NEWTON'S LAW & MOTION

INTRODUCTION

We described motion in terms of position, velocity and acceleration without considering what might cause that motion . Now we consider what might cause one object to remain at rest and another object to accelerate ? The two main factor we need to consider are the forces acting on as object and the mass of the object. We discuss the three basic law of motion, which deal with forces and masses and were formulated more than three centuries ago by Issac Newton. Once we understand these laws, We can answer such question as what mechanism changes motion and why do some objects accelerate more than other ?

MOTION

Motion of a body is its movement and is identified by change in either its location or orientation or both, related to other objects.

Location

Location of a rigid body tells where it is placed and can be measure by position coordinates of any particle of the body or its mass centre. It is also known as position.

Orientation

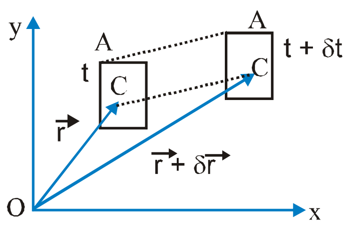
Orientation of a body tell us how it is placed with respect Coordinate axes by any linear dim of the body or a straight line draw on it, provide suitable measure of orientation

TRANSLATION AND ROTATION MOTION

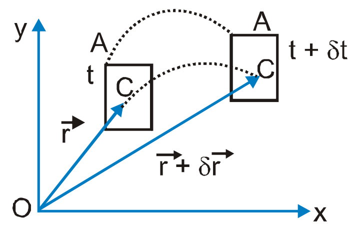
If a body changes its location without changes in orientation, it is in pure translation motion and if it changes orientation with change in location, it is in pure rotation motion.

TRANSLATION MOTION

Let us consider the motion of a plate, which involves only change in position without change in orientation. It is in pure translation motion. The plate is shown at two different instants t and t +dt. The coordinate axes shown are in the plane of the plate and represent the reference frame. A careful observation makes the following points obvious



Rectilinear Translation



Curvilinear Translation

None of the linear dimension or any line drawn on the body changes its angles with the coordinate. Therefore there is no rotation motion.

All the particles of the body including its mass center move on identical parallel trajectories. Here trajectories of corner A and center C are shown by dashed lines

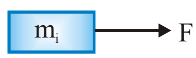
All the particles and mass center of the body cover identical segments of their trajectories in a given time interval. Therefore, at any instant of time all of them have identical velocities and acceleration.

Pure transition motion of a body can be represented by motion of any of its particle, this is way, we usually consider a body in pure transition motion as a particle.

MASS

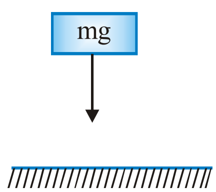
The mass of a body is the quantity of matter Contained in it. Its S.I. unit is Kg. The mass of the body which determines its inertia in translatory motion is called its inertial mass. This is the mass the appears in Newton's second law, which can be written as

$$F = m_i a_i \qquad \text{or} \qquad m_i = \frac{F}{a_i}$$



The mass of body which determines the gravitation pull due to the earth is called its gravitational mass. This is the mass that appears in Newton's law of gravitation which we can write as

$$F = \frac{GMm}{R^2} \qquad \text{or} \qquad m_g = \frac{FR^2}{GM}$$



Earth surface

FORCE

Everyone has a basic understanding of the concept of force. In a vague language, Force is push or pull which is an attempt to change the state of rest or motion of an object, or merely deform it. The effect of force on the state of motion of body was first understood by Isaac Newton. Force is the interaction between the object and the source (providing the pull or push) It is a vector quantity

Effect of resultant force

(i) May change only speed

(ii) may change only direction of motion

(iii) may change both the speed and direction of motion

(iv) may change size and shape of a body

Unit of force

(i) Newton and $\frac{Kg.m}{s^2}$ (MKS System)

(ii) dyne and $\frac{g.cm}{s^2}$ (CGS system)

(iii) 1 Newton = 10^5 dyne

Kilogram Force (Kgf)

The force with which earth attracts a 1k body towards its centre is called kilogram force, thus

$$\text{Kgf} = \frac{\text{Force in newton}}{g}$$

Dimensional formula of force : $\left[M^1 L^1 T^{-2}\right]$

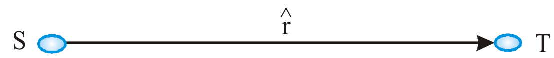
FUNDAMENTAL FORCE

All the forces observed in nature such as muscular force, tension, reaction, Friction elastic , weight electric, magnetic, nuclear etc., can be explained in term of only Following four basic interactions.

[A] Gravitational Force

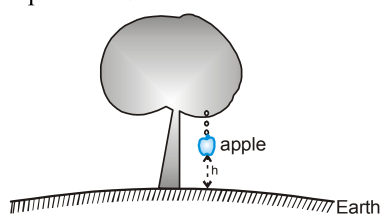
The force of interaction which exists between two particles of masses m_1 and m_2, due to their masses is called gravitational force

$$F = -G\frac{m_1 m_2}{r^2}\hat{r}$$



$\hat{r}$ = Unit position vector of test particle 'T' with respect to source particle 'S'

G = Universal Gravitational Constant = 6.67×10^{-11} Nm^2/kg^2.



(i) It is the weakest force and is always attractive.

(ii) It is a long range force as it acts between any two particles situated at any distance in the universe.

(iii) It is independent of the nature of medium between the particles.

An apple is freely falling as shown in figure, When it is at a height h, force between earth and apple is given by

$$F = \frac{GM_e m}{(R_e + h)^2} \quad \text{where } M_e = \text{mass of earth}, \quad R_e = \text{radius of earth}$$

It acts towards earth's centre. Now rearranging above result,

$$F = m.\frac{GM_e}{R_e^2}.\left(\frac{R_e}{R_e + h}\right)^2 \quad \Rightarrow \quad F = mg\left(\frac{R_e}{R_e + h}\right)^2 \quad \left\{g = \frac{GM_e}{R_e^2}\right\}$$

$$\text{Here} \quad h << R_e, \text{ so } \frac{R_e}{R_e + h} \simeq 1 \quad \therefore \quad F = mg$$

This is the force exerted by earth on any particle of mass m near the earth surface. The value of g = 9.81 $m/s^2 \simeq 10$ $m/s^2 \simeq p^2$ $m/s^2 \simeq 32$ ft/s^2. It is also called acceleration due to gravity near the surface of earth.

[B] Electromagnetic Force

Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force

(a) These can be attractive or repulsive.

(b) These are long range forces

(c) These depend on the nature of medium between the charged particles.

(d) All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electro magnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experi enced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between atoms/molecules.

[C] Nuclear Force

It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspect of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

[D] Weak Force

It acts between any two elementary particles. Under its action a neutron can change into a proton emitting an electron and a particle called antineutrino. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.

It has been found that for two protons at a distance of 1 Fermi :

$F_N:F_{EM}:F_W:F_G::1:10^{-2}:10^{-7}:10^{-38}$

CLASSIFICATION OF FORCES ON THE BASIS OF CONTACT

(A) Field Force:

Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called field force. Examples

(a) Gravitation force **(b)** Electromagnetic force

(B) Contact Force:

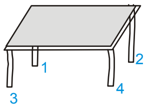
Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

Examples:

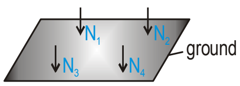
(a) Normal force (N):

It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force.

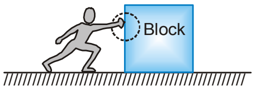
A table is placed on Earth as shown in figure



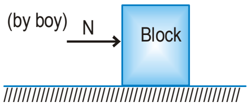
Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.



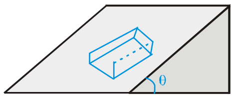
Now a boy pushes a block kept on a frictionless surface.



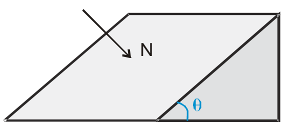
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.

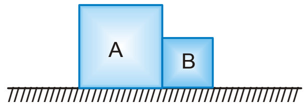


Normal force exerted by block on the surface of inclined plane is shown in figure.



Force acts perpendicular to the surface

Ex. Two blocks are kept in contact on a smooth surface as shown in figure. Draw normal force exerted by A on B.

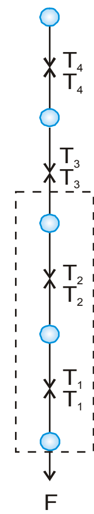


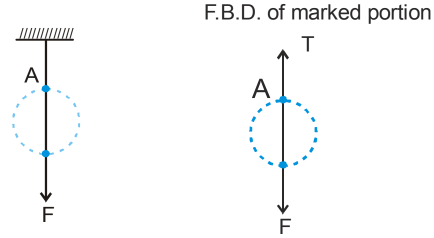
Sol. In above problem, block A does not push block B, so there is no molecular interaction between A and B. Hence normal force exerted by A on B is zero.

Note Normal is a dependent force, it comes in role when one surface presses the other.

(b) Tension :

Tension in a string is a electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force 'F' as shown in figure, for calculating the tension at point 'A' we draw F.B.D. of marked portion of the string; Here string is massless.

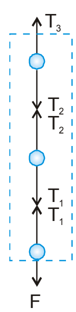




$\Rightarrow \qquad T = F$

String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature as shown in figure. The attraction force between two segments is equal and opposite due to Newton's third law.

For calculating tension at any segment, we consider two or more than two parts as a system.



$\boxed{T_3 = F}$

Here interaction between segments are considered as internal forces, so they are not shown in F.B.D.

(C) Frictional force :

It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact.

Basic characteristic of a forces

Force is vector quantity therefore has magnitude as well as direction. To predict how a force affects motion of a body we must know its magnitude, direction and point on the body where the force is applied. This point is known a point of application of the force. The direction and the point of application of a force both decide line of action of the force. Magnitude and direction decide effect on translation motion and magnitude and line of action decides effects on rotation motion.

Momentum : Amount of Motion

Amount of motion in a body depends on its velocity and mass

Linear momentum of a body is defined as product of its mass and velocity. It provides measure of amount of motion

Linear momentum $\vec{p}$ of a body of mass m, moving with velocity by $\vec{v}$ is expressed by the following equation

$$\vec{p} = m\vec{v}$$

SI unit of momentum is kg-m/s.

Dimension of momentum are $[MLT^{-1}]$

NEWTON'S LAWS OF MOTION

Newton has published three laws, which describe how forces affect motion of a body on which they act. These laws are fundamental in nature in the sense that the first law gives concept of force, inertia and the inertial frames; the second law defines force and the third law action and reaction as two aspects of mutual interaction between two bodies

1. The First Law of motion

Every material body has tendency to preserve its state of rest, or of uniform motion in straight line, unless it is compelled to change that state by external forces impressed on it.

Inertia

The tendency of a material body to preserve its present state of uniform motion of rest in known as inertia of the body. It was first conceived by Galileo

Inertia is a physical quantity and mass of a material body is measure of its inertia

Inertial frame of Reference

The first law requires a frame of reference in which only the forces acting on a body can be responsible for any acceleration produced in the body and not the acceleration of the frame of reference. These frames of reference are known as inertial frames

2. The Second Law of motion

The rate of change in momentum of a body is equal to, and occurs in the direction of the net applied force. A body of mass m in translational motion with velocity $\vec{v}$, if acted upon with a net external force $\vec{F}$, the second law suggests:

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

If mass of the body is constant, the above equation relates the acceleration $\vec{a}$ of the body with the net force $\vec{F}$ acting on it

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\vec{a}$$

The first law provides concept of force and the second law provides the quantitative definition of force, therefore the second law is also valid only in inertial frames.

SI unit of force in newton. It is abbreviated as N. One newton equals to one kilogram-meter per second square

$$1\text{ N} = 1\text{ kg-m/s}^2$$

Dimension of force are $[MLT^{-2}]$

(i) The second law is obviously consistent with the first law as F = 0 Implies a = 0

(ii) The second law of motion is a vector law it is actually a combination of three equations

$$F_x = \frac{dp_x}{dt} = ma_x \, , F_y = \frac{dp_y}{dt} = ma_y$$

$$F_z = \frac{dp_z}{dt} = ma_z$$

This means that if a force is not parallel to the velocity of the body, but makes some angles with it, it changes only the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.

(iii) The second law of motion given above is strictly applicable to a single point mass the force F in the law stand for the net external force on the particle and a stands for the acceleration of the particle. Any internal forces in the system are not to be include in F.

(iv) The second law of motion is a local relation what this means is that the force F at a point in space (location of the particle) at a certain instant of time is related to a at the same point at the same instant that is acceleration here and now is determined by the force here and now not by any history of the motion of the particle.

3. Newton Third Law of motion

Force is always a two-body interaction. The first law describes qualitatively and the second law describes quantitatively what happens to a body if a force acts on it, but do not reveal anything about what happens to the other body participating in the interaction responsible for the force.

The third law accounts for this aspect of the force and states the every action on a body has equal and opposite reaction on the other body participating in the interaction.

(i) The term 'action' and reaction in the third law mean nothing also but force A simple and clear way of standing the third law is a follows force always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.

(ii) The terms 'action' and reaction' in the third law may give a wrong interaction that action comes before reaction i.e. action is the cause and reaction the effect. There is no such cause effect reaction implied in the third law. The force on A by B and the force on B by A act at the same instant Any one of them may be called action and other reaction.

(iii) Action and reaction forces act on different bodies, not on the same body thus if we are considering the motion of any one body (A or B) they add up to give a null Force. internal forces in a body or a system of particles thus cancle away in pairs.

This is an important fact that enables the second law to be applicable to a body or a system of particles.

SYSTEM

Two or more than two objects which interact with each other form a system.

Classification of forces on the basis of boundary of system

(A) Internal forces

Forces acting each with in a system among it constituents

(B) External forces

Forces extended on the constituents of a system by the outside surroundings are called as external forces.

(C) Real force

Force which act on an object due to other object is called as real force An isolated object (for away from all objects) does not experience any real force.

CONCEPT OF FREE BODY DIAGRAM (FBD)

A force on a body can only exists when there is another body to create it, therefore in every physical situation of concern there must be two or more bodies applying force on each other. On the other hand the three laws of Newton, describe motion of a single body under action of several forces, therefore, to analyze a given problem, we have to consider each of the bodies separately one by one. This idea provides us with the concept of free body diagram.

A free body diagram is pictorial representation in which the body under stud is assumed free from rest of the system i.e. assumed separated from rest of the interacting bodies and is drawn in its actual shape and orientation and all the force acting on the body are shown.

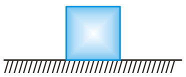
How to draw a Free Body Diagram (FBD)

Separate the body under consideration from the rest of the system and draw it separately in actual shape and orientation.

Show all the forces whether known or unknown acting on the body at their respective points of application

For the purpose count every contact where we separate the body under stud from other bodies. At every such point, there may be a contact force. After showing, all the contact forces shown all the field forces.

Ex. A block of mass 'm' is kept on the ground as shown in figure.

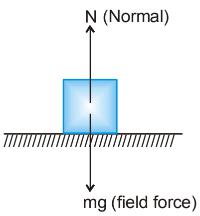


(i) Draw F.B.D. of block.

(ii) Are forces acting on block action–reaction pair.

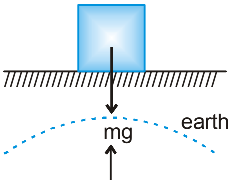
(iii) If answer is no, draw action reaction pair.

Sol. **(i)** F.B.D. of block

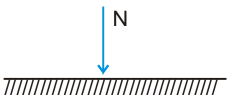


(ii) 'N' and mg are not action-reaction pair. Since pair act on different bodies, and they are of same nature.

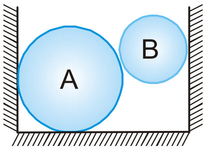
(iii) Pair of 'mg' of block acts on earth in opposite direction.



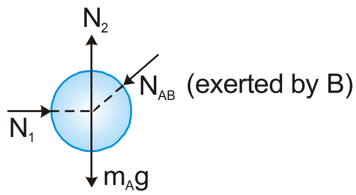
and pair of 'N' acts on surface as shown in figure.



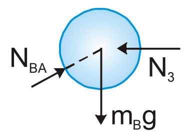
Ex. Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.



Sol. : **F.B.D.** of sphere 'A' :



F.B.D. of sphere 'B' :



(exerted by A)

Note : Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

APPLICATIONS OF NEWTON'S LAWS

(a) When objects are in equilibrium

To solve problems involving objects in equilibrium:

Step 1: Make a sketch of the problem.

Step 2: Isolate a single object and then draw the **free-body diagram** for the object. Label all external forces acting on it.

Step 3: Choose a convenient coordinate system and resolve all forces into rectangular components along x and y direction.

Step 4: Apply the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

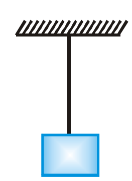
Step 5: Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.

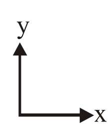
Eventually at step 5 you will have enough equations to solve for all unknown quantities.

Ex. A 'block' of mass 10 kg is suspended with string as shown in figure.

Find tension in the string. ($g = 10\ m/s^2$)



Sol. F.B.D. of block

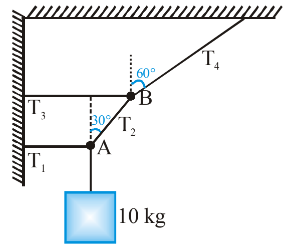


$\Sigma Fy = 0$

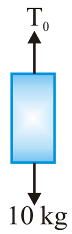
$T - 10\,g = 0$

$\therefore \quad T = 100\ N$

Ex. The system shown in figure is in equilibrium. Find the magnitude of tension in each string ; T_1, T_2, T_3 and T_4. ($g = 10\ m/s^{-2}$)



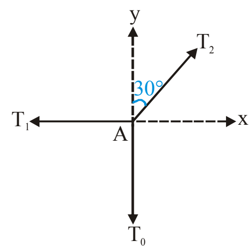
Sol. F.B.D. of block 10 kg



$T_0 = 10\ g$

$T_0 = 100\ N$

F.B.D of point 'A'



$\Sigma Fy = 0$

$T_2 \cos 30° = h\ T_0 = 100\ N$

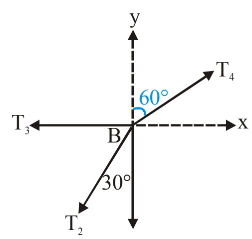
$$T_2 = \frac{200}{\sqrt{3}}\ N$$

$\Sigma F_x = 0$

$T_1 = T_2 \sin 30°$

$$= \frac{200}{\sqrt{3}} \cdot \frac{1}{2}$$

F.B.D. of point of 'B'



$\Sigma F_y = 0 \Rightarrow T_4\cos 60° = T_2\cos 30°$

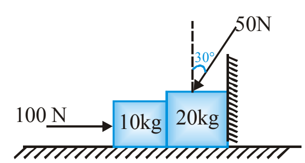
and $\Sigma F_x = 0 \Rightarrow T_3 + T_2 \sin 30° = T_4 \text{sins} 60°$

$\therefore$ $T3 = \dfrac{200}{\sqrt{3}}\ N$, $T_4 = 200\ N$

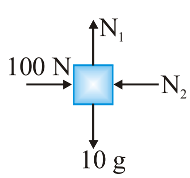
Ex. Two block are kept in contact as shown in figure. Find

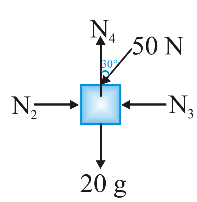
(a) forces exerted by surfaces (floor and wall) on blocks.

(b) contact force between two blocks.



Sol. F.B .D. of 10 kg block





$N_1 = 10\ g = 100\ N$ ....**(1)**

$N_2 = 100\ N$ ....**(2)**

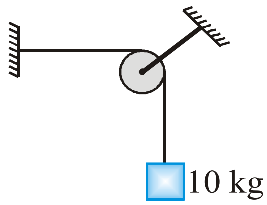
$N_2 = 50 \sin 30° + N_3$

$\therefore$ $N_3 = 100 - 25 = 75\ N$ ....**(3)**

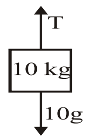
and $N_4 = 50 \cos 30° + 20\ g$

$N_4 = 243.30\ N$

Ex. Find magnitude of force exerted by string on pulley.

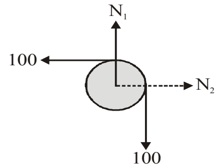


Sol. F.B.D. of 10 kg block :



$T = 10g = 100$ N

F.B.D. of pulley :



Since string is massless, so tension in both sides of string is same.

Force exerted by string

$$= \sqrt{(100)^2 + (100)^2} = 100\sqrt{2} \text{ N}$$

Note : Since pulley is in equilibrium position, so net forces on it is zero.

Hence force exerted by hinge on its is $100\sqrt{2}$ N.

(b) Accelerating Objects

To solve problems involving objects that are in accelerated motion:

Step 1: Make a sketch of the problem.

Step 2 : Isolate a single object and then draw the **free-body diagram** for that object. Label all external forces acting on it. Be sure to include all the forces acting on the chosen body, but be equally carefully not include any force exerted by the body on some other body. Some of the forces may be unknown; label them with algebraic symbols.

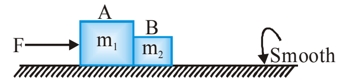
Step 3: Choose a convenient coordinate system, show location of coordinate axis exactly in the free-body diagram, and then determine components of forces with reference to these axes and resolve all forces into x and y components.

Step 4: Apply the equations $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$.

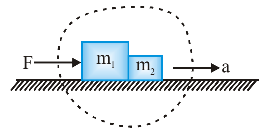
Step 5: Step 4 will give two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equation for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps. Eventually at step 5 you will have enough equations to solve for all unknown quantities.

Ex. A force F is applied horizontally on mass m_1 as shown in figure. Find the constant force between m_1 and m_2.



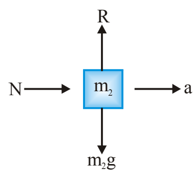
Sol. Considering both blocks as a system to find the common acceleration.



$$a = \frac{F}{(m_1 + m_2)} \qquad \text{.........(1)}$$

To find the contact force between 'A' and 'B' we draw F.B.D. of mass m_2.

F.B.D. of mass m_2



$$\Sigma F_x = ma_x$$

$$N = m_2.a$$

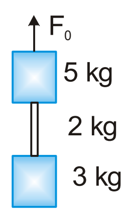
$$N = \frac{m_2 F}{(m_1 + m_2)}$$

Ex. The velocity of a particle of mass 2 kg is given by $\vec{v} = at\hat{i} + bt^2\hat{j}$. Find the force acting on the particle.

Sol. From second law of motion :

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) = 2.\frac{d}{dt}(at\hat{i} + bt^2\hat{j}) \Rightarrow \vec{F} = 2a\hat{i} + 4bt\hat{j}$$

Ex. A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at 2 m/s² by an external force F_0.



(a) What is F_0 ?

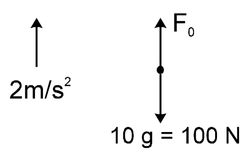
(b) What is the net force on rope ?

(c) What is the tension at middle point of the rope ? $(g = 10\ m/s^2)$

Sol. For calculating the value of F_0, consider two blocks with the rope as a system.

F.B.D. of whole system

(a)

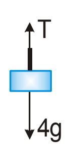


$F_0 - 100 = 10 \times 2 \qquad F = 120\ N \qquad$ .....**(1)**

(b) According to Newton's second law, net force on rope.

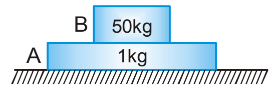
$F = ma = (2)(2) = 4\ N \qquad$ .....**(2)**

(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.



$T - 4g = 4.(2)\ ; \qquad T = 48\ N$

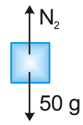
Ex. A block of mass 50 kg is kept on another block of mass 1 kg as shown in figure. A horizontal force of 10 N is applied on the 1kg block. (All surface are smooth). Find ($g = 10\ m/s^2$)



(a) Acceleration of block A and B.

(b) Force exerted by B on A.

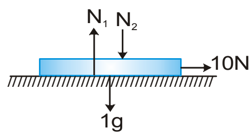
Sol. **(a) F.B.D. of 50 kg**



$N_2 = 50\ g = 500\ N$

along horizontal direction, there is no force $a_B = 0$

(b) F.B.D. of 1 kg block :



along horizontal direction

$10 = 1\ a_A$.

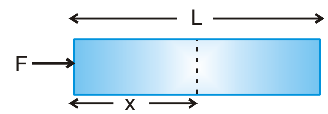
$a_A = 10\ m/s^2$

along vertical direction

$\therefore \quad N_1 = N_2 + 1g$

$= 500 + 10 = 510\ N$

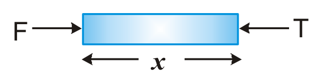
Ex. A horizontal force is applied on a uniform rod of length L kept on a frictionless surface. Find the tension in rod at a distance 'x' from the end where force is applied.



Sol. Considering rod as a system, we find acceleration of rod

$$a = \frac{F}{M}$$

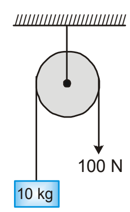
now draw F.B.D. of rod having length 'x' as shown in figure.



Using Newton's second law

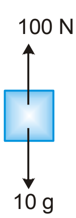
$$F - T = \left(\frac{M}{L}\right)x.a \Rightarrow \quad T = F - \frac{M}{L}x.\frac{F}{M} \quad \Rightarrow \quad T = F\left(1 - \frac{x}{L}\right).$$

Ex. One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass. ($g = 9.8\ m/s^2$)



Sol. Since string is pulled by 100 N force. So tension in the string is 100 N.

F.B.D. of 10 kg block

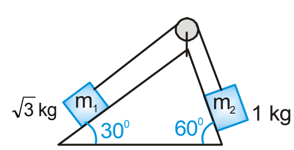


$100 - 10\ g = 10\ a$

$100 - 10 \times 9.8 = 10\ a$

$a = 0.2\ m/s^2$.

Ex. Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find :

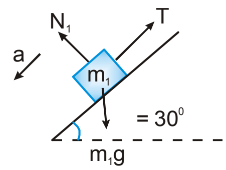


(i) Acceleration of mass m_1 and m_2

(ii) Tension in the string

(iii) Net force on pulley exerted by string

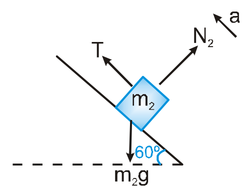
Sol. **F.B.D. of m_1 :**



$m_1 g\sin\theta - T = m_1 a$

$$\frac{\sqrt{3}}{2}g - T = \sqrt{3}\,a \qquad \text{..........(i)}$$

F.B.D. of m_2 :



$T - m_2 g\sin\theta = m_2 a$

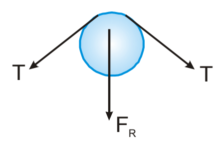
$$T - 1.\frac{\sqrt{3}}{2}g = 1.a \qquad \text{.........(ii)}$$

Adding eq.(i) and (ii) we get a = 0

Putting this value in eq.(i) we get

$$T = \frac{\sqrt{3}g}{2},$$

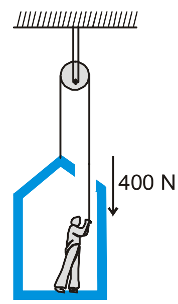
F.B.D. of pulley



$$F_R = \sqrt{2}\,T$$

$$F_R = \frac{\sqrt{3}}{2}g$$

Ex. A 60 kg painter stands on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.

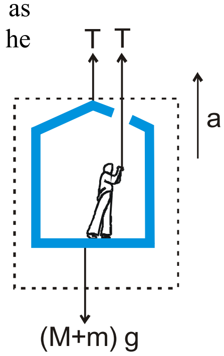


(i) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.

(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1s?

(iii) What force should he apply now to maintain the constant speed of 1 m/s?

Sol. The free body diagram of the painter and the platform as a system can be drawn as shown in the figure. Note that the tension in the string is equal to the force by which he pulls the rope.



(i) Applying Newton's Second Law

$$2T - (M+m)g = (M+m)a \quad \text{or} \qquad a = \frac{2T-(M+m)g}{M+m}$$

Here M = 60 kg; m = 15 kg; T = 400 N

$g = 10\ m/s^2$

$$a = \frac{2(400) - (60+15)(10)}{60+15} = 0.67\ m/s^2$$

(ii) To attain a speed of 1 m/s in one second, the acceleration a must be 1 m/s^2.

Thus, the applied force is

$$F = \frac{1}{2}(M+m)(g+a) = \frac{1}{2}(60+15)(10+1) = \mathbf{412.5\ N}$$

(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium.

Thus, $2F - (M+m)g = 0$

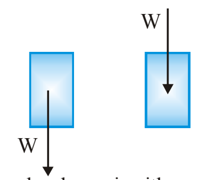
or $$F = \frac{(M+m)g}{2} = \frac{(60+15)(10)}{2} = \mathbf{375\ N}$$

VARIOUS FIELD FORCES

Field forces include the gravitational force (weight) electrostatic force and magnetic forces, which can easily be identified. At present, we consider only gravitational pull from the earth i.e. weight of the body

Weight : The net gravitational pull of the Earth

The gravitational pull from the earth acts on every particle of the body hence it is a distributed force. The net gravitational pull of the Earth on a body may be considered as weight of the body. It is assumed to act on the center of gravity of the body. For terrestrial bodies or celestial bodies of small size, this force can be assumed uniform throughout its volume. Under such circumstances, center of gravity and center of mass coincide and the weight is assumed to act on them. Further more, center of mass of uniform bodies lies at their geometrical center. At present, we discuss only uniform bodies and assume their weight to act on their geometrical center. In the figure weight of a uniform block is shown acting on its geometrical centre that coincides with the center of mass and the centre of gravity of the body.



Weight can be shown in either of the above two ways

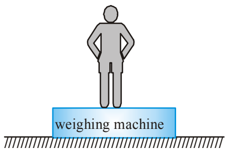
VARIOUS CONTACT FORCES

At every point where a body under consideration is supposed to be separated from other bodies to draw its free-body diagram, there may be a contact force. Most common contact forces, which we usually encounter, are tension force of a string, normal reaction on a surface in contact, friction, spring force etc.

1. Weighing Machine

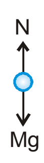
A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

Ex. A man of mass 60 Kg is standing on a weighing machine placed on ground. Calculate the reading of machine ($g = 10\ m/s^2$).



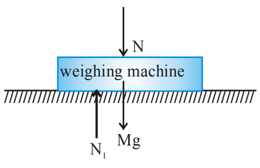
Sol. For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately.

F.B.D. of man



$N = Mg$

F.B.D. of weighing machine



Here force exerted by object on upper surface is N

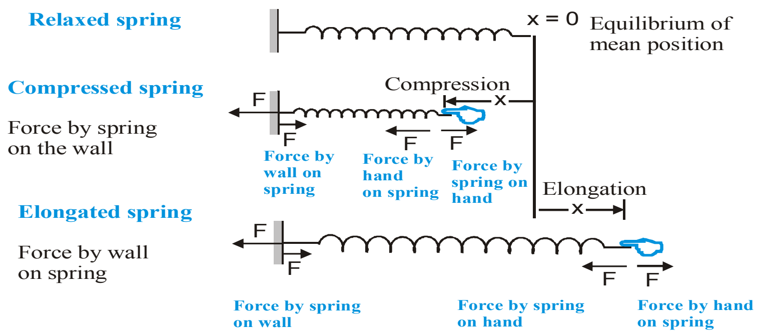
Reading of weighing machine

$N = Mg = 60 \times 10$

$N = 600\ N.$

2. Spring Force

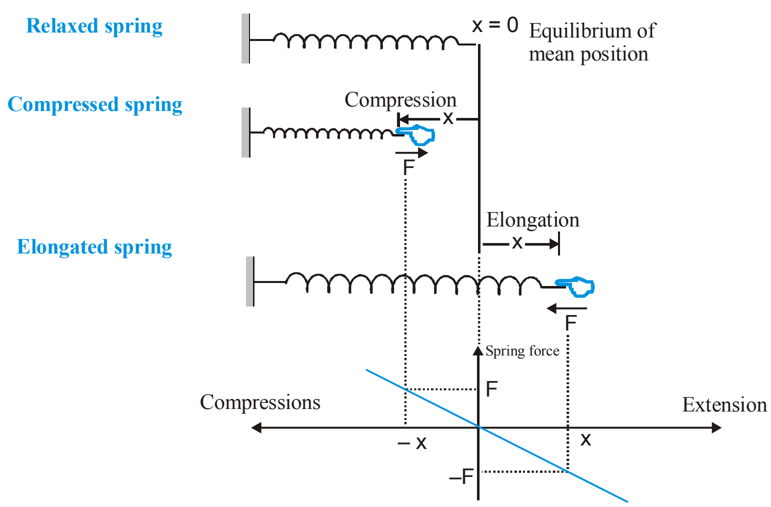
When no force acts on a spring, its is in relaxed condition i.e. neither compressed nor elongated. Consider a spring attached to a fixed support at one of its end and the other end is free. If we neglect gravity, it remains in relaxed state. When it is pushed by a force F, it is compressed and displacement x of its free end is called compression. When the spring is pulled by a force F, it is elongated and displacement x of its free end is called elongation. Various forces developed in these situations are shown in the following figure.



The force applied by the spring on the wall and the force applied by the wall on the spring make a third law action-reaction pair. Similarly, force by hand on the spring and the force by spring on the hand make another third law action -reaction pair

Hooke's Law :

How spring force varies with deformation in length x of the spring is also shown in the following figure.



The force F varies linearly with x and acts in a direction opposite to x. Therefore, it is expressed by the following equation

$$F = -kx$$

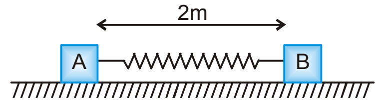
Here, the minus (–) sign represents the fact that force F is always opposite to x.

The constant of proportionality k is known as force constant of the spring or simply as spring constant. The slope modulus of the graph equals to the spring constant.

SI unit of spring constant is newton per meter of (N/m).

Dimensions of spring constant are MT^{-2} .

Ex. Two blocks are connected by a spring of natural length 2 m.
The force constant of spring is 200 N/m.
Find spring force in following situations :

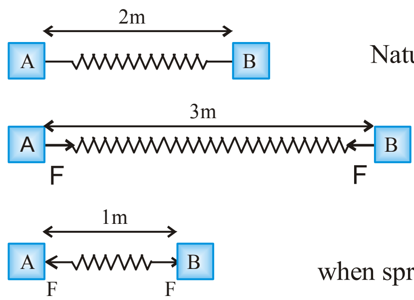


(a) If block 'A' and 'B' both are displaced by 0.5 m in same direction.

(b) If block 'A' and 'B' both are displaced by 0.5 m in opposite direction.

Sol. **(a)** Since both blocks are displaced by 0.5 m in same direction, so change in length of spring is zero. Hence, spring force is zero.

(b) In this case, change in length of spring is 1 m. In case of extension or compression of spring, spring force is $F = Kx = (200) . (1)$ $\quad F = 200\text{ N}$



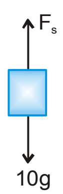
Natural length

when spring is extended

when spring is compressed

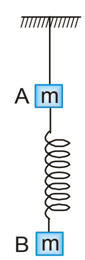
Ex. Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring. ($g = 10\text{ m/s}^2$)

Sol. In this situation, spring is in extended state so spring force acts in upward direction.
Let x be the extension in the spring.
F.B.D. of 10 kg block :



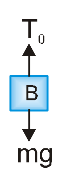
$$F_s = 10g$$
$$\Rightarrow \quad Kx = 100$$
$$\Rightarrow \quad (100)x = (100)$$
$$\Rightarrow \quad x = 1\text{m}$$

Ex. Two blocks 'A' and 'B' of same mass 'm' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block 'A' and 'B' just after the string is cut.



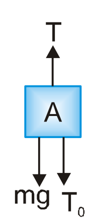
Sol. When block A and B are in equilibrium position

F.B.D of 'B'



$T_0 = mg$ .... **(i)**

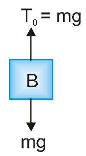
F.B.D of 'A'



$T = mg + T_0$
$T = 2\text{ mg}$ ....**(ii)**

when string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw F.B.D. of blocks A and B as shown in figure

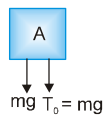
F.B.D. of 'B'



$T_0 - mg = ma_B$

$a_B = 0$

F.B.D. of 'A'

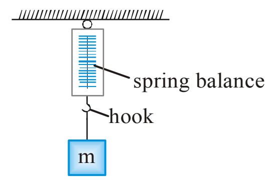


$mg + T_0 = ma_A$

$2mg = ma_A$

$a_A = 2g$ (downwards)

3. Spring Balance :

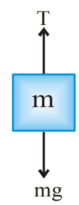


It does not measure the weight. It measures the force exerted by the object at the hook.

Symbolically, it is represented as shown in figure.

A block of mass 'm' is suspended at hook.

When spring balance is in equilibrium, we draw the F.B.D. of mass m for calculating the reading of balance.



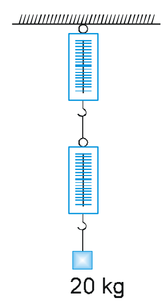
F.B.D. of 'm'.

$mg - T = 0$

$T = mg$

Magnitude of T gives the reading of spring balance.

Ex. A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the :

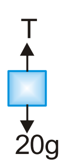


(1) reading of spring balance (1).

(2) reading of spring balance (2).

Sol. For calculating the reading, first we draw F.B.D. of 20 kg block.

F.B.D of 20 kg.



$mg - T = 0$

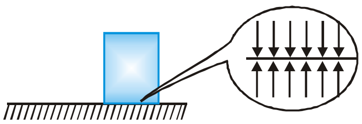
$T = 20\,g = 200\,N$

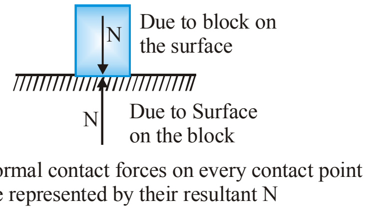
Since both balances are light so, both the scales will read 20 kg.

4. Normal Reaction

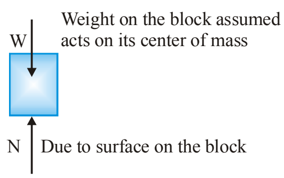
Two bodies in contact, when press each other, must apply equal and opposite forces on each other. These forces constitute a third law action-reaction pair. If surfaces of the bodies in contact are frictionless, this force acts along normal to the surface at the point of contact. Therefore, it is known as normal reaction.

consider a block of weight W placed on a frictionless floor. Because of its weight it presses the floor at every point in contact and the floor also supplies equal and opposite reaction forces on every point of contact. We show all of them by a single resultant N obtained by their vector addition.

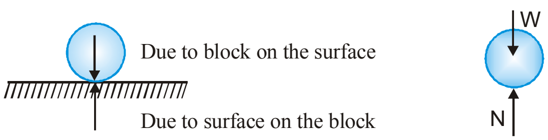




To apply Newton's laws of motion (NLM) on the block, its weight W and normal reaction N applied by the floor on the block must be considered as shown in the following figure. It is the FBD of the block.

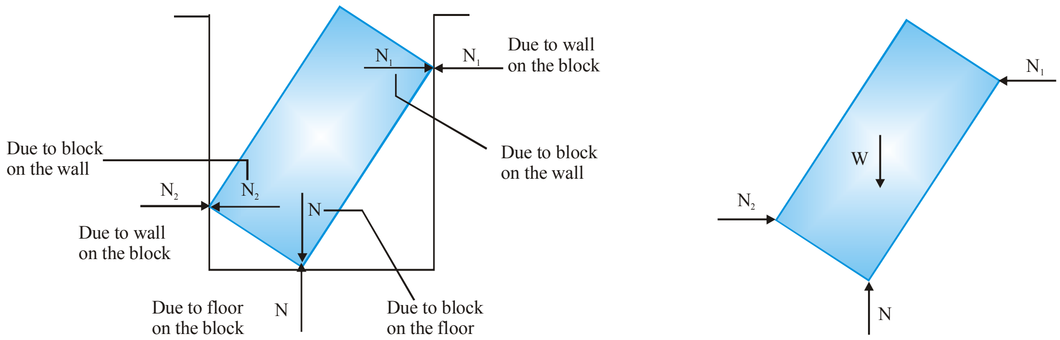


Consider a spherical ball of weight W placed on a floor. The reaction from the floor on the ball and from the ball on the floor makes third law action-reaction pair. These forces are shown in the left figure



To apply Newton's laws of motion (NLM) on the ball, its weight W and normal reaction N applied by the follow on the ball must be considered as shown in the above right figure. It is the FBD of the ball.

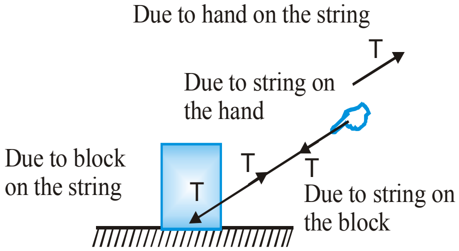
When two surfaces make contact, the normal reaction acts along the common normal and when a surface and placed in a rectangular trough as shown in the figure .



To apply Newton's laws of motion (NLM on the block, its free body diagram (FBD) is shown in the above right figure.

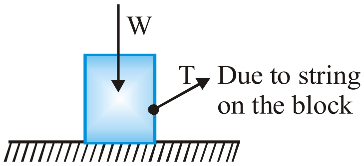
5. Tension force of strings

A string or similar flexible connecting links as a thread or a chain etc. we use to transmit a force. Due to flexibility, a string can be used only to pull a body connected to it by applying a force always along the string. According to the third law, the connected body must also apply an equal and apposite force on the string which makes the string taut. Therefore, this force is known as tension force T of the string. In the given figure is shown a block pulled by a string, which is being pulled by a person.

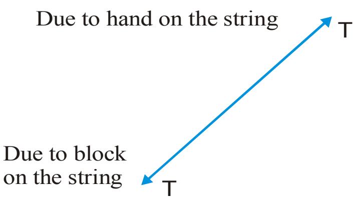


The tension force applied by string on the block and the force applied by the block on the string shown in the figure constitute a third law action-reaction pair similarly, tension force applied by the string on hand and force applied by the hand on string is another third law action-reaction pair.

While studying motion of the block, the force applied by the string on it, weight of the block and a reaction from the floor has to be considered. In the figure only weight and tension of string are shown.



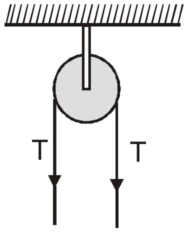
To study motion of the string, the force applied by the block on the string and the force applied by the hand on the string must be considered. These forces are shown in the FBD of string



To study conditions of motion of the person, the force applied by the string on the hand has to be considered as shown in the figure.

6. String passing over a pulley

A pulley is device consisting of a wheel, which can rotate freely on its axil. A single pulley changes direction of tension force. At present for simplicity, we discuss only ideal pulley, which is massless i.e. has negligible mass and rotates on its axil without any friction. An ideal pulley offers no resistance to its rotation, therefore tension force in the string on both sides of it are equal in magnitude, Such a pulley is known as ideal pulley.



CONSTRAINED MOTION:

1. String Constraint :

When two objects are connected through a string and if the string have the following properties :

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them.

Steps for String Constraint

Step 1. Identify all the objects and number of strings in the problem.

Step 2. Assume variable to represent the parameters of motion such as displacement, velocity acceleration etc.

(i) Object which moves along a line can be specified by one variable.

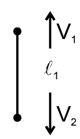
(ii) Object moving in a plane are specified by two variables.

(iii) Objects moving in 3-D requires three variables to represent the motion.

Step 3. Identify a single string and divide it into different linear sections and write in the equation format.

$$\ell_1+\ell_2+\ell_3+\ell_4+\ell_5+\ell_6=\ell$$

Step 4. Differentiate with respect to time



$$\frac{d\ell_1}{dt}+\frac{d\ell_2}{dt}+\frac{d\ell_3}{dt}+....=0$$

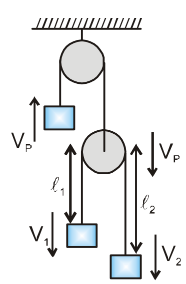
$\frac{d\ell_1}{dt}$ = represents the rate of increment of the portion 1, end points are always in contact with some object

So take the velocity of the object along the length of the string $\frac{d\ell_1}{dt}=V_1+V_2$

Take positive sign if it tends to increase the length and negative sign if it tends to decrease the length. Here $+V_1$ represents that upper end is tending to increase the length at rate V_1 and lower end is tending to increase the length at rate V_2.

Step 5. Repeat all above steps for different-different strings.

Let us consider a problem given below



Here $l_1 + l_2$ = constant

$$\frac{d\ell_1}{dt}+\frac{d\ell_2}{dt}=0$$

$$(V_1-V_P)+(V_P-V_2)=0$$

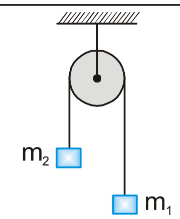
$$V_p=\frac{V_1+V_2}{2}$$

Similarly, $a_P=\frac{a_1+a_2}{2}$ Remember this result

Ex. Two blocks of masses m_1 and m_2 are attached at the ends of an inextensible string which passes over a smooth massless pulley. If $m_1 > m_2$, find :

(i) the acceleration of each block

(ii) the tension in the string.



Sol. The block m_1 is assumed to be moving downward and the block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction. If the values of a_1 and a_2 come out to be positive then only the assumed directions are correct; otherwise the body moves in the opposite direction. Since the pulley is smooth and massless, therefore, the tension on each side of the pulley is same.

The free body diagram of each block is shown in the figure.

F.B.D. of m_2 **F.B.D. of m_1**

F.B.D. of m_2 **F.B.D. of m_1**

Applying Newton's second Law on blocks m_1 and m_2

Block m_1 $\quad m_1g - T = m_1a_1$ ....**(1)**

Block m_2 $\quad -m_2g + T = m_2a_2$ ....**(2)**

Number of unknowns : T, a_1 and a_2 (three)

Number of equations: only two

Obviously, we require one more equation to solve the problem. Note that whenever one finds the number of equations less than the number of unknowns, one must think about the constraint relation. Now we are going to explain the mathematical procedure for this.

How to determine Constraint Relation ?

(1) Assume the direction of acceleration of each block, e.g. a_1 (downward) and a_2 (upward) in this case.

(2) Locate the position of each block from a fixed point (depending on convenience), e.g. centre of the pulley in this case.

(3) Identify the constraint and write down the equation of constraint in terms of the distance assumed. For example, in the chosen problem, the length of string remains constant is the constraint or restriction.

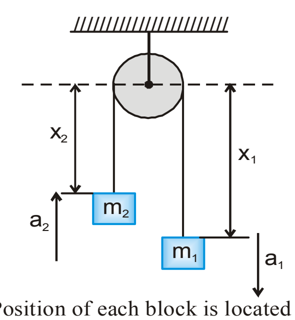
Thus, $x_1 + x_2 = \text{constant}$

Differentiating both the sides w.r.t. time we get $\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$

Each term on the left side represents the velocity of the blocks.

Since we have to find a relation between accelerations,

therefore we differentiate it once again w.r.t. time.



Position of each block is located w.r.t. centre of the pulley

Thus $\quad \frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$

Since, the block m_1 is assumed to be moving downward (x_1 is increasing with time)

$$\therefore \qquad \frac{d^2x_1}{dt^2} = +a_1$$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

$$\therefore \qquad \frac{d^2x_2}{dt^2} = -a_2$$

Thus $\quad a_1 - a_2 = 0$

or $\quad a_1 = a_2 = a$ (say) is the required constraint relation.

Substituting $a_1 = a_2 = a$ in equations **(1)** and **(2)** and solving them, we get

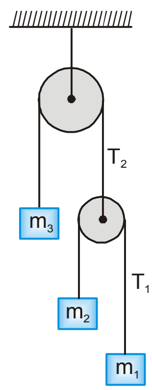
(i) $\quad a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] g \qquad$ **(ii)** $\quad T = \left[\frac{2m_1m_2}{m_1 + m_2}\right] g$

Ex. A system of three masses m_1, m_2 and m_3 are shown in the figure.

The pulleys are smooth and massless; the strings are massless and inextensible.

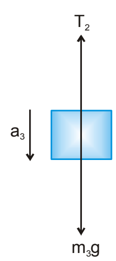
(i) Find the tensions in the strings.

(ii) Find the acceleration of each mass.

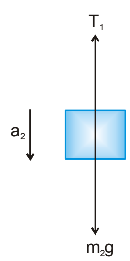


Sol. All the blocks are assumed to be moving downward and the free body diagram of each block is shown in figure.

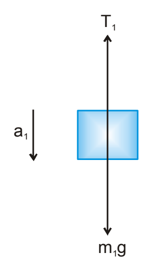
F.B.D. m_3



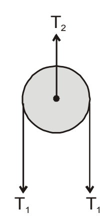
F.B.D. m_2



F.B.D. m_1



F.B.D. of pulley



Applying Newton's Second Law to

Block m_1 : $\quad m_1g - T_1 = m_1a_1 \qquad$ ....**(1)**

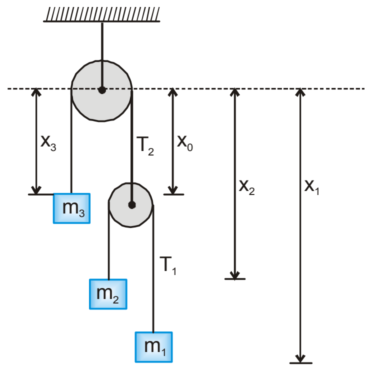
Block m_2 : $\quad m_2g - T_1 = m_2a_2 \qquad$ ....**(2)**

Block m_3 : $\quad m_3g - T_2 = m_3a_3 \qquad$ ....**(3)**

Pulley : $\quad T_2 = 2T_1 \qquad$ ....**(4)**

Number of unknowns a_1, a_2, a_3, T_1 and T_2 (Five)

Number of equations : Four



The constraint relation among accelerations can be obtained as follows

For upper string $\quad x_3 + x_0 = c_1$

For lower string $\quad x_2 - x_0) + (x_1 - x_0) = c_2$

$$x_2 + x_1 - 2x_0 = c_2$$

Eliminating x_0 from the above two relations,

We get $\quad x_1 + x_1 + 2x_3 = 2c_1 + c_2 = \text{constant.}$

Differentiating twice with respect to time,

We get $$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} + 2\frac{d^2x_3}{dt^2} = 0 \quad \text{or} \quad a_1 + a_2 + 2a_3 = 0 \qquad \text{......(5)}$$

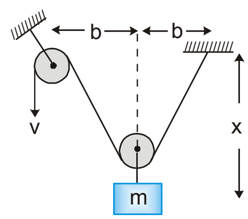
Solving equations **(1)** to **(5)**, we get

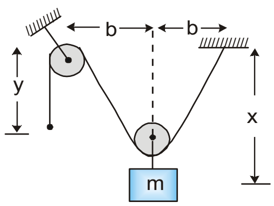
(i) $$T_1 = \left[\frac{4m_1m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g \quad ; \quad T_2 = 2T_1$$

(ii) $$a_1 = \left[\frac{4m_1m_2 + m_1m_3 - 3m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g \quad ; \quad a_2 = \left[\frac{3m_1m_3 - m_2m_3 - 4m_1m_2}{4m_1m_2 + m_3(m_1+m_2)}\right]g$$

$$a_3 = \left[\frac{4m_1m_2 - m_3(m_1+m_2)}{4m_1m_2 + m_3(m_1+m_2)}\right]g$$

Ex. The figure shows one end of a string being pulled down at constant velocity v. Find the velocity of mass 'm' as a function of 'x'.





Sol. Using constraint equation

$2\sqrt{x^2 + b^2} + y$ = length of string = constant

Differentiating w.r.t. time :

$$\frac{2}{2\sqrt{x^2+b^2}} \cdot 2x\left(\frac{dx}{dt}\right) + \left(\frac{dy}{dt}\right) = 0$$

$$\Rightarrow \left(\frac{dy}{dt}\right) = v \Rightarrow \left(\frac{dx}{dt}\right) = -\frac{v}{2x}\sqrt{x^2+b^2}$$

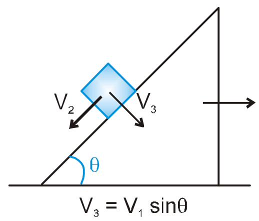
2. Wedge Constraint :

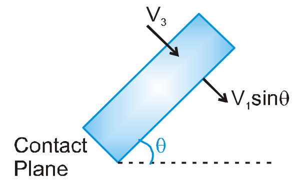
Conditions :

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.

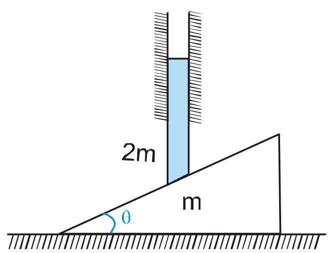




In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

Ex. A rod of mass 2m moves vertically downward on the surface of wedge of mass as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.



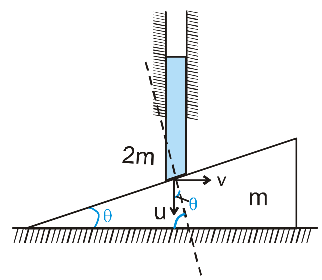
Sol. Using wedge constraint.

Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.

u cos q = v sin q

$$\frac{u}{v} = \tan q$$

u = v tan q



perpendicular to contact of two blocks

TRANSLATION EQUILIBRIUM

A body in state of rest or moving with constant velocity is said to be in translational equilibrium. Thus if a body is in translational equilibrium in a particular inertial frame of reference, it must have no linear acceleration. When it is at rest, it is in static equilibrium, whereas if it is moving at constant velocity it is in dynamic equilibrium.

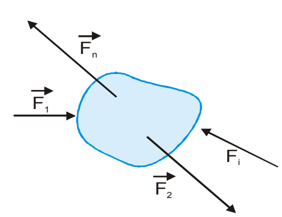
Conditions for translational equilibrium

For a body to be in translational equilibrium, no net force must act on it i.e. vector sum of all the forces acting on it must be zero.

If several external forces $\vec{F}_1, \vec{F}_2......\vec{F}_1.....$ and $\vec{F}_n$ act simultaneously on a body and the body is in translational equilibrium, the resultant of these forces must be zero.

$$\sum \vec{F}_i = \vec{0}$$

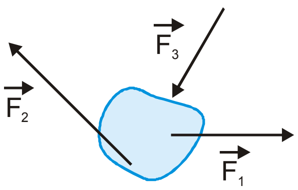
If the forces $\vec{F}_1, \vec{F}_2......\vec{F}_1.....$ and $\vec{F}_n$ are expressed in Cartesian components, we have



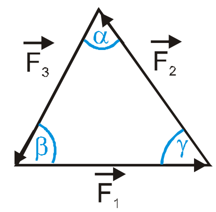
$$\sum F_{tx} = 0 \qquad \sum F_{ty} = 0 \qquad \sum F_{tx} = 0$$

If a body is acted upon by a single external force, it cannot be in equilibrium.

If a body is in equilibrium under the action of only two external forces, the forces must be equal and opposite.
If a body is in equilibrium under action of three forces, their resultant must be zero; therefore, according to the triangle law of vector addition they must be coplanar and make a closed triangle.

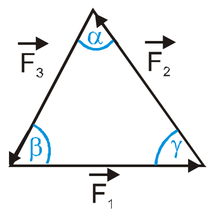


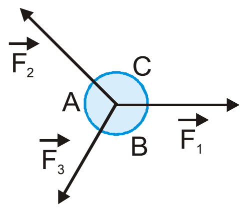
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0} \Rightarrow$$



The situation can be analyzed by either graphical method or analytical method.
Graphical method makes use of sine rule or Lami's theorem

Sine rule : $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$ **Lami's theorem** : $\dfrac{F_1}{\sin A} = \dfrac{F_2}{\sin B} = \dfrac{F_3}{\sin C}$

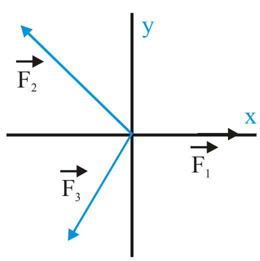




Analytical method makes use of Cartesian components. Since the forces involved make a closed triangle, they lie in a plane and two-dimensional Cartesian frame can be used to resolve the forces. As far as possible orientation of the x-y frame is selected in such a manner that angles made by forces with axes should have convenient values.

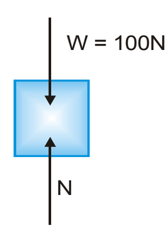
$$\sum F_x = 0 \Rightarrow F_{1x} + F_{2x} + F_{3x} = 0$$
$$\sum F_y = 0 \Rightarrow F_{1y} + F_{2y} + F_{3y} = 0$$

Problems involving more than three forces should be analyzed by analytical method. However in, in some situations, there may be some parallel or anti-parallel forces and they should be combined first to minimize the number of forces. This may sometimes lead a problem involving more than three forces to a three-force system.



Ex. Consider a box of mass 10 kg resting on a horizontal table and acceleration due to gravity to be 10 m/s^2.
(a) Draw the free body diagram of the box.
(b) Find value of the force exerted by the table on the box.
(c) Find the value of the force exerted by the box on the table
(d) Are force exerted by table on the box and weight of the box third law action-reaction pair ?

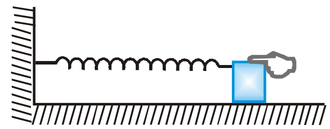
Sol. **(a)** N : Force exerted by table on the box.



(b) The block is in equilibrium, $\sum \vec{F} = \vec{0} \Rightarrow W - N = 0 \Rightarrow N = 100$ N

(c) N = 100 N : Because force by table on the box and force by box on table make Newton's third law pair.
(d) No

Ex. Consider a spring attached at one of its ends to a fixed support and at other end to a box, which rests on a smooth floor as shown in the figure. Denote mass of the box by m, force constant of the spring by k and acceleration due to gravity by g.



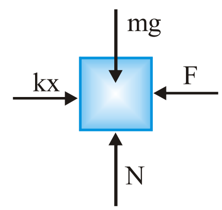
The box is pushed horizontally displacing it by distance x towards the fixed support and held at rest.

(a) Draw free body diagram of the box.

(b) Find force exerted by hand on the box.

(c) Write all the third law action-reaction pairs.

Sol. **(a)** F is push by hand



(b) Since the block is in equilibrium $\Sigma F_x = 0 \Rightarrow F = kx$

(i) Force by hand on box and force by box on hand.

(ii) Force by spring on box and force by box on spring.

(iii) Normal reaction by box on floor and normal reaction by floor on box.

(iv) Weight of the box and the gravitational force by which box pulls the earth.

(v) Force by spring on support and force by support on spring

Ex. **(a)** A box of weight $10\sqrt{3}$ N is held in equilibrium with the help of two strings OA and OB as shown in figure-I, The string OA is horizontal, Find the tensions in both the strings.

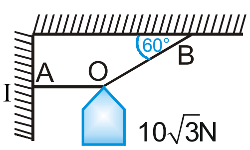
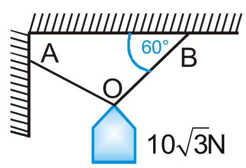
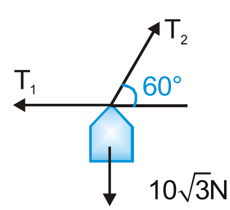
Fig. I


Fig. II


(b) If you can change location of the point A on the wall and hence the orientation of the string OA without altering the orientation of the string OB as shown in figure-II. What angle should the string OA make with the wall so that a minimum tension is developed in it?

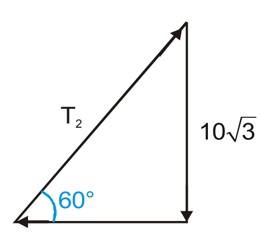
Sol. **(a)** Free body diagram of the box



Graphical Method : Use triangle law

$$T_2 \sin 60° = 10\sqrt{3} \Rightarrow T_2 = 20\text{N}$$

$$T_1 \tan 60° = 10\sqrt{3} \Rightarrow T_1 = 10\text{ N}$$

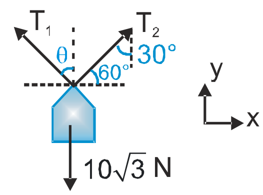


Analytical Method : Use Cartesian components

$$\sum F_x = 0 \Rightarrow T_2 \cos 60° = T$$

$$\sum F_y = 0 \Rightarrow T_2 \sin 60° = 10\sqrt{3}$$

From equation **(i)** & **(ii)** we have $T_1 = 10N$ and $T_2 = 20N$

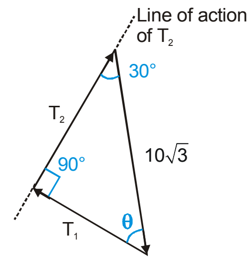


(b) Free body diagram of the box

Graphical Method : Use triangle law

For T_1 to be minimum, it must be perpendicular to T_2.

From figure $\theta = 60°$



Analytical Method : Use Cartesian components

$$\sum F_x = 0 \Rightarrow T_2\cos 60° = T_1\sin\theta \qquad ....\text{(i)}$$

$$\sum F_y = 0 \Rightarrow T_1\cos\theta + T_2\sin 60° = 10\sqrt{3} \qquad ....\text{(ii)}$$

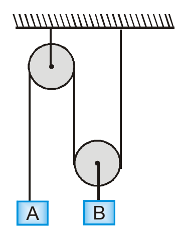
From equation **(i)** and **(ii)**, we have $T_1 = \dfrac{10\sqrt{3}}{\sqrt{3}\sin\theta + \cos\theta}$

If T_1 is minimum, $\sqrt{3}\sin\theta + \cos\theta$ must be maximum. maximum value of $\sqrt{3}\sin\theta + \cos\theta$ is 2.

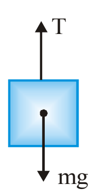
$$\sqrt{3}\sin\theta + \cos\theta = 2$$

Solving the above equation we get $\theta = 60°$

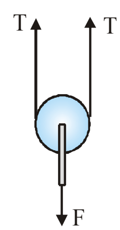
Ex. Two boxes A and B massed m and M are suspended by a system of pulleys are in equilibrium as how. Express M in terms of m.



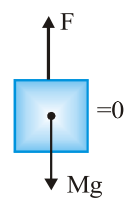
Sol. Since tension on both sides of a pulley are equal and string is massless therefore tension everywhere on the string must have same magnitude



FBD of block A



FBD of pulley



FBD of block B

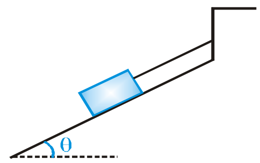
For equilibrium of block A $\quad \sum \vec{F} = \vec{0} \Rightarrow T = mg \qquad$ ...**(i)**

For equilibrium of pulley attached to block B $\quad \sum \vec{F} = \vec{0} \Rightarrow F = 2T \qquad$ ...**(ii)**

For equilibrium of block B $\quad \sum \vec{F} = \vec{0} \Rightarrow F = Mg \qquad$ ...**(iii)**

From equation **(i)**, **(ii)** and **(iii)**, we have $M = 2m$

Ex. A box of mass m rests on a smooth slope with help of a thread as shown in the figure. The thread is parallel to the incline plane.

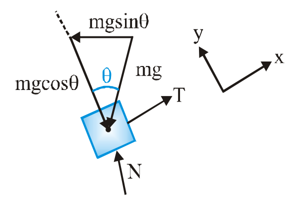


(a) Draw free body diagram of the box.

(b) Find tension in the thread.

(c) If the thread is replaced by a spring of force constant k, find extension in the spring.

Sol. **(a)** Free body diagram of the block



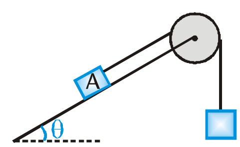
(b) The block is in equilibrium, therefore $\sum F_x = 0 \Rightarrow T = mgsin\theta$ ...**(i)**

(c) If the thread is replaced by a spring of force must be equal to T, therefore

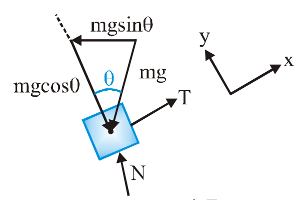
$T = kx$ ...**(ii)**

From equation **(i)** and **(ii)**, we have $x = \dfrac{mg\sin\theta}{k}$

Ex. Block A of mass m placed on a smooth slope is connected by a string with another block B of mass M as shown in the figure. If the system is in equilibrium, express M in terms of m.



Sol.



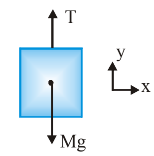
For equilibrium of the block A, net force on it must be zero

N : Normal reaction from slope

$$\sum F_x = 0 \Rightarrow T = mgsin\theta \quad ...\textbf{(i)}$$

$$\sum F_y = 0 \Rightarrow N = mgcos\theta \quad ...\textbf{(ii)}$$

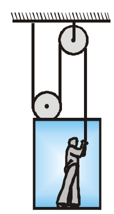
For equilibrium of block B, the net force on it must be zero



$$\sum F_y = 0 \Rightarrow T = mg \quad ...\textbf{(iii)}$$

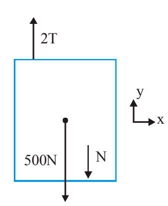
From equation **(i)** and **(i)**, we have M = msinq

Ex. A 70 kg man standing on a weighing machine in a 50 kg lift pulls on the rope, which supports the lift as shown in the figure. Find the force with which the man should pull on the rope to keep the lift stationary and the weight of the man as shown by the weighing machine.



Sol.

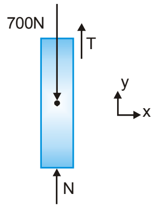
Tension magnitude everywhere in the string is same. For equilibrium of the lift



$$\sum F_y = 0 \Rightarrow 500 + N = 2T \quad ...\textbf{(i)}$$

To analyse equilibrium of the man let us assume him as a block

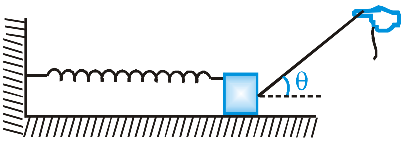
$$\sum F_y = 0 \Rightarrow N + T = 700$$ ...(ii)



From equation (i) & (ii), we have T = 400 N and N = 300 N

Here, T is the pull of mass and N is reading of the weighing machine.

Ex. A block of mass m placed on a smooth floor is connected to a fixed support with the help of a spring of force constant k. It is pulled by a rope as shown in the figure. Tension force T of the rope is increased gradually without changing its direction, until the block losses contact form the floor. The increase in rope tension T is so gradual the acceleration in the block can be neglected.

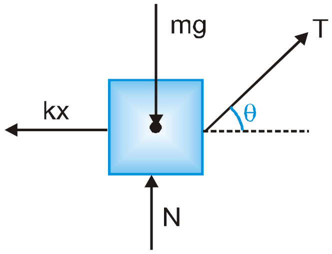


(a) Well before the block losses contact from the floor, draw its free body diagram.

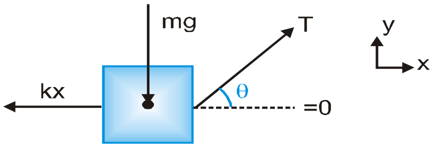
(b) What is the necessary tension in the rope so that the block loses contact from the floor ?

(c) What is the extension in the spring. When the block looses contact with the floor ?

Sol. **(a)** Free body diagram of the block, well before it looses contact with the floor.



(b) When the block is about the leave the floor, it is not pressing the floor. Therefore N = 0 and the block is in equilibrium.



$$\sum F_x = 0 \Rightarrow 0 \Rightarrow T\cos\theta = kx$$ ...(i)

$$\sum F_y = 0 \Rightarrow 0 \Rightarrow T\sin\theta = mg$$ ...(ii)

From equation (ii), we have T = mg cosec θ

(c) From equation (i) and (ii), we have $x = \dfrac{mg\cot\theta}{k}$

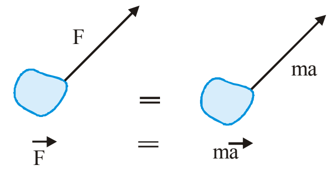
DYNAMICS OF PARTICLES

Translation motion of accelerated bodies

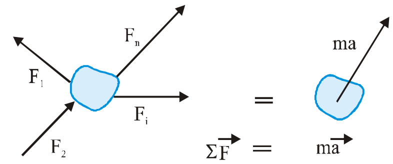
Newton's laws are valid in inertial frames, which are un-accelerated frames. At present we are interested in motion of terrestrial bodies and for this purpose; ground can be assumed a satisfactory inertial frame.

In particle dynamics, according to Newton's second law, forces acting on the body are considered as cause and rate of change in momentum as effect. For a rigid body of constant mass, the rate of change in momentum equals to product of mass and acceleration vector. Therefore, forces acting on it are the cause and product of mass and acceleration vector is the effect.

To write the equation of motion it is recommended to draw the free body diagram, put a sign of equality and in front of it draw the body attached with a vector equal to mass times acceleration produced. In the figure is shown a body of mass m on which a single force $\vec{F}$ acts and an observer in an inertial frame of reference observes the body moving with acceleration $\vec{a}$.



Acceleration imparted to a body by a force is independent of other forces, therefore when several forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_n$ act simultaneously on a body, the acceleration imparted to the body is the same as a single force equal to the vector sum of these forces could produce. The vector sum of these force is known as the net resultant of these forces.



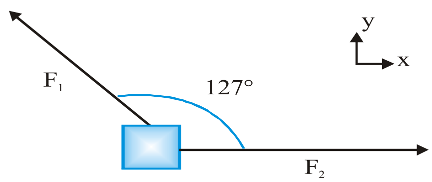
$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \ldots\ldots\ldots\ldots\ldots\ldots + \vec{F}_n = m\vec{a}$$

$$\sum \vec{F} = m\vec{a}$$

In cartesian coordinate system the vector quantities in the above equation is resolved into their components along x, y and z axes as follows :

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_x = ma_z$$

Ex. Two forces F_1 and F_2 of magnitudes 50 N act on a free body of mass m = 5 kg in directions shown in the figure. What is acceleration of object with respect to the free space?



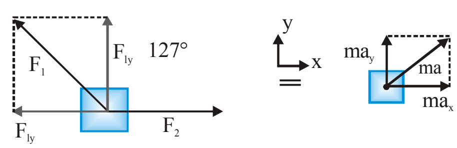
Sol. In an inertial frame of reference with its x-axis along the force F_2 the forces are expressed in Cartesian components.

$\vec{F}_1\left(-30\hat{i} + 40\hat{j}\right)$N and $\vec{F}_2 60\hat{i}$ N

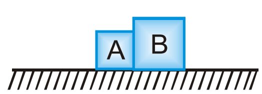
$\sum F_x = ma_x \Rightarrow a_x = 6\,m/s^2$

$\sum F_y = ma_y \Rightarrow a_y = 8\,m/s^2$

$\vec{a} = \left(6\hat{i} + 8\hat{j}\right)$ m/s²



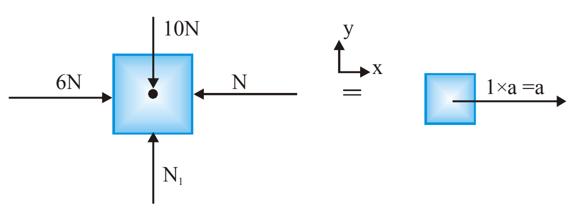
Ex. Boxes A and B of mass m_A = 1 kg and m_B = 2 kg are placed on a smooth horizontal plane,. A man pushes horizontally the 1 kg box with a force F = 6N. Find the acceleration and the reaction force between the boxes.



Sol. Since both the blocks move in contact it is obvious that both of them have same acceleration. Say it is 'a'. Applying NLM to block A

N : Normal reaction from B

N_1 : Normal reaction from floor



$$\sum F_x = ma_x \Rightarrow 6 - N = a \qquad \ldots(i)$$

$$\sum F_y = 0 \Rightarrow N_1 = 10\,N \qquad \ldots(ii)$$

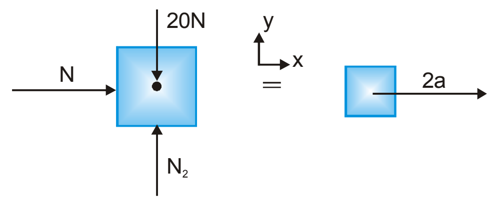
Applying NLM to block B

N : Normal reaction from A

N_2 : Normal reaction from ground

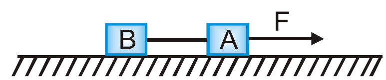
$\sum F_x = ma_x \Rightarrow N = 12a$ ...**(iii)**

$\sum F_y = 0 \Rightarrow N_2 = 20N$ ...**(iv)**



From equation **(i)** & **(iii)**, we have a = 2 m/s² and N = 4 N

Ex. Two blocks A and B of masses m_1 and m_2 connected by light strings are placed on a smooth floor as shown in the figure. If the block A is pulled by a constant force F, find accelerations of both the blocks and tension in the string connecting them.



Sol.

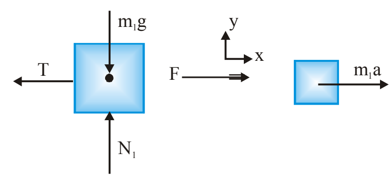
String connecting the blocks remain taut keeping separation between them constant. Therefore it is obvious that both of them move with the same acceleration Say it is 'a'.

T : Tension of string

N_1 : Normal reaction from ground

$\sum F_x = ma \Rightarrow F - T = m_1 a$ ...**(i)**

$\sum F_y = 0 \Rightarrow N_1 = m_1 g$ ...**(ii)**



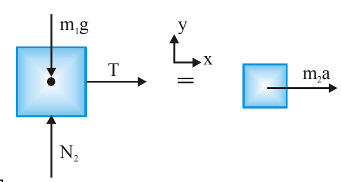
Applying NLM to block B.

T : Tension of string

N_1 : Normal reaction from ground

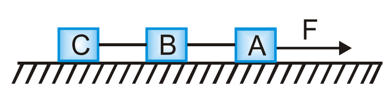
$\sum F_x = ma_x \Rightarrow T = m_2 a$ ...**(iii)**

$\sum F_y = 0 \Rightarrow N_2 = m_2 g$ ...**(iv)**



From equation **(i)** and **(iii)**, we have $a = \dfrac{F}{m_1 + m_2}$ and $T = \dfrac{m_2 F}{m_1 + m_2}$

Ex. Three identical blocks A, B and C, each of mass 2.0 kg are connected by light strings as shown in the figure. If the block A is pulled by an unknown force F, the tension in the string connecting blocks A and B is measured to be 8.0 N. Calculate magnitude of the force F, tension in the string connecting blocks B and C, and accelerations of the blocks.



Sol. It is obvious that all the three blocks move with the same acceleration say it is 'a'.

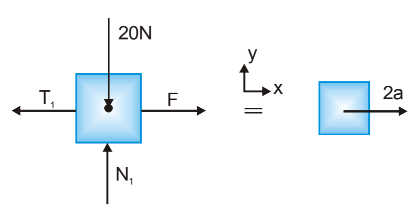
Applying NLM to the block A.

T_1 : Tension of string connecting blocks A and B

N_1 : Normal reaction from floor.

$\sum F_x = ma_x \Rightarrow F - T_1 = 2a$ ...**(i)**

$\sum F_y = 0 \Rightarrow N_1 = 20N$ ...**(ii)**

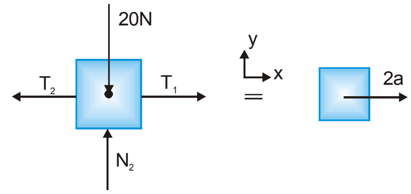


Applying NLM to the block B.

T_1 : Tension of string connecting blocks A and B

T_2 : Tension of string connecting B & C.

N_1 : Normal reaction from floor.



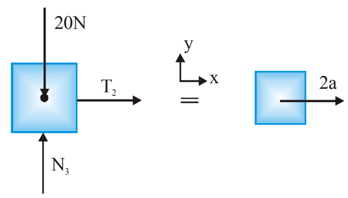
$$\sum F_x = ma_x \Rightarrow T_1 - T_2 = 2a \qquad \text{...(iii)}$$

$$\sum F_y = 0 \Rightarrow N_2 = 20aN \qquad \text{...(iv)}$$

Applying NLM to the block C.

T_3 : Tension of string connecting B & C

N_3 : Normal reaction from floor.



$$\sum F_x = ma_x \Rightarrow T_2 = 2a \qquad \text{...(v)}$$

$$\sum F_y = 0 \Rightarrow N_3 = 20N \qquad \text{...(vi)}$$

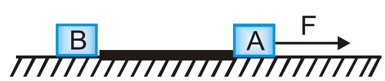
From equation **(i)**, **(iii)** and **(v)**, we have $F = 6a$

Now using the fact that $T_1 = 8N$ with equation **(i)**, we have $a = 2\ m/s^2$

Now from equation **(i)** we have $F = 12$ N

From equation **(iii)**, we have $T_2 = 4$ N

Ex. Two blocks A and B of masses m_1 and m_2 connected by uniform string of mass m and length l are placed on smooth floor as shown in the figure. The string also lies on the floor. The block A is pulled by constant force F.



(a) Find acceleration a of both the blocks and tensions T_A and T_B at the ends of the string.

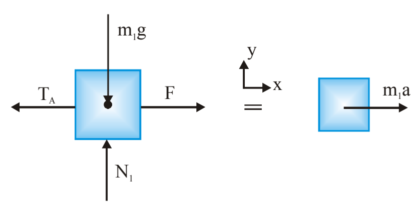
(b) Find an expression for tension T in the string at a distance x from the rear block in terms of T_A, T_B, m,l and x.

Sol. It is obvious that both the blocks and the whole string move with the same acceleration say it is 'a'. Since string has mass it may have different tensions at different points.

(a) Applying NLM to block A.

T_A : Tension of the string at end connected to block A.

N_1 : Normal reaction of floor



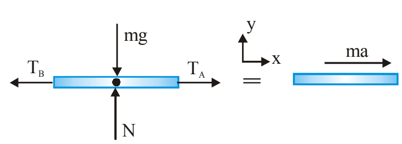
$$\sum F_x = ma_x \Rightarrow F - T = m_1a \qquad \text{...(i)}$$

$$\sum F_y = 0 \Rightarrow N_1 = m_1g \qquad \text{...(ii)}$$

Applying NLM to the rope

T_B : Tension of string at end connected to block B.

