HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. Work $= \vec{F}.\vec{dr}$, Work $= -\int_0^{\theta}(0.5)(5)R d\theta$ $\therefore$ F = mN

$\Rightarrow$ [work] = (2.5) (R) (2π) = –5 J

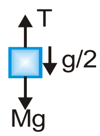
2. By applying work energy theorem change in kinetic energy = $W_g + W_{ext.P}$

$0 = mg(\ell \cos 37° - \ell \cos 53°) + W_{ext\,P}$

$= 50 \times 10 \times 1\left[\frac{3}{5} - \frac{4}{5}\right] + W_{ext\,P}$

$W_{ext} = 100$ joule

3. $W = \vec{f}.\vec{d}$



$mg - T = \frac{Mg}{2}$; $T = \frac{Mg}{2}$

$W = \left(-\frac{Mg}{2}\right)x$

4. By applying work energy theorem

$\Delta KE = \vec{f}.\vec{d} = m\left(\frac{v}{t_1}\right)\frac{1}{2}\left(\frac{v}{t_1}\right)t^2 \Rightarrow \Delta K.E. = \frac{mv^2}{2t_1^2}t^2$

5. For conservation force work done is independent of the path

$W_{AB} + W_{BC} = W_{AC}$, 3+4 = W_{AC} = 7 J

6. By applying work energy theorem

$\frac{1}{2}m\frac{v^2}{4} - \frac{1}{2}mv^2 = -\frac{1}{2}kx^2$

$\Rightarrow \frac{-3mv^2}{8} = \frac{-1}{2}kx^2$; $k = \frac{3mv^2}{4x^2}$

7. By applying work energy theorem ΔKE = work done by all the forces

New kinetic energy = $\frac{1}{2}mv_f^2 = \frac{mv^2}{8}$

$\Rightarrow v_f = \frac{v_0}{2} \Rightarrow v = u - \mu g t_0 \Rightarrow \mu \Rightarrow \frac{v_0}{2gt_0}$

8. Slope of v–t graph Acceleration ