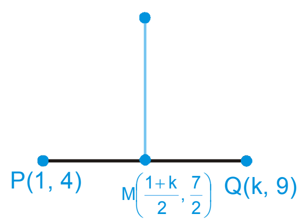
If $y=x$ is Bisector, then

$mx^2+(1-m^2)x^2-mx^2=0 \quad \Rightarrow \quad m+1-m^2-m=0$

$\Rightarrow \quad m^2=1 \quad \Rightarrow \quad m=\pm 1.$

18. Slope of $PQ=\frac{1}{1-k}$

Hence equation of $\perp$ to line PQ



$y-\frac{7}{2}=(k-1)\left(x-\frac{(1+k)}{2}\right)$

Put $x=0$

$y=\frac{7}{2}+\frac{(1-k)(1+k)}{2}=-4$

$7+(1-k^2)=-8$

$\Rightarrow \quad k^2=16 \quad \Rightarrow \quad k=\pm 4.$

Hence possible answer = –4.

19. $p(p^2+1)x-y+q=0$

$(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular for a common line

$\Rightarrow$ lines are parallel $\quad \Rightarrow$ slopes are equal

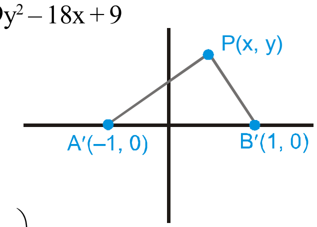
$\therefore \quad \frac{p(p^2+1)}{1}=-\frac{(p^2+1)^2}{(p^2+1)} \quad \Rightarrow \quad p=-1$

20. $\therefore \quad \frac{PA'}{PB'} = \frac{3}{1}$

$\therefore \quad (x+1)^2 + y^2 = 9((x-1)^2 + y^2)$

$x^2 + 2x + 1 + y^2 = 9x^2 + 9y^2 - 18x + 9$

$8x^2 + 8y^2 - 20x + 8 = 0$



$x^2 + y^2 - \frac{10}{4}x + 1 = 0$

$\therefore$ circumcentre $\left(\frac{5}{4}, 0\right)$.

21. $\frac{x}{5} + \frac{y}{b} = 1$

$\frac{13}{5} + \frac{32}{b} = 1 \quad \Rightarrow \quad \frac{32}{b} = -\frac{8}{5} \quad \Rightarrow \quad b = -20$

$\frac{x}{5} - \frac{y}{20} = 1 \quad \Rightarrow \quad 4x - y = 20$

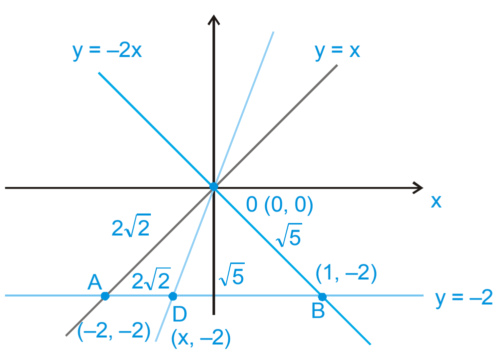
Line K has same slope $\Rightarrow \; -\frac{3}{c} = 4$

$c = -\frac{3}{4} \;\Rightarrow\; 4x - y = -3$

distance $= \frac{23}{\sqrt{17}}$

Hence correct option is (3)

22.



$\therefore \quad AD : DB = 2\sqrt{2} : \sqrt{5}$

$\because \quad$ OD is angle bisector

of angle AOB

$\therefore \quad$ St : 1 true

St. 2 false (obvious)

23. $x + y = |a|$

$ax - y = 1$

If $\quad a > 0$

$x + y = a$

$ax - y = 1$

$x(1 + a) = 1 + a \;$ as $x = 1$

$y = a - 1$

It is in the first quadrant

So $\quad a - 1 \geq 0$

$a \geq 1$

$a \in [1, \infty)$

If $\quad a < 0$

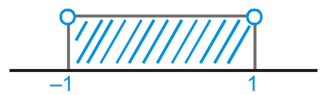
$x + y = -a$

$ax - y = 1$

+

$x(1 + a) = 1 - a$

$x = \frac{1-a}{1+a} > 0 \quad \Rightarrow \quad \frac{a-1}{a+1} < 0$



...... (i)

$y = -a - \frac{1-a}{1+a}$

$= \frac{-a - a^2 - 1 + a}{1+a} > 0$

$-\left(\frac{a^2+1}{a+1}\right) > 0 \quad \Rightarrow \quad \frac{a^2+1}{a+1} < 0$

...... (ii)

from (1) and (2) $a \in \{\phi\}$

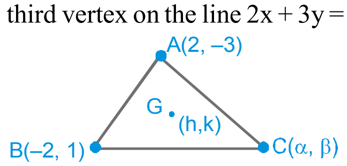
correct answer is $a \in [1, \infty)$

24. $\alpha = 3h$

$\beta - 2 = 3k$

$\beta = 3k + 2$

third vertex on the line $2x + 3y = 9$



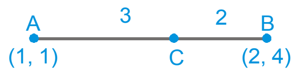
$2\alpha + 3\beta = 9$

$2(3h) + 3(3k + 2) = 9$

$2h + 3k = 1$

$2x + 3y - 1 = 0$

25. $\therefore \quad C\left(\frac{8}{5}, \frac{14}{5}\right)$



Line $2x + y = k$ passes $C\left(\frac{8}{5}, \frac{14}{5}\right)$

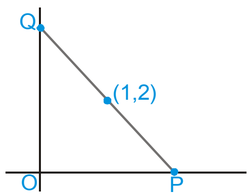
$$\frac{2\times 8}{5} + \frac{14}{5} = k$$