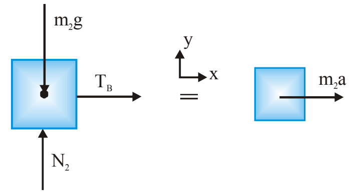
N : Normal reaction of floor



$$\sum F_x = ma_x \Rightarrow T_A - T_B = ma \qquad \text{...(iii)}$$

$$\sum F_y = 0 \Rightarrow N = mg \qquad \text{...(iv)}$$

Applying NLM to the block B



T_B : Tension of string ...**(v)**

N_2 : Normal reaction from floor ...**(vi)**

$$\sum F_x = ma_x \Rightarrow T_B = m_2a$$

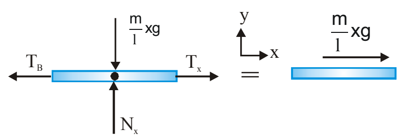
$$\sum F_y = 0 \Rightarrow N_2 = m_2g$$

From equation **(i)**, **(iii)** and **(v)**, we have

$$a = \frac{F}{m + m_1 + m_2} \quad \text{...(vii)} \qquad T_A = \frac{(m + m_2)F}{m + m_1 + m_2} \quad \text{...(viii)} \qquad T_B = \frac{m_2F}{m + m_1 + m_2} \quad \text{...(ix)}$$

(b) To find tension at a point x distance away from block B, we can consider string of length x or $l - x$.
Let as consider length of string x and apply NLM.

$\frac{mx}{\ell}$: mass of length x.



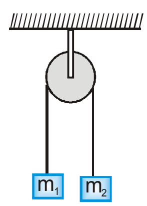
T_x = Tension at distance x

N_x = Normal reaction of floor

$$\sum F_x = ma_x \Rightarrow T_x - T_B = \frac{mx}{\ell}a \qquad \text{....(x)}$$

From equation **(vii)**, **(viii)**, **(ix)** and **(x)**0, we have $T_x = \left(\frac{m}{\ell}x + m_2\right)\frac{F}{m + m_1 + m_2}$

Ex. The system shown in the figure is released from rest. Assuming mass m_2 more than the mass m_1, find the accelerations of the blocks and the tension in the string.



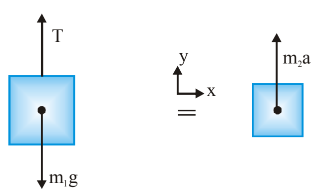
Sol.

In obvious that both blocks move with same acceleration magnitudes. Say it is 'a'. Since m_2 is heavier, it moves downwards and m_1 moves upwards.

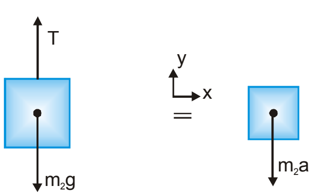
Tension at both the ends of the string has same magnitude. Say it is 'T'

Apply NLM to block A of mass m_1

$$\sum F_y = ma_y \Rightarrow T - m_1g = m_1a \qquad \text{...(i)}$$



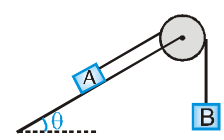
Apply NLM to block of mass m_2



$$\sum F_y = ma_y \Rightarrow T - m_2g - T = m_2a \qquad \text{...(ii)}$$

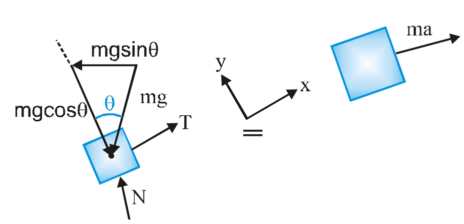
From equation **(i)** & **(ii)**, we have $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g, \; T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$

Ex. Block A of mass m placed on a smooth slope is connected by a string with another block B of mass M (> msinq) as shown in the figure. Initially the block A is held at rest and then let free. Find acceleration of the blocks and tension in the string.



Sol.

Both the blocks must move with the same magnitude of acceleration
Since M > m sinq , block B move downward pulling block A up the plane. Let acceleration magnitude is 'a'.
Tension at both the ends of the string is same. Say it is 'T'.



Apply NLM to block A

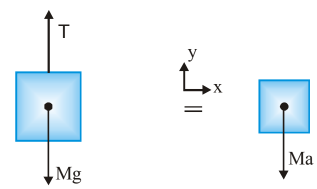
N: Normal reaction from slope

T : Tension of string

$$\sum F_x = ma_x \Rightarrow T - mg\sin\theta = ma \qquad \text{...(i)}$$

$$\sum F_y = 0 \Rightarrow N = mg\cos\theta \qquad \text{...(ii)}$$

Apply NLM to block B



T : Tension of string

$$\sum F_y = ma_y \Rightarrow Mg - T = Ma \qquad \text{...(iii)}$$

From equation (i) & (iii), we have $a = a = \dfrac{(M - m\sin\theta)}{M+m}$, $T = \dfrac{(1+\sin\theta)mMg}{m+M}$

NEWTON'S LAW FOR A SYSTEM

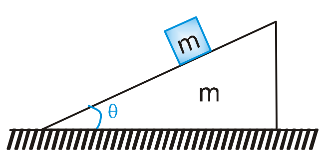
$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$$

$\vec{F}_{ext}$ = Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and

$\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects respectively.

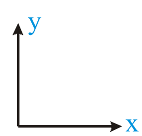
Ex. The block of mass m slides on a wedge of mass 'm' which is free to move on the horizontal ground. Find the accelerations of wedge and block. (All surfaces are smooth).



Sol. : Let . a $\Rightarrow$ acceleration of wedge

b $\Rightarrow$ acceleration of block with respect to wedge

Taking block and wedge as a system and applying Newton's law in the horizontal direction



$F_x = m_1\,\vec{a}_{1x} + m_2\,\vec{a}_{2x} = 0$

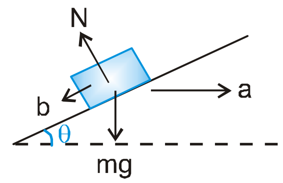
$0 = ma + m(a - b\cos\theta)$ ....(i)

Here 'a ' and 'b' are two unknowns , So for making second equation, we draw F.B.D. of block.

F.B.D of block.

using Newton's second law along inclined plane

mg sin θ = m (b – a cos θ) ....**(ii)**



Now solving equations **(1)** and **(2)** we will get

$$a = \frac{mg\sin\theta\cos\theta}{m(1+\sin^2\theta)} = \frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)} \quad \text{and} \quad b = \frac{2g\sin\theta}{(1+\sin^2\theta)}$$

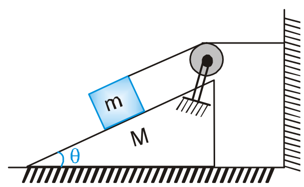
So in vector form :

$$\vec{a}_{\text{wedge}} = a\hat{i} = \left(\frac{g\sin\theta\cos\theta}{1+\sin^2\theta}\right)\hat{i}$$

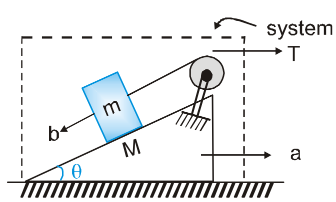
$$\vec{a}_{\text{block}} = (a - b\cos\theta)\,\hat{i} - b\sin\theta\,\hat{j}$$

$$\vec{a}_{\text{block}} = -\frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}\hat{i} - \frac{2g\sin^2\theta}{(1+\sin^2\theta)}\hat{j}\,.$$

Ex. For the arrangement shown in figure when the system is released, find the acceleration of wedge. Pulley and string are ideal and friction is absent.



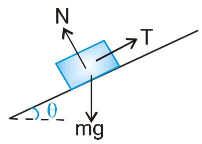
Sol. Considering block and wedge as a system and using Newton' s law for the system along x– direction



T = Ma + m (a – b cos θ) ....**(i)**

F.B.D of m along the inclined plane

mg sin θ – T = m (b – a cos θ) ....**(ii)**

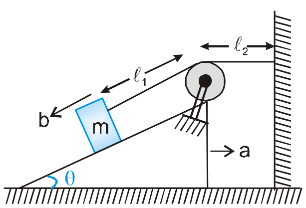


using string constraint equation.

$$\ell_1 + \ell_2 = \text{constant}$$

$$\frac{d^2\ell_1}{dt^2} + \frac{d^2\ell_2}{dt^2} = 0$$

b – a = 0 ....**(iii)**



Solving above equations **(i)**,**(ii)** & **(iii)** , we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

NEWTON'S LAW FOR NON INERTIAL FRAME :

$$\vec{F}_{Real} + \vec{F}_{Pseudo} = m\vec{a}$$

Net sum of real and pseudo force is taken in the resultant force.

$\vec{a}$ = Acceleration of the particle in the non inertial frame

$$\vec{F}_{Pseudo} = -\text{m}\ \vec{a}_{Frame}$$

Pseudo force is always directed opposite to the direction of the acceleration of the frame.

Pseudo force is an imaginary force and there is no action-reaction for it. So it has nothing to do with Newton's Third Law.

Reference Frame:

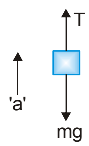
A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

(a) **Inertial reference frame:** Frame of reference either stationary or moving with constant velocity.

(b) **Non-inertial reference frame:** A frame of reference moving with non-zero acceleration.

Ex. A lift having a simple pendulum attached with its ceiling is moving upward with constant acceleration 'a'. What will be the tension in the string of pendulum with respect to a boy inside the lift and a boy standing on earth, mass of bob of simple pendulum is m.

Sol. **F.B.D** . of bob (with respect to ground)



$$\text{T} - \text{mg} = \text{ma}$$

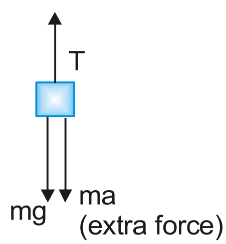
$$\text{T} = \text{mg} + \text{ma} \qquad \text{........(i)}$$

With respect to boy inside the lift, the acceleration of bob is zero.

So he will write above equation in this manner.

$$\text{T} - \text{mg} = \text{m. (0)} . \qquad \therefore \qquad \text{T} = \text{mg}$$

He will tell the value of tension in string is mg. But this is 'wrong' . To correct his result, he makes a free body diagram in this manner, and uses Newton's second law.



$$\text{T} = \text{mg} + \text{ma} \text{ ....(ii)}$$

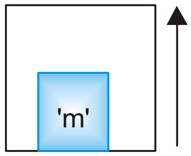
By using this extra force, equations **(i)** and **(ii)** give the same result . This extra force is called pseudo force. This pseudo force is used when a problem is solved with a accelerating frame (Non-inertial)

Note : **Magnitude of Pseudo force** = mass of system × acceleration of frame of reference .

Direction of force:

Opposite to the direction of acceleration of frame of reference, (not in the direction of motion of frame of reference)

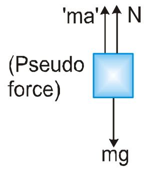
Ex. A box is moving upward with retardation 'a' <g, find the direction and magnitude of " pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block



Sol. **Pseudo force** acts opposite to the direction of acceleration of reference frame.

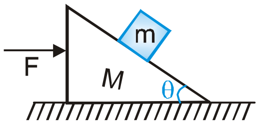
pseudo force = ma in upward direction

F.B.D of 'm' w.r.t. box (non–inertial)



$N + ma = mg$
$N = mg - ma$

Ex. All surfaces are smooth in the adjoining figure. Find F such that block remains stationary with respect to wedge.

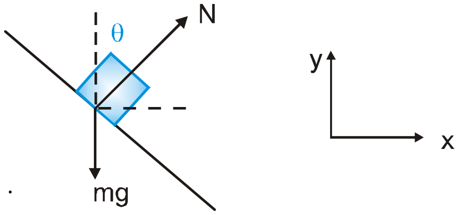


Sol. Acceleration of (block + wedge) is $a = \dfrac{F}{(M+m)}$

Let us solve the problem by using both frames.

From inertial frame of reference (Ground)

F.B.D. of block w.r.t. ground (Apply real forces) :



with respect to ground block is moving with an acceleration 'a' .

$\therefore \ \Sigma F_y = 0 \Rightarrow N\cos\theta = mg$ ....**(i)**

and $\Sigma F_x = ma \Rightarrow N\sin\theta = ma$ .....**(ii)**

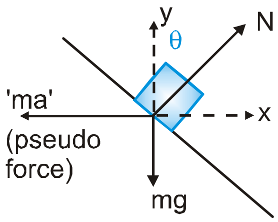
From Eqs. **(i)** and **(ii)**

$a = g\tan\theta$

$\therefore \ F = (M+m)a \ = (M+m)g\tan\theta$

From non–inertial frame of reference (Wedge) :

F.B.D. of block w.r.t . wedge (real forces + pseudo force)



w.r.t. wedge, block is stationary

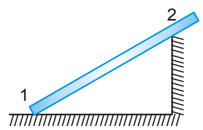
$\therefore \ \Sigma F_y = 0 \Rightarrow N\cos\theta = mg$ ....**(iii)**

$\Sigma F_x = 0 \Rightarrow N\sin\theta = ma$ ....**(iv)**

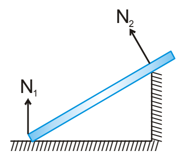
From Eqs. **(iii)** and **(iv)** , we will get the same result

i.e. $F = (M+m)g\tan\theta$.

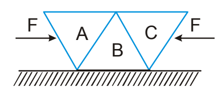
Ex. Draw normal forces on the massive rod at point 1 and 2 as shown in figure.



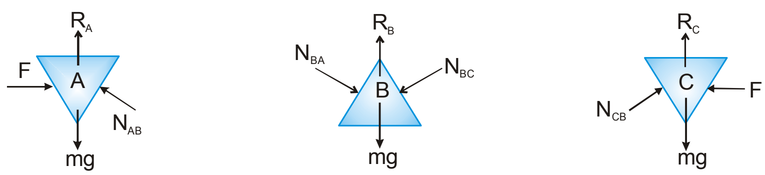
Sol. Normal force acts perpendicular to extended surface at point of contact.



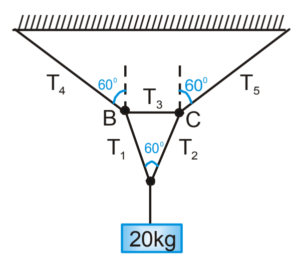
Ex. Three triangular blocks A,B and C of equal masses 'm' are arranged as shown in figure. Draw F.B.D. of blocks A,B and C. Indicate action–reaction pair between A,B and B,C.



Sol.

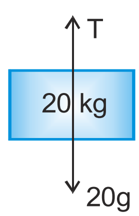


Ex. The system shown in figure is in equilibrium, find the tension in each string ; T_1,T_2,T_3, T_4 and T_5.



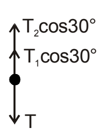
Ans. $T_1 = T_2 = \dfrac{200}{\sqrt{3}}$ N, $T_4 = T_5 = 200$ N, $T_3 = \dfrac{200}{\sqrt{3}}$ N.