$k = 6$

26. $(y-2) = m(x-1)$

$OP = 1 - \frac{2}{m}$

$OQ = 2 - m$



$$\text{Area of } \Delta POQ = \frac{1}{2}(OP)(OQ) = \frac{1}{2}\left(1-\frac{2}{m}\right)(2-m)$$

$$= \frac{1}{2}\left[2 - m - \frac{4}{m} + 2\right]$$

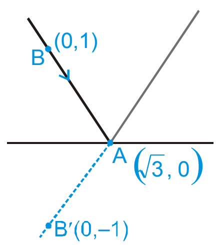
$$= \frac{1}{2}\left[4 - \left(m + \frac{4}{m}\right)\right]$$

$m = -2$

27. Take any point B(0, 1) on given line

Equation of AB′

$$y - 0 = \frac{-1-0}{0-\sqrt{3}}\left(x - \sqrt{3}\right)$$

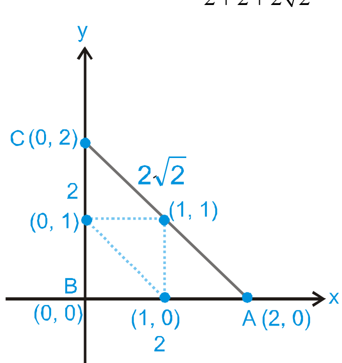


$-\sqrt{3}y = -x + \sqrt{3}$

$x - \sqrt{3}y = \sqrt{3}$

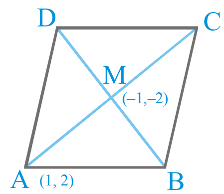
$\Rightarrow \quad \sqrt{3}y = x - \sqrt{3}$

28. x - coordinate of incentre $= \frac{2\times 0 + 2\sqrt{2}.0 + 2.2}{2+2+2\sqrt{2}} = \frac{2}{2+\sqrt{2}}$



$= 2 - \sqrt{2}$

31. Point of intersection of sides



$x - y + 1 = 0$ and $7x - y - 5 = 0$

$\therefore \quad x = 1, y = 2$

Slope of AM $= \frac{4}{2} = 2$

$\therefore$ Equation of BD : $y + 2 = -\frac{1}{2}(x+1)$

$\Rightarrow \quad x + 2y + 5 = 0$

Solving $x + 2y + 5 = 0$ and $7x - y - 5 = 0$

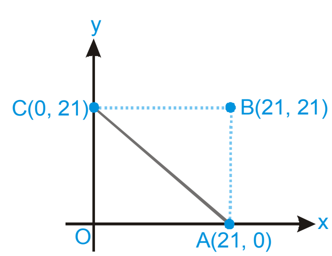
$$x = \frac{1}{3}, y = -\frac{8}{3} \quad \Rightarrow \quad \left(\frac{1}{3}, -\frac{8}{3}\right)$$

Part # II : IIT-JEE ADVANCED

1. The number of integral points that lie in the interior of square OABC is 20 × 20. These points are (x, y) where x, y = 1, 2, ..., 20. Out of these 400 points 20 lie on the line AC. Out of the remaining exactly half lie in ΔABC.

$\therefore$ number of integral point in the triangle OAC

$$= \frac{1}{2}[20 \times 20 - 20] = 190$$

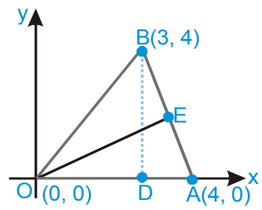


Alternative Solution

There are 19 points that lie in the interior of ΔABC and on the line x = 1, 18 point that lie on the line x = 2 and so on. Thus, the number of desired points is

$$19 + 18 + 17 + ... + 2 + 1 = \frac{20\times 19}{2} = 190.$$

2. Refer Figure



Equation of altitude BD is $x = 3$.

slope of AB is $\dfrac{4-0}{3-4} = -4$

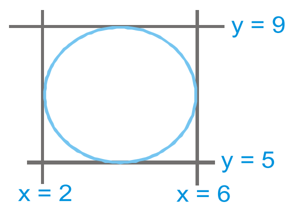
$\therefore$ slope of OE is 1/4

Equation of OE is

$$y = \frac{1}{4}x.$$

Lines BD and OE meets at (3, 3/4)

3. The lines given by $x^2 - 8x + 12 = 0$ are $x = 2$ and $x = 6$.



The lines given by $y^2 - 14y + 45 = 0$ are $y = 5$ and $y = 9$

Centre of the required circle is the centre of the square.

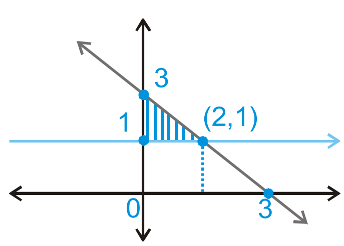
$\therefore$ Required centre is

$$\left(\frac{2+6}{2}, \frac{5+9}{2}\right) = (4, 7).$$

4. $x^2 - y^2 + 2y = 1$

$x = \pm(y - 1)$

Bisector of above lines are $x = 0$, $y = 1$



so Area between $x = 0$, $y = 1$ and $x + y = 3$

$$= \frac{1}{2} \times 2 \times 2 = 2 \text{ squ. units}$$

5. A line passing through P(h, k) and parallel to x-axis is $y = k$. .....**(i)**

The other lines given are $y = x$ .....**(ii)**

and

$y + x = 2$ .....**(iii)**

Let ABC be the Δ formed by the points of intersection of the lines **(i)**, **(ii)** and **(iii)**.

$\therefore$ A(k, k), B(1, 1), C(2–k, k)

$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2-k & k & 1 \end{vmatrix} = 4h^2$$

$$C_1 \to C_1 - C_2 \quad \frac{1}{2}\begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2-2k & k & 1 \end{vmatrix} = 4h^2$$

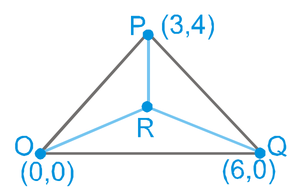
$\Rightarrow \dfrac{1}{2}|(2-2k)(k-1)| = 4h^2$

$\Rightarrow (k-1)^2 = 4h^2 \quad \Rightarrow \quad k - 1 = 2h,\ k - 1 = -2h$

$\Rightarrow k = 2h + 1 \qquad k = -2h + 1$

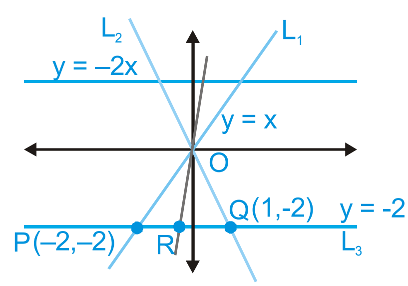
$\therefore$ locus of (h, k) is $y = 2x + 1$ $\quad y = -2x + 1$

6.



R is centroid hence $R \equiv \left(3, \dfrac{4}{3}\right)$

7. $\dfrac{PR}{RQ} = \dfrac{OP}{OQ}$



$$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$$

but statement – 2 is false

$\therefore$ Ans. **(C)**

8. $P \equiv (-\sin(\beta-\alpha), -\cos\beta)$

$Q \equiv (\cos(\beta-\alpha), \sin\beta)$

$R \equiv (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$

$0 < \alpha, \beta, \theta < \frac{\pi}{4}$

$x_R = \cos(\beta-\alpha)\cos\theta - \sin(\beta-\alpha)\sin\theta$

$\Rightarrow\ x_R = x_Q . \cos\theta + x_P . \sin\theta$

$y_R = \sin\beta\cos\theta - \cos\beta\sin\theta$

$\Rightarrow\ y_R = y_Q . \cos\theta + y_P . \sin\theta$

For P, Q, R to be collinear

$\sin\theta + \cos\theta = 1$

$\Rightarrow\ \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow$ not possible for the given interval $\theta \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow$ non collinear

9. $(1+p)x - py + p(1+p) = 0$ ...... **(i)**

$(1+q)x - qy + q(1+q) = 0$ ...... **(ii)**

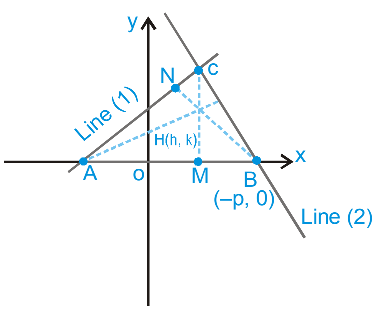
on solving (1) and (2), we get $C(pq, (1+p)(1+q))$

$\therefore$ equation of altitude CM passing through C and perpendicular to AB is $x = pq$ ...... **(iii)**

$\because$ slope of line (2) is $= \left(\frac{1+q}{q}\right)$

$\therefore$ slope of altitude BN (as shown in figure) is $= \frac{-q}{1+q}$

$\therefore$ equation of BN is $y - 0 = \frac{-q}{1+q}(x+p)$



$\Rightarrow\ y = \frac{-q}{(1+q)}(x+p)$ .... **(4)**

Let orthocentre of triangle be H(h, k) which is the point of intersection of (3) and (4)

$\therefore$ on solving (3) and (4), we get

$x = pq$ and $y = -pq$ $\Rightarrow$ $h = pq$ and $k = -pq$

$\therefore\ h + k = 0$

$\therefore$ locus of H(h, k) is $x + y = 0$

10. Let slope of line L = m

$\therefore\ \left|\frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}\right| = \tan 60^\circ = \sqrt{3} \Rightarrow \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$

taking positive sign, $m + \sqrt{3} = \sqrt{3} - 3m$

$m = 0$

taking negative sign

$m + \sqrt{3} + \sqrt{3} - 3m = 0$

$m = \sqrt{3}$

As L cuts x-axis $\Rightarrow$ $m = \sqrt{3}$

So L is $y + 2 = \sqrt{3}(x-3)$

11. (A) or (C) or Bonus

As $a > b > c > 0$

$\Rightarrow\ a - c > 0$ and $b > 0$

$\Rightarrow\ a - c > 0$ and $b > 0$

$\Rightarrow\ a + b - c > 0$

$\Rightarrow$ option **(A)** is correct

Further $a > b$ and $c > 0$

$\Rightarrow\ a - b > 0$ and $c > 0$

$\Rightarrow\ a - b > 0$ and $c > 0$

$\Rightarrow\ a - b + c > 0$ $\Rightarrow$ option **(C)** is correct

Aliter

$(a-b)x + (b-a)y = 0 \Rightarrow x = y$

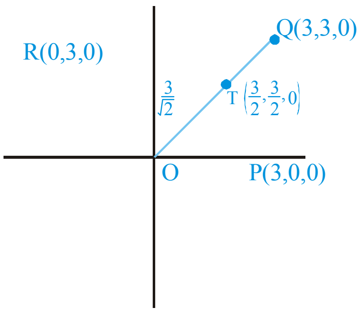
$\Rightarrow$ Point of intersection $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

Now $\sqrt{\left(1+\frac{c}{a+b}\right)^2 + \left(1+\frac{c}{a+b}\right)^2} < 2\sqrt{2}$

$\Rightarrow\ \sqrt{2}\left(\frac{a+b+c}{a+b}\right) < 2\sqrt{2}$

$\Rightarrow\ a + b - c > 0$

12.



$$S \equiv \left(\frac{3}{2},\frac{3}{2},3\right)$$

$$\overline{OQ} = 3\hat{i}+3\hat{j} \quad \Rightarrow \quad \overline{OS} = \frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+3\hat{k}$$

$$\cos\theta = \frac{\frac{1}{2}+\frac{1}{2}}{\sqrt{2}\sqrt{\frac{1}{2}+\frac{1}{4}+1}} = \frac{1}{\sqrt{2}\sqrt{\frac{3}{2}}} = \frac{1}{\sqrt{3}}$$

$$\vec{n} = \overline{OQ}\times\overline{OS} = (\hat{i}+\hat{j})\times(\hat{i}+\hat{j}+2\hat{k})$$