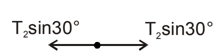
Sol. **FBD of 20 kg block →**



So, $T = 20 \times g = 200$ N .... **(1)**

From figure →





$T = T_1\cos30° + T_2\cos30°$ .... **(2)**

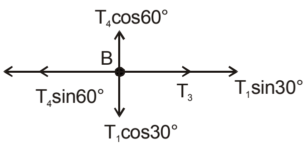
$T_1\sin30° = T_2\sin30°$

$T_1 = T_2$ .... **(3)**

So from equation **(3)** $T = 2T_1\cos30°$

$T_1 = \dfrac{200}{\sqrt{3}} = T_2$ .... **(4)**

From figure FBD of point B →



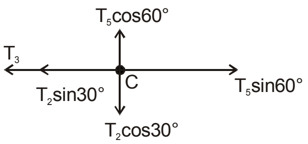
In vertical direction

So, $T_4\cos60° = T_1\cos30°$

$$T_4 \times \frac{1}{2} = \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 200\ \text{N}$$

So, $T_4 = 200$ N

FBD of point C →



Equating forces In vertical direction -

$T_5\cos60° = T_2\cos30°$

$$T_5 \times \frac{1}{2} = \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$

$T_5 = 200$ N

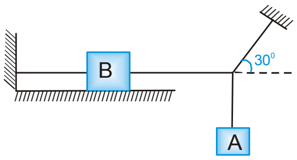
For T_3 →

Equating forces in horizontal direction -

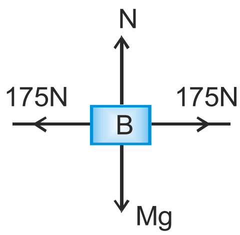
$T_3 + T_2\sin30° = T_5\sin60°$

$$T_3 = 200 \times \frac{\sqrt{3}}{2} - \frac{200}{\sqrt{3}} \times \frac{1}{2} \Rightarrow T_3 = \frac{200}{\sqrt{3}} N$$

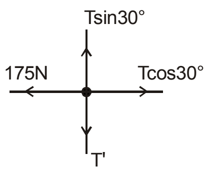
Ex. The breaking strength of the string connecting wall and block B is 175 N, find the magnitude of weight of block A for which the system will be stationary. The block B weighs 700 N.
($g = 10\ m/s^2$)



Sol. **FBD of block B →**



FBD of point in figure →

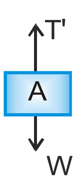


Equating forces in horizontal direction → $T\cos30° = 175 \Rightarrow T = \frac{175 \times 2}{\sqrt{3}}$ N

In vertical direction → $T\sin30° = T'$

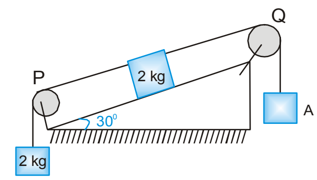
$$\text{So, } T' = \frac{175 \times 2}{\sqrt{3}} \times \frac{1}{2} = \frac{175}{\sqrt{3}} \text{ N}$$

FBD of block A →



So, $T' = W = \frac{175}{\sqrt{3}}$ N

Ex. In the arrangement shown in figure, what should be the mass of block A so that the system remains at rest. Also find force exerted by string on the pulley Q. ($g = 10\ m/s^2$)

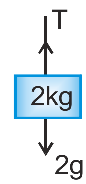


Ans. $m = 3$ kg, $30\sqrt{3}$ N.

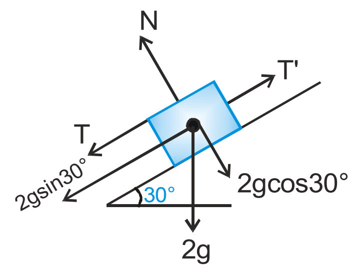
Sol. From figure

FBD of 2 kg block hanging vertically →

$T = 20$ N .... **(1)**



FBD of 2kg block on incline plane →



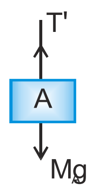
Along the plane →

$T + 2g\sin 30° = T'$

$T' = 20 + 20 \times \frac{1}{2} = 30$ N

FBD of block A →

So $T' = M_A g$

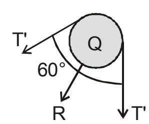


$M_A = \frac{T'}{g} = \frac{30}{10} = 3$ kg

$M_A = 3$ kg

FBD of pulley Q →

So, $R = 2T'\cos\frac{\theta}{2}$



$R = 2 \times 30\cos 30°$

$R = 2 \times 30 \times \frac{\sqrt{3}}{2}$ ⇒ $R = 30\sqrt{3}$ N

Ex. Two blocks with masses $m_1 = 0.2$ kg and $m_2 = 0.3$ kg hang one under other as shown in figure. Find the tensions in the strings (massless) in the following situations : ($g = 10\ m/s^2$)

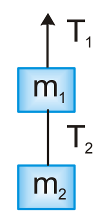
(a) the blocks are at rest

(b) they move upward at 5 m/s

(c) they accelerate upward at 2 m/s^2

(d) they accelerate downward at 2 m/s^2

(e) if maximum allowable tension is 10 N. What is maximum possible upward acceleration ?



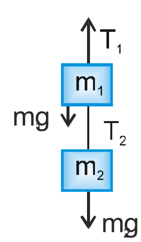
Ans. **(a)** 5 N, 3 N **(b)** 5 N, 3 N **(c)** 6 N, 3.6 N **(d)** 4 N, 2.4 N **(e)** 10 m/s^2

Sol. **(a)** At rest $a = 0$

$T_2 = m_2 g = 0.3 \times 10 = 3N$

$T_1 = m_1 g + T_2$

$T_1 = 0.2 \times 10 + 3 = 5$ N



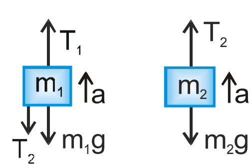
(b) same as above

$a = 0,\ T_2 = 3N,\ T_1 = 5N$

(c) $a = 2m/s^2$ ↑ (upward)

$T_2 - m_2 g = m_2 a$

⇒ $T_2 - m_2 g = m_2 a$



$\Rightarrow \quad T_2 - 0.3 \times 10 = 0.3 \times 2$

$\Rightarrow \quad T_2 = 0.6 + 3 = 3.6\,N$

$T_1 - m_1 g - T_2 = m_1 a \quad \Rightarrow T_1 - 0.2 \times 10 - 3.6 = 0.2 \times 2 \quad \Rightarrow T_1 = 0.4 + 5.6 = 6N$

(d) $a = 2\ m/s^2$ (downward)

$m_2 g - T_2 = m_2 a$

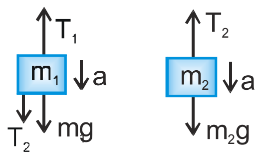
$\Rightarrow 0.3 \times 10 - T_2 = 0.3 \times 2$

$\Rightarrow T_2 = 3 - 0.6 = 2.4\,N$

$T_2 + m_1 g - T_1 = m_1 a$

$\Rightarrow 2.4 + 2 - T_1 = 0.2 \times 2$

$\Rightarrow T_1 = 4.4 - 0.4 =$ **4 N Ans.**



(e) Chance of breaking is of upper string means $T_1 < 10\,N$

For m_1 –

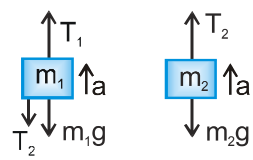
$T_1 - m_1 g - T_2 = m_1 a$

$10 - 2 - T_2 = 0.2\,a$ .... **(1)**

For m_2 –

$T_2 - m_2 g = m_2 a$

$\Rightarrow T_2 - 3 = 0.3\,a$ .... **(2)**



Adding equation **(1)** and **(2)**

$$8 - 3 = 0.5\,a \quad \Rightarrow a = \frac{5}{0.5} = \mathbf{10\ m/s^2}$$

Ex. A block of mass 50 kg is kept on another block of mass 1 kg as shown in figure. A horizontal force of 10 N is applied on the 50kg block. (All surface are smooth). Find ($g = 10\ m/s^2$)

(a) Acceleration of block A and B.

(b) Force exerted by B on A.

Ans. **(a)** $0.2\ m/s^2$, 0 **(b)** 500 N

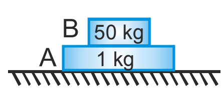
Sol. **(a) FBD of A block**

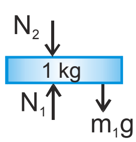
$N_2 = 50 \times 10 = 500\,N$

$m_1 g = 1 \times 10 = 10\,N$

$N_1 = N_2 + m_1 g = 500 + 10 = 510\,N$

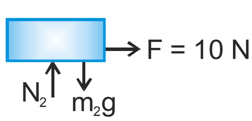
$a_A = 0$ (No horizontal force)





FBD of B block

Vertical $N_2 = m_2 g = 500\,N$

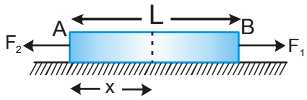


Horizontal force

$$F = ma \quad \Rightarrow 10 = 50 \times a \Rightarrow a = \frac{1}{5}\ m/s^2$$

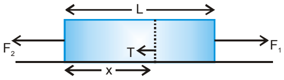
(b) Force exerted By B on A $\rightarrow \; = N_2 = 500N$ (Vertically downwards)

Ex. Two forces F_1 and F_2 ($F_2 > F_1$) are applied at the free ends of uniform rod kept on a horizontal frictionless surface. Find tension in rod at a distance x from end 'A',

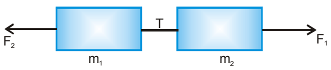


Ans. $T = F_2 - \frac{(F_2 - F_1)}{L}.x$

Sol. $a = \frac{F_2 - F_1}{m}$



$T - F_1 = m_2 a$



$\Rightarrow \quad T - F_1 = \frac{m}{L}(L - x)\frac{F_2 - F_1}{m} \quad (m_2 = \frac{m}{L}(L - x))$

$\Rightarrow \quad T = F_1 + \left(1 - \frac{x}{L}\right)(F_2 - F_1)$

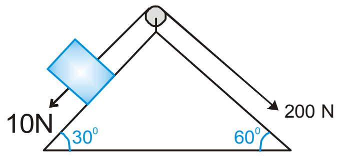
$= F + F_2 - F_1 - \frac{x}{L}(F_2 - F_1) = F_2 - \frac{x}{L}(F_2 - F_1)$

Ex. A 10 kg block kept on an inclined plane is pulled by a string applying 200 N force. A 10 N force is also applied on 10 kg block as shown in figure.

Find : **(a)** tension in the string.

(b) acceleration of 10 kg block.

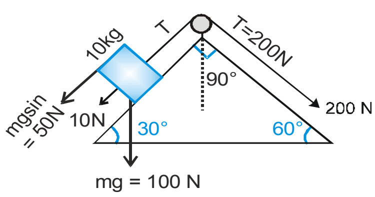
(c) net force on pulley exerted by string



Ans. **(a)** 200 N, **(b)** 14 m/s^2, **(c)** $200\sqrt{2}$ N

Sol. **(a)** $T = 200$ N

(b) $T - 10 - mg\sin\theta = ma$

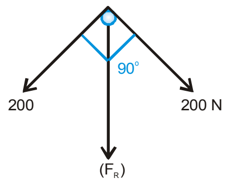


$\Rightarrow \quad T - 10 - 50 = 10a$

$\Rightarrow \quad 200 - 60 = 10a$

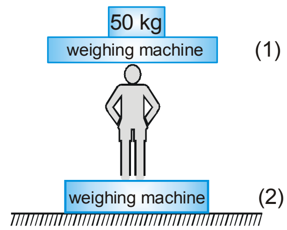
$\Rightarrow \quad a = \frac{140}{10} = $ **14 m/s^2**

(c)



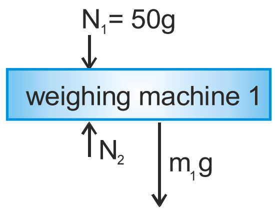
$(F_R) = \sqrt{(200)^2 + (200)^2} = 200\sqrt{2}$ N **Ans.**

Ex. A man of mass 60 kg is standing on a weighing machine (2) of mass 5kg placed on ground. Another similar weighing machine is placed over man's head. A block of mass 50kg is put on the weighing machine (1) . Calculate the readings of weighing machines (1) and (2) ($g = 10\ m/s^2$)



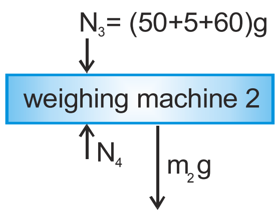
Ans. **500 N, 1150 N**

Sol.



$R_1 = N_1 = 50 \times g = 500\ N$

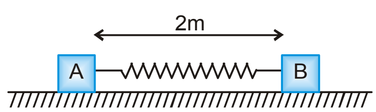
where R_1 = reading in weighing machine 1



$R_2 = N_3 \quad = (50 + 5 + 60)\ g \quad = 115 \times 10 \quad = 1150\ N$

where R_2 = reading in weighing machine 2

Ex. Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 N/m. Find spring force in following situations :



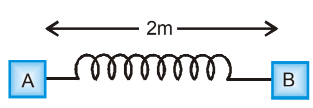
(a) A is kept at rest and B is displaced by 1 m in right direction.

(b) B is kept at rest and A is displaced by 1m in left direction.

(c) A is displaced by 0.75 m in right direction, and B is 0.25 m in left direction.

Ans. **(a)** F = 200 N, **(b)** 200 N, **(c)** 200 N

Sol. **(a)** Extension in spring = 1 m.



$\therefore \quad F_{spring} = Kx$

$= \quad 200 \times 1 = 200\ N$

(b) Same extension same spring force in both directions -

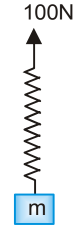
$F_{spring} = 200\ N.$

(c) Both displacements of A of B are compressing the spring

total compressing $= 0.75 + 0.25 = 1$ m.

$\therefore \quad F_{spring} = k\,x \quad = 200 \times 1 = 200\ N.$

Ex. If force constant of spring is 50 N/m. Find mass of the block, if it is at rests in the given situation . ($g = 10\ m/s^2$)

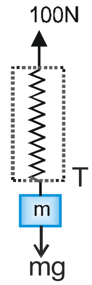


Ans. $m = 10\ kg$

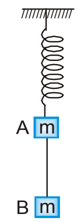
Sol. $T = 100\ N$

$\Rightarrow mg = 100\ N$

$$m = \frac{100}{g} = 10\ kg.$$



Ex. Two blocks 'A' and 'B' of same mass 'm' attached with a light string are suspended by a spring as shown in figure Find the acceleration of block 'A' and 'B' just after the string is cut.

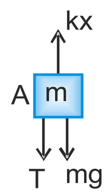


Ans. g (upwards), g (downwards)

Sol. **When string is not cut :**

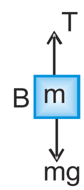
FBD of 'A' block

$kx = mg + T$ ....(i)



F. B.D of 'B' block

$T = mg$ ....(ii)



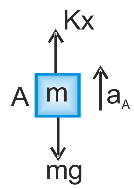
When string is cut :

FBD of 'A' block

$kx - mg = ma_A$

$2mg - mg = ma_A$

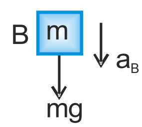
$= a_A = g$ (upwards)



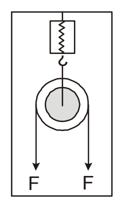
FBD of 'B' block

$ma_B = mg$

$a_B = g$ (downwards)

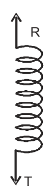


Ex. Find the reading of spring balance in the adjoining figure, pulley and strings are ideal.



Ans. 2F

Sol. **FBD of spring balance**

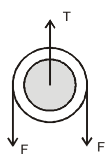


$R = T$ ....(i)

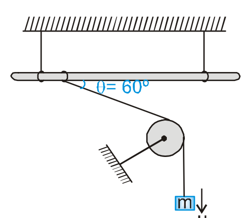
FBD of pulley

$T = 2\ F$ ....(ii)

$R = 2F$



Ex. The figure shows mass m moves with velocity u. Find the velocity of ring at that moment. Ring is restricted to move on smooth rod.

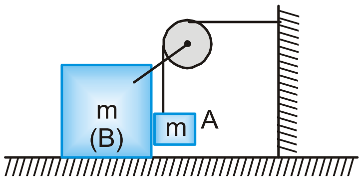


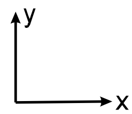
Ans. $V_R = \frac{u}{\cos\theta}$, $V_R = 2u$

Sol. Velocity along string remains same .