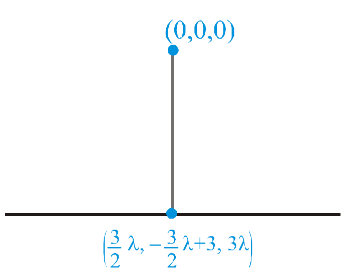
$$= \hat{k}-2\hat{j}-\hat{k}+2\hat{i} \quad \Rightarrow \quad 2\hat{i}-2\hat{j}$$

$$x-y=\lambda \quad \Rightarrow \quad x=y \quad \Rightarrow \quad \perp(3,0,0) \quad \Rightarrow \quad \frac{3}{\sqrt{2}}$$

$$RS \to \frac{x-0}{\frac{3}{2}} = \frac{y-3}{-\frac{3}{2}} = \frac{z-0}{3} = \lambda$$

$$\Rightarrow \quad x=\frac{3}{2}\lambda,\ y=-\frac{3}{2}\lambda+3,\ z=3\lambda$$

$$\text{T distance} \quad \Rightarrow \quad \sqrt{\frac{3}{2}-3+9} \quad \Rightarrow \quad \sqrt{\frac{15}{2}}$$



$$D = \frac{9}{4}\lambda^2+\left(3-\frac{3}{2}\lambda\right)^2+9\lambda^2 = \frac{27}{2}\lambda^2-9\lambda+9$$

$$\Rightarrow \quad \lambda = \frac{9}{27} = \frac{1}{3}$$

MOCK TEST

1. Condition for concurrency $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$3bc - 4bc - 2a(c-b) + a(4c-3b) = 0$

$-bc + 2ac - ab = 0 \qquad \Rightarrow \quad \frac{2}{b} = \frac{1}{a}+\frac{1}{c}$

So a, b, c are in H.P.

2. (A)
point of intersection is $A(-2, 0)$. The required line will be one which passes through $(-2, 0)$ and is perpendicular to the line joining $(-2, 0)$ and $(2, 3)$

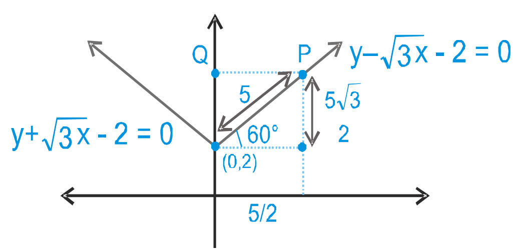
3. $x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$

$|m_1 - m_2| = \sqrt{(m_1+m_2)^2 - 4m_1m_2}$

$$\sqrt{\left(\frac{2\tan\theta}{\sin^2\theta}\right)^2 - 4\left(\frac{\sec^2\theta-\sin^2\theta}{\sin^2\theta}\right)} = 2$$

4. (B)
for $x>0$ $\ y-\sqrt{3}\,x-2=0$

$x<0 \qquad y+\sqrt{3}\,x-2=0$



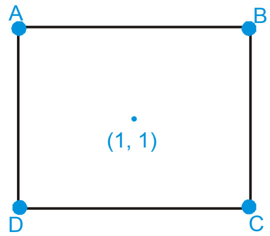
$$P = \left(\frac{5}{2}, \frac{4+5\sqrt{3}}{2}\right) \text{ or } \left(-\frac{5}{2}, \frac{4+5\sqrt{3}}{2}\right)$$

distance of p on its angle bisector i.e.

y-axis is $\left(0, \frac{4+5\sqrt{3}}{2}\right)$

5. To find equations of AB and CD

$\because$ AB and CD are parallel to $3x - 4y = 0$ and at a distance of 2 units from (1, 1)



$\therefore \quad 3x - 4y + k = 0$ and $\left|\dfrac{3-4+k}{5}\right| = 2$

$\Rightarrow \quad k - 1 = \pm 10 \;\Rightarrow\; k = 11, -9$

$\therefore$ equations of two sides of the square which are parallel to $3x - 4y = 0$ are

$3x - 4y + 11 = 0$ and $3x - 4y - 9 = 0$

Now the remaining two sides will be perpendicular to $3x - 4y = 0$ and at a distance of 2 unit from (1, 1)

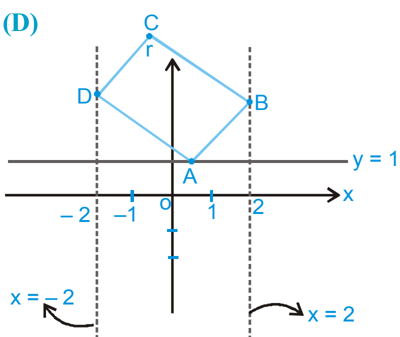
$\therefore \quad 4x + 3y + k = 0$

and $\left|\dfrac{4+3+k}{5}\right| = 2 \;\Rightarrow\; k + 7 = \pm 10 \;\Rightarrow\; k = 3, -17$

$\therefore$ remaining two sides are

$4x + 3y + 3 = 0$ and $4x + 3y - 17 = 0$

6. (D)



Let equation of AB be $y = x + a$

$\therefore \quad A(1 - a, 1)$ and $B(2, 2 + a)$

$\therefore$ equation of AD is

$y - 1 = -1(x - 1 + a)$

$\therefore \quad D(-2, 4 - a)$

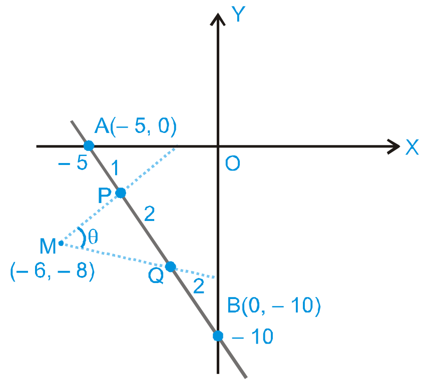
Let C(h, k) mid point of AC = mid point of BD

$\Rightarrow \quad h + 1 - a = 2 - 2 \quad \Rightarrow \quad h = a - 1$

and $k + 1 = 2 + a + 4 - a \quad \Rightarrow \quad k = 5$

$\therefore$ Locus of C(h, k) is $y = 5$

7.



$\therefore \quad P \equiv (-4, -2)$ and $Q \equiv (-2, -6)$

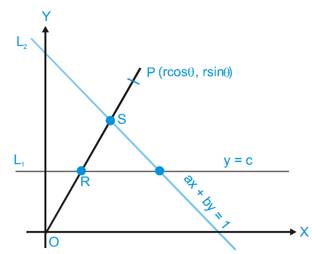
$\therefore$ Let slopes of PM and QM be m_1 and m_2 respectively.

$\therefore \quad m_1 = 3$ and $m_2 = \dfrac{1}{2}$.

Let 'θ' be the acute angle between PM and QM

8. (A)

Let O be taken as the origin and a line through O parallel to L_1 as the x-axis and the line through O perpendicular to x-axis as y-axis (figure).



Let equations of L_1 and L_2 in this system of coordinates be $y = c$ and $ax + by = 1$ respectively, where a, b, c are fixed constants.

Let equation of the variable line through O be

$$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$$

Then $(r\cos\theta, r\sin\theta)$ are the coordinates of a point on this line at a distance r from the origin O.

Let $OP = r$, $OR = r_1$ and $OS = r_2$ so that coordinates of P, R and S are respectively

$(r\cos\theta, r\sin\theta)$, $(r_1\cos\theta, r_1\sin\theta)$ and $(r_2\cos\theta, r_2\sin\theta)$.

Since R lies on L_1, $r_1\sin\theta = c$ and S lies on L_2, $a.\, r_2\cos\theta + b.\, r_2\sin\theta = 1$.

so that $r_1 = \dfrac{c}{\sin\theta}$ and $r_2 = \dfrac{1}{a\cos\theta + b\sin\theta}$ ..... **(1)**

Now we are given $\frac{m+n}{OP} = \frac{m}{OR} + \frac{n}{OS}$

$\Rightarrow \quad \frac{m+n}{r} = \frac{m}{r_1} + \frac{n}{r_2}$

$\Rightarrow \quad \frac{m+n}{r} = \frac{m \sin \theta}{c} + n\,(a\cos\theta + b\sin\theta)$

[from (1)]

$\Rightarrow \quad (m+n)\,c = mr\sin\theta + cnar\cos\theta + cnbr\sin\theta$

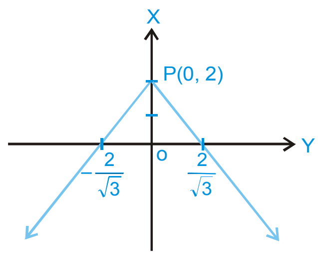
Therefore locus of P $(r\cos\theta, r\sin\theta)$ is

$cn\,(ax + by - 1) + m(y - c) = 0$

which is a straight line passing through the intersection of $L_1 : y - c = 0$ and $L_2 : ax + by = 1$

9. $\because$ point of intersection of the two ray is P(0, 2)

$\therefore$ Point A is $\left(\frac{2}{\sqrt{3}}, 0\right)$ or $\left(-\frac{2}{\sqrt{3}}, 0\right)$



and PO is bisector of the angle between two rays

$\therefore$ required point is (0, 0)

$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

10. (D)

S_1 : Image of (2, 1) in the line $x + 1 = 0$ is (–4, 1)

$\therefore$ S_1 is false

S_2 : $\ell + m = 4$

$\therefore \quad \frac{\ell + m}{2} = 2 \qquad \therefore$ S_2 is true

S_3 : A (10, 20), B(22, 25), C(10, 25)

$AB^2 = (22 - 10)^2 + (25 - 20)^2 = 169, BC^2 = 12^2 + 0 = 144$,

$CA^2 = 0^2 + 5^2 = 25$

ABC is right angled triangle

Hence (10, 25) is orthocentre $\quad \therefore$ S_3 is true

S_4 : Equation of pair of bisectors of angles between lines $ax^2 - 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{-h}$$

$\Rightarrow \quad -h\,(x^2 - y^2) = (a - b)\,xy$

but y = mx is one of these lines, then it will satisfy it. Substituting y = mx in (1)

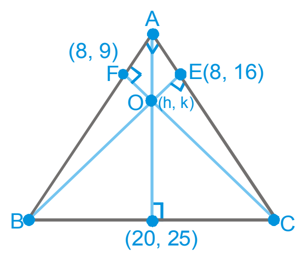
$-h\,(x^2 - m^2 x^2) = (a - b)\,x.\,mx$

Dividing by x^2, $h(1 - m^2) + m(a - b) = 0$

11. Orthocentre O of the ΔABC is the incentre of the pedal ΔDEF.

$ED = \sqrt{(20-8)^2 + (25-16)^2} = 1$

$FD = 20, \; EF = 7$



$$H = \frac{7\times20 + 20\times8 + 15\times8}{7 + 20 + 15} = 10$$

$$K = \frac{7\times25 + 20\times16 + 15\times9}{7 + 20 + 15} = 15$$

0(10, 15)