$V_R \cos\theta = u \quad \Rightarrow \quad V_R = \frac{u}{\cos\theta} \quad \Rightarrow \quad \theta = 60^o \quad \Rightarrow \quad V_R = 2u$

Ex. In the system shown in figure, the block A is released from rest. Find :

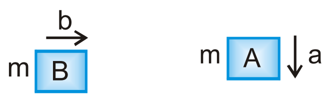




(i) the acceleration of both blocks 'A' and 'B'.
(ii) Tension in the string.
(iii) Contact force between 'A' and 'B'.

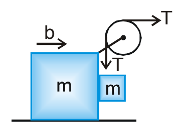
Ans. **(i)** $\frac{g}{3}\hat{i}-\frac{g}{3}\hat{j},\ \frac{g}{3}\hat{i}$ **(ii)** $\frac{2mg}{3}$ **(iii)** $\frac{mg}{3}$.

Sol. **(i)** Let acceleration of blocks in x & y directions are

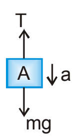


Taking both blocks as a system

$T = 2mb$ .... **(i)**



Taking A block :



$mg - T = ma$ ....**(ii)**

From equations **(i)** & **(ii)** ; $ma + 2\,mb = mg$

$a + 2\,b = g$ ....**(iii)**

From string constraint ;

$a = b$ ....**(iv)**

From equations **(iii)** & **(iv)**; $a = b = \frac{g}{3}$

hence, acceleration of block A

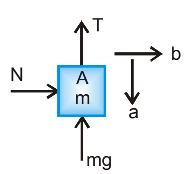
$a_A = b\hat{i} - a\hat{j}$ $\qquad a_A = \frac{g}{3}\hat{i} - \frac{g}{3}\hat{j}$

acceleration of block B $\qquad a_B = b\hat{i} = \frac{g}{3}\hat{i}$

(ii) $T = 2mb = \frac{2mg}{3}$

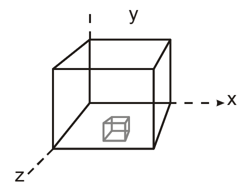
(iii) For contact force between 'A' and 'B'

FBD of block 'A'



$N = mb$ $\qquad N = \frac{mg}{3}$

Ex. A block of mass 2 kg is kept at rest on a big box moving with velocity $2\hat{i}$ and having acceleration $-3\hat{i}+4\hat{j}$ m/s^2. Find the value of 'Pseudo force' acting on block with respect to box



Ans. $\vec{F}_{pseudo} = -m\vec{a}_{frame} = -2(-3\hat{i}+4\hat{j})$

$F = 6\hat{i}-8\hat{j}$.

SYSTEM OF INTERCONNECTED BODIES

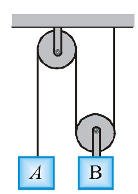
In system of interconnected bodies, several bodies are interconnected in various manners through some sort of physical links. Sometimes these physical links includes ropes and pulleys and sometimes the bodies under investigation are pushing each other through direct contact. In systems consisting of bodies interconnected through ropes and pulleys, relation between their accelerations depends on the arrangement of the ropes and pulleys. In addition, in system where bodies push each other, they affect relation between their accelerations due to their shapes.

In kinematics while dealing with dependant motion or constrained motion, we have already learn how to find relations between velocities and accelerations of interconnected bodies.

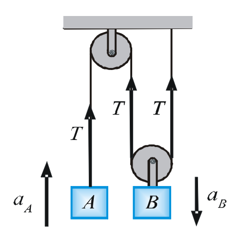
Analysis of physical situations involving interconnected bodies often demands relation between accelerations of these bodies in addition to the equations obtained by application of Newton's laws of motion. Therefore, while analyzing problems of interconnected bodies, it is recommended to explore first the relations between accelerations and then apply Newton's laws of motion.

In following few examples, we learn how to deal with problems of interconnected bodies.

Ex. Two boxes A and B of masses m and M interconnected by an ideal rope and ideal pulleys, are held at rest as shown. When it is released, box B accelerates downwards. Find accelerations of the blocks.



Sol. We first show tension forces applied by the string on the box A and the pulley connected to box B. Since the string, as well as the pulleys, both are ideal; the string applies tension force of equal magnitude everywhere. Denoting the tension force by T, we show it in the adjacent figure.

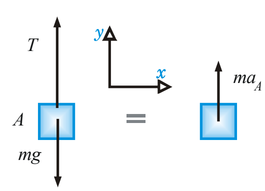


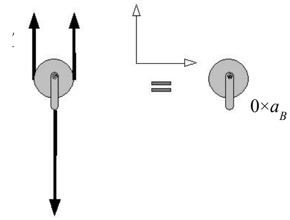
We first explore relation between accelerations a_A and a_B of the boxes A and B, which can be written either by using constrained relation or method of virtual work or by inspection.

$a_A = 2a_B$ ...**(i)**

Applying Newton's Laws of motion to box A

$$\sum F_y = ma_y \rightarrow T - mg = ma_A \quad \text{...(ii)}$$

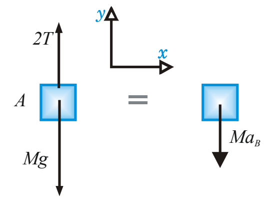




Applying Newton's Laws of motion to the pulley

$\sum F_y = ma_y \rightarrow 2T - F = 0 \times a_B$

$F = 2T$ ...**(iii)**



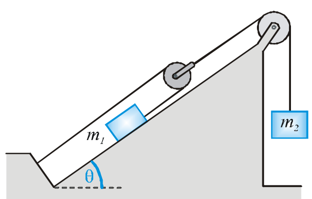
Applying Newton's Laws of motion to box B

$\sum F_y = ma_y \rightarrow Mg - 2T = Ma_B$ ...**(iv)**

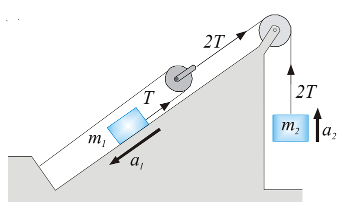
From equations **(i)**, **(ii)**, **(iii)** and **(iv)**, we have

$$a_A = 2\left(\frac{M-2m}{M+4m}\right)g \text{ and } a_B = \left(\frac{M-2m}{M+4m}\right)g$$

Ex. In the system shown in figure, block m_1 slides down a friction less inclined plane. The pulleys and strings are ideal. Find the accelerations of the blocks.



Sol. Tension forces applied by the strings are shown in the adjacent figure.

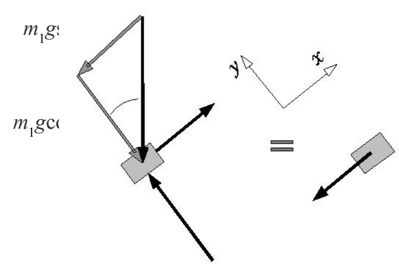


Let the block m_1 is moving down the plane with an acceleration a_1 and m_2 is moving upwards with accelerations a_2. Relation between accelerations a_1 and a_2 of the blocks can be obtained easily by method of virtual work.

$a_1 = 2a_2$ ...**(i)**

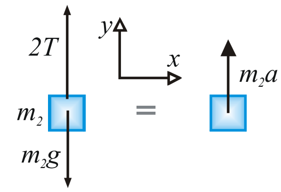
Applying Newton's laws to analyze motion of block m_1

$\sum F_x = ma_x \rightarrow m_1 g \sin\theta - T = m_1 a_1$ ...**(ii)**



Applying Newton's laws to analyze motion of block m_2

$$\sum F_y = ma_y \rightarrow 2T - m_2g = m_2a_2 \qquad \text{...(iii)}$$



From equation (i), (ii) and (iii), we have

$$a_1 = \frac{2(2m_1 \sin\theta - m_2)}{4m_1 + m_2}g \quad a_2 = \frac{2m_1 g \sin\theta - m_2 g}{4m_1 + m_2}$$

PSEUDO FORCE

Motion in Accelerated frames :

Till now we have restricted ourselves to apply Newton's laws of motion, only to describe observations that are made in an inertial frame of reference. In this part, we learn how Newton's laws can be applied by an observer in a non inertial reference frame. For example, consider a block kept on smooth surface of a compartment of train.

If the train accelerates, the block accelerates toward the back of the train. When observed from the train we may conclude based on Newton's second law F = ma that a force is acting the block to cause it to accelerate, but the Newton's second law is not applicable from this non-inertial frame. So we can not relate observed acceleration with the force acting on the block.

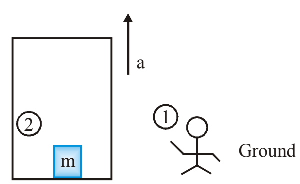
If we still want to use Newton's second law we need to apply a pseudo force, acting in back-ward direction, i.e. opposite to the acceleration of non inertial reference frame. This force explains the motion of block towards the back of train. The friction force is equal to - $\overline{ma}$, where $\vec{a}$ is the acceleration of the non inertial reference frame. Fictitious source appears to act on an object in the same way as a real force, but real forces are always interactions between two objects, on the other hand there is no second object for a fictitious force

pseudo force, $\vec{F}_P = -m\vec{a}_0$

where a_0 is acceleration of non inertial reference frame

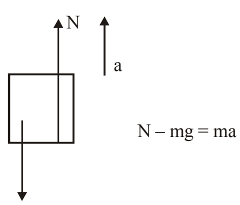
Thus, we may conclude that pseudo force is not a real force. When we draw the free body diagram of a mass, with respect to an inertial frame of reference we apply only the real forces (forces which are actually acting on the mass), but when the free body diagram is drown from a non-inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation

$\vec{F} = -m\vec{a}_0$, valid in this frame also.

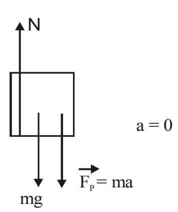


Suppose a block A of mass m is placed on a lift ascending with an acceleration a_0. Let N be the normal reaction between the block and the floor of the lift free body diagram of A is shown in figure.

for observer I (Inertial reference frame):



For observer 2(Non inertial reference frame) :



$\therefore$ N = mg + ma

Ex. A small ball of mass m hangs by a cord from the ceiling of a compartment of a train that is accelerating to the right as shown in Figure. Analyze the situations for two observers A & B.

Sol. The observer A on the ground is inertial frame. He sees the compartment is accelerating and knows that the horizontal comp. of tension in the cord provides the ball, required horizontal force. The non inertial observer on the compartment can not see the car's motion, he is not aware of its acceleration. He will say that Newton's second law is not valid as the object has net horizontal force (the horizontal component of tension) but no horizontal acceleration.

For the inertial observer, ball has a net force in the horizontal direction and is in equilibrium in the vertical direction. For the non inertial observer, we apply fictitious force towards left and consider it to be in equilibrium.

According to the inertial observer A, the ball experience two forces, T exerted by the cord and the weight.

Applying Newton's second law in horizontal and vertical direction we get

Inertial observer $\quad T\sin\theta - mg = 0 \quad$ ....**(i)**

$T\cos\theta = ma \quad$ ....**(ii)**

According to the non inertial observer B riding in the car (Fig. g), the ball is always at rest and so its acceleration is zero. The non inertial observer applies a fictitious force in the horizontal direction of magnitude ma towards left. This friction force balances the horizontal component of T and thus the net force on the ball is zero.

Apply Newton's second law in horizontal and vertical direction we get

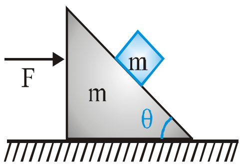
Non inertial observer $\quad T\sin\theta - mg = 0$

$T\cos\theta = ma = 0$

These expressions are equivalent to Equation **(1)** and **(2)**

The non inertial observer B obtains the same equations as the inertial observer. The physical explanation of the cord's deflection, however, differs in the two frames of reference.

Ex. All surfaces are smooth in following figure. Find F, such that block remains stationary with respect to wedge



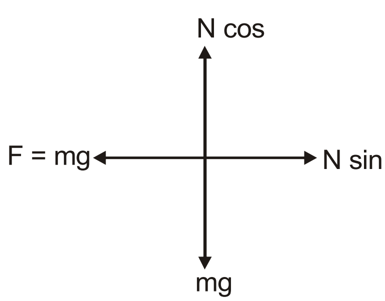
Sol. Acceleration of (block + wedge) $a = \dfrac{F}{(M+m)}$

Let us solve the problem by both the methods.

From inertial frame of reference (Ground)

FBD of block w/r/t ground (Apply real forces)

With respect to ground block is moving with an acceleration 'a'.



$\sum F_y = 0 \Rightarrow N\cos\theta = mg \quad$ ...**(i)**

and $\quad \sum F_x = ma \Rightarrow N\sin\theta = ma \quad$ ...**(ii)**

From Equation **(i)** and **(ii)**

$a = g \tan\theta$

$F = (M + m)a$

$= (M + m)g \tan\theta$

From non-inertial frame of reference (Wedge)

FBD of block w.r.t. wedge (real force + pseudo force)

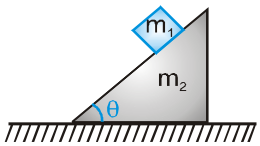
w.r.t wedge block is stationary

$\therefore \quad \sum F_x = 0 \Rightarrow N\cos\theta = mg$ ...**(iii)** and $\sum F_x = ma \Rightarrow N\sin\theta = ma$ ...**(iv)**

From equation **(iii)** and **(iv)**, we will get the same result

$F = (M + m)g \tan\theta$

Ex. There is no friction at any contact. Wedge is free to move. Find force acting on wedge due to block. Also find acceleration of wedge



Sol. You many want to directly reach to conclusion that answer is m, g cos θ. but it is being solved in reference frame of wedge which amy be accelerating. Horizontal component of normal contact force applied by block on wedge will accelerate the wedge. Thus reference frame attached to wedge is non-inertial reference frame.

Acceleration vector of block in ground frame is sum of acceleration of wedge and acceleration of block w.r.t. $(\vec{a}_{b/w})$

$$\vec{a}_b = \vec{a}_{b/w} + \vec{a}_w$$

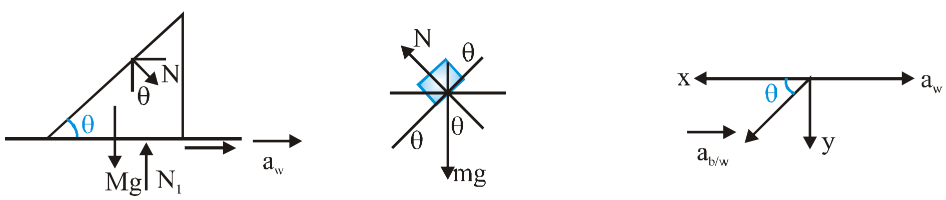
Consider F.B.D of wedge. Take horizontal component of normal contact force and apply Newton's 2nd Law

$N \sin\theta = m_2 a_w$

consider F.B.D of block and acceleration vector of block. Take horizontal and vertical component of forces and acceleration and apply Newton's second Law.

$$a_x = a_{b/w}\cos\theta - a_w \quad \Rightarrow \quad a_y = a_{b/w}\sin\theta$$

$$N\sin\theta = m_1\left(a_{b/w}\cos\theta - a_w\right) \quad \Rightarrow \quad m_1 g - N\cos\theta = m_1\left(a_{b/w}\sin\theta\right)$$



Solving we get

$$N = \frac{m_1 m_2 g\cos\theta}{\left(m_2 + m_1\sin^2\theta\right)} \Rightarrow a_w = \frac{m_1 g\cos\theta\sin\theta}{\left(m_2 + m_1\sin^2\theta\right)}, a_{b/w} = \frac{\left(m_1 + m_2\right)g\sin\theta}{\left(m_1\sin^2\theta + m_2\right)}$$