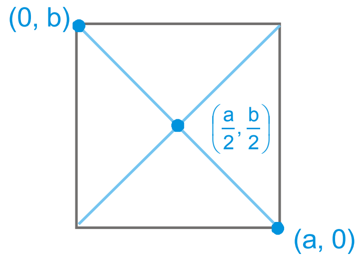
$AC \equiv y - 2x = 0$

$AB \equiv 3y + x - 35 = 0$

$BC \equiv x + y - 45 = 0 \Rightarrow A(5, 10), B(50, –5)C(15, 30)$

12. (A,C)

$$\frac{x - \frac{a}{2}}{\frac{b}{\sqrt{a^2+b^2}}} = \frac{y - \frac{b}{2}}{\frac{a}{\sqrt{a^2+b^2}}} = \pm\frac{\sqrt{a^2+b^2}}{2}$$



$$x = \frac{a}{2} + \frac{b}{2}, y = \frac{b}{2} + \frac{a}{2}$$

and $x = \frac{a}{2} - \frac{b}{2}, y = \frac{b}{2} - \frac{a}{2}$

$\therefore$ the required points are $\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$

and $\left(\frac{a-b}{2}, \frac{b-a}{2}\right)$

13. Take A(0, 0), B(a, 0), C(a , a) and D(0, a) then M(a, a/2) and P(a/2, a)

$$\Delta AMP = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8}$$

$$\Delta MAP = \frac{a^2}{8} \Rightarrow \Delta ABM = \Delta ARP = \frac{a^2}{4}$$

Area of quad. AMCP $= \frac{3a^2}{8} + \frac{a^2}{8} = \frac{a^2}{2}$

14. (A,C)

$\tan\alpha \tan\beta = -1$

$\Rightarrow \cos(\alpha - \beta) = 0$

$\Rightarrow \alpha - \beta = \frac{\pi}{2}$

16. (B)

Put $2h = -(a+b)$ in $ax^2 + 2hxy + by^2 = 0$

$\Rightarrow ax^2 - (a+b)xy + by^2 = 0$

$\Rightarrow (x-y)(ax-by) = 0$

$\Rightarrow$ one of the line bisects the angle between co-ordinate axes in positive quadrant.

Also put $b = -2h - a$ in $ax - by$ we have $ax - by = ax - (-2h - a)y = ax + (2h + a)y$

Hence $ax + (2h + a)y$ is a factor of $ax^2 + 2hxy + by^2 = 0$

17. (D)

Statement-II is true (standard result from high school classes)

Statement-I :

Since AB may not be equal to AC,

$\therefore$ perpendicular drawn from A to BC may not bisects BC

$\therefore$ statement is false

18. (B)

$ax^3 + bx^2y + cxy^2 + dy^3 = 0$

$\Rightarrow d\left(\frac{y}{x}\right)^3 + c\left(\frac{y}{x}\right)^2 + b\left(\frac{y}{x}\right) + a = 0$

$\Rightarrow dm^3 + cm^2 + bm + a = 0$ ..... (i)

$m_1 m_2 m_3 = -a/d$

$\Rightarrow m_3 = a/d$

as two lines are perpendicular, put $m_3 = a/d$ in ..... (i)

7 $\Rightarrow a^2 + ac + bd + d^2 = 0$

19. (A)

ABC is a right angled triangle, right angled at C as (m_{AC})

$(m_{BC}) = \left(\frac{-4+2}{5+5}\right)\left(\frac{-4-6}{5-7}\right) = -1$

Hence circumcentre is mid pt. of AB $\equiv (1, 2)$

20. (B)

Bisector at C $\frac{|3x+2y|}{\sqrt{13}} = \frac{|2x+3y+6|}{\sqrt{13}}$

$\Rightarrow x - y - 6 = 0$ and $5x + 5y + 6 = 0$

according to given equations of sides, internal angle bisector at C will have negative slope.

Image of A will lie on BC with respect to both bisectors.

21. (A) $\rightarrow$ (t), (B) $\rightarrow$ (s), (C) $\rightarrow$ (p), (D) $\rightarrow$ (q)

(A) For point (a, a^2) to lie inside the triangle must satisfy

$a > 0$ ..... (i)

$a^2 > 0$ ..... (ii)

and $a + 2a^2 - 3 < 0$ ..... (iii)

$(2a+3)(a-1) < 0$

$\Rightarrow a < 1$

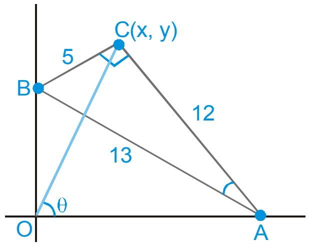
$\Rightarrow a \in (0, 1)$

Hence correct answer is t

(B) Since $\angle BCA = 90°$

Points A, O, B, C are concyclic

Let $\angle AOC = \theta$



$\angle BOC = \angle BAC$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{5}{12}$$

$$\frac{x}{y} = \frac{5}{12}$$

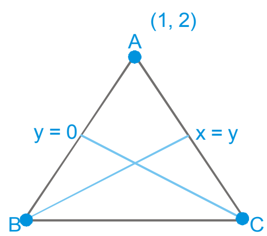
$\Rightarrow$ $12x - 5y = 0$

(C) Slope of line joining the point $(t-1, 2t+2)$ and its image $(2t+1, t)$ is $\frac{(2t+2)-t}{t-1-2t-1} = \frac{t+2}{-(t+2)} = -1$.

So slope of line is 1

(D) Image of point A(1, 2) in bisector of angles B and C lie on the line BC.

Image of A in x = y is (2, 1) and image of A in y = 0 is (1, –2).



So equation of line BC is $y = 3x - 5$

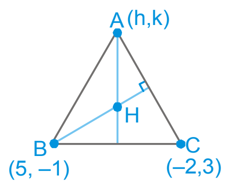
So $d(A, BC) = \frac{4}{\sqrt{10}}$ So $\sqrt{10}\ d(A, BC) = 4$.

22. (A) → (p), (B) → (q), (C) → (s), (D) → (s)

(A) AH ⊥ BC. $\Rightarrow$ $\left(\frac{k}{h}\right)\left(\frac{3+1}{-2-5}\right) = -1$

$4k = 7h$ ...... **(i)**

BH ⊥ AC. $\Rightarrow$ $\left(\frac{0+1}{0-5}\right)\left(\frac{k-3}{h+2}\right) = -1$



$k - 3 = 5(h + 2)$ ...... **(ii)**

$\Rightarrow$ $7h - 12 = 20h + 40$

$13h = -52$

$h = -4$ $\therefore$ $k = -7$

$\therefore$ A (–4,–7)

(B) $x + y - 4 = 0$ ......**(i)**

$4x + 3y - 10 = 0$ ......**(ii)**

Let (h, 4 – h) be the point on **(i)**,

then $\left|\frac{4h + 3(4-h) - 10}{5}\right| = 1$ i.e. $h + 2 = \pm 5$

i.e. h = 3 ; h = –7

$\therefore$ required point is either (3,1) or (–7, 11)

(C) Orthocentre of the triangle is the point of intersection of the lines

$x + y - 1 = 0$ and $x - y + 3 = 0$ i.e. (–1, 2)

(D) Since a, b, c are in A.P.

$$\therefore\ b = \frac{a+c}{2}$$

$\therefore$ the family of lines is $ax + \frac{a+c}{2}y = c$

i.e. $a\left(x + \frac{y}{2}\right) + c\left(\frac{y}{2} - 1\right)$ 7

$\therefore$ point of concurrency is (–1, 2)

23.

1. (B)

$\omega = 60°, m = 2$

$$\tan\theta = \frac{m\sin\omega}{1 + m\cos\omega} = \frac{2\sin 60°}{1 + 2\cos 60°} = \frac{2\times\sqrt{3}/2}{1 + 2\times 1/2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow\ \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

2. (D) $\omega = 60°, m_1 = 2, m_2 = -\frac{1}{2}$

$$\tan\theta_1 = \frac{m_1\sin\omega}{1 + m_1\cos\omega} = \frac{2\times\sqrt{3}/2}{1 + 2\times 1/2} = \frac{\sqrt{3}}{2}$$

$$\tan\theta_2 = \frac{-1/2\times\sqrt{3}/2}{1 - 1/2\times 1/2} = \frac{-\sqrt{3}}{4}\times\frac{4}{3} = -\frac{1}{\sqrt{3}}$$

Let angle between the lines be ϕ then

$$\tan\phi = \left|\frac{\tan\theta_1 - \tan\theta_2}{1+\tan\theta_1\tan\theta_2}\right| = \left|\frac{\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{3}}}{1-\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}}}\right|$$

$\Rightarrow$ $\phi = \tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$

3. **(C)** $m = \frac{\sin 60^\circ}{\sin(30^\circ - 60^\circ)} = -\sqrt{3}$

$\therefore$ equation of the line is $y - 0 = -\sqrt{3}\,(x-2)$

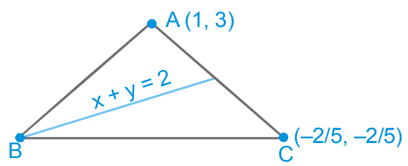
i.e. $\sqrt{3}\,x + y = 2\sqrt{3}$

24.

1. **(B)**

Image of A(1, 3) in line x + y = 2 is

$$\left(1-\frac{2(2)}{2},\ 3-\frac{2(2)}{2}\right) \equiv (-1, 1)$$



So line BC passes through (–1, 1) and $\left(-\frac{2}{5}, -\frac{2}{5}\right)$.

Equation of line BC is $y - 1 = \frac{-2/5-1}{-2/5+1}(x+1)$

$\Rightarrow$ $7x + 3y + 4 = 0$

2. (C) Vertex B is point of intersection of 7x + 3y + 4 = 0 and x + y = 2 i.e B = (– 5/2, 9/2)

3. **(A)** Line AB is $y - 3 = \frac{3-9/2}{1+5/2}(x-1)$

$\Rightarrow$ $3x + 7y = 24$

25.

1. $\frac{x-2}{3} = \frac{y-3}{-4} = -15\ \frac{6-12+1}{25} = 3$

$\therefore$ $x = 11,\ y = -9$

$\therefore$ $\alpha = 2$

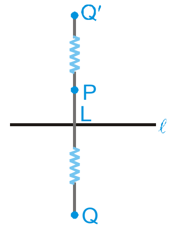
2. $\frac{x-1}{-5} = \frac{y-1}{12} = 26\ \frac{-5+12+6}{169} = 2$

$x = -9, y = 25$

$\therefore$ $\beta = 16$

3. since PQ = 16PL, therefore, LQ = 15 PL and so PQ′ = 14PL.

Thus n = 14 for the point Q′.



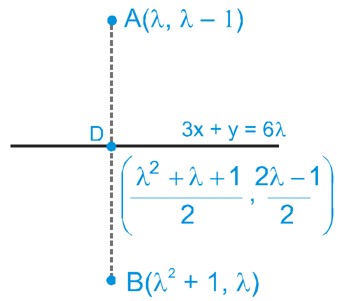
Since L and Q′ are on opposites sides of P

$\therefore$ $\frac{x-2}{1} = \frac{y+1}{-1} = 14.\ \frac{2+1+1}{2} = 28$ $\therefore$ Q′(30,–29)

26. D is mid point of AB and lies on the line $3x + y = 6\lambda$

$\Rightarrow$ $3\cdot\frac{\lambda^2+\lambda+1}{2} + \frac{2\lambda-1}{2} = 6\lambda$

$3\lambda^2 - 7\lambda + 2 = 0$ ....**(1)**



$\lambda = \frac{1}{3}, 2$

multiplication of slope of AB & line = –1

$\frac{-1}{\lambda-\lambda^2-1}(-3) = -1$

$\lambda^2 - \lambda - 2 = 0$ ....**(2)**

$\lambda = -1, 2$

$\lambda = 2$ satisfies both (1) & (2)

27. Let the line (L) through the origin is

$x = r\cos\theta$

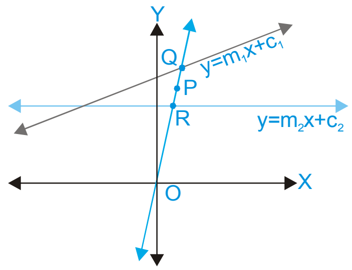
$y = r\sin\theta$

as L intersects L_1 at Q and $OQ = r_1$

$\therefore$ $r_1\sin\theta = m_1r_1\cos\theta + c_1$ ....**(1)**

similarly, L intersects L_2 at R and $OR = r_2$

$r_2\sin\theta = m_2r_2\cos\theta + c_2$ ....**(2)**



Let $P \equiv (h, k)$ & $OP = r$

$\therefore \quad r^2 = r_1 r_2$ ....(3)

& $\quad h = r\cos\theta$ ....(4)

$\quad k = r\sin\theta$ ....(5)

putting the values of r_1 and r_2 from (1) and (2) in (3)

$\therefore \quad r^2 = \dfrac{c_1}{(\sin\theta - m_1\cos\theta)} \cdot \dfrac{c_2}{(\sin\theta - m_2\cos\theta)}$ ....(6)

putting the value of cosθ and sinθ from (4) and (5) in (6), we get

$\Rightarrow \quad r^2 = \dfrac{c_1 c_2}{\left(\dfrac{k}{r} - m_1\dfrac{h}{r}\right)\left(\dfrac{k}{r} - m_2\dfrac{h}{r}\right)}$

$\Rightarrow \quad (k - m_1 h)(k - m_2 h) = c_1 c_2$

replacing (h, k) by (x, y) we get the desired locus

as $\quad (y - m_1 x)(y - m_2 x) = c_1 c_2$

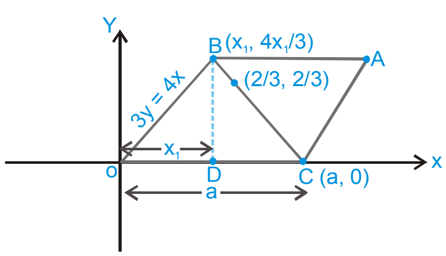
28. (6)

Let $OC = a$

$\therefore \quad OC = CA = AB = BO = a$

Let $\left(x_1, \dfrac{4x_1}{3}\right) \quad \therefore \quad A\left(a + x_1, \dfrac{4x_1}{3}\right)$

$\because \quad x_1^2 + \dfrac{16x_1^2}{9} = a^2$ ($\because$ ODB is a right angle triangle)



$\therefore \quad a = \dfrac{5x_1}{3}$

$\because \quad$ equation of BC is

$y - 0 = \dfrac{\dfrac{4x_1}{3} - 0}{x_1 - a}(x - a) \quad \because \quad a = \dfrac{5x_1}{3}$

$\therefore \quad y = -2x + \dfrac{10x_1}{3}$

$\therefore \quad$ BC passes through $\left(\dfrac{2}{3}, \dfrac{2}{3}\right)$

$\therefore \quad x_1 = 3/5 \qquad \therefore \quad a = 1$

$\therefore \quad A\left(1 + \dfrac{3}{5}, \dfrac{4}{3} \times \dfrac{3}{5}\right)$

$\therefore \quad A\left(\dfrac{8}{5}, \dfrac{4}{5}\right) \qquad \therefore \quad \dfrac{5}{2}(\alpha + \beta) = 6.$

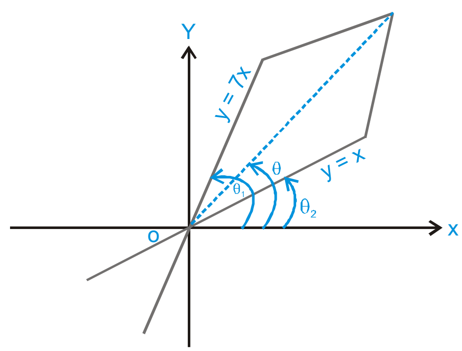
29. (2)

$\because \quad \theta_1 - \theta = \theta - \theta_2 \qquad \Rightarrow \quad 2\theta = \theta_1 + \theta_2$

$\therefore \quad \tan 2\theta = \dfrac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$

$\Rightarrow \quad \tan 2\theta = \dfrac{7 + 1}{1 - 7} \qquad \Rightarrow \quad \tan 2\theta = \dfrac{-4}{3}$

$\Rightarrow \quad \dfrac{2\tan\theta}{1 - \tan^2\theta} = -\dfrac{4}{3}$



$\Rightarrow \quad \tan\theta = 2$ or $-\dfrac{1}{2}$

$\therefore \quad$ slope of longer diagonal is = 2

30. Consider region OABC, x-coordinate downword very from 0 to – 10

$(0, 0) \quad (0, 1) \quad \ldots\ldots \quad (0, 10) \to 11$

$(-1, 0) \quad (-1, 1) \quad \ldots\ldots \quad (-1, 10) \to 11 \qquad 11^2$

$\vdots$

$(-10, 0) \quad (-10, 1) \quad (-10, 2), (-10, 10) \to 11$

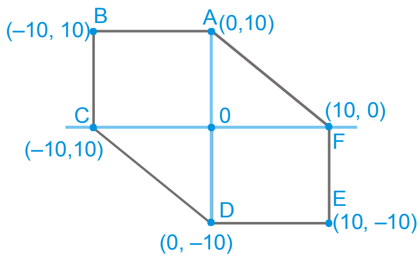
Similarly ODEF $= 11^2$

origin is common to both $\Rightarrow$ integral point in region O ABC and ODEF $= 11^2 + 11^2 - 1 = 241$ .... **(1)**

consider region OAF excluding OA & OF

$(1, 9), (1, 8) \ldots.(1, 1) \to 9$

$(2, 8), (2, 7) \ldots.(2, 1) \to 8$



$(9, 1) \to 1$

$=$ total points $1 + 2 + \ldots\ldots. + 8 + 9 = \frac{9 \times 10}{2} = 45$ points .... **(2)**

similarly region OCD $= 45$ points .... **(3)**

total integral points $= 241 + 45 + 45 = 331$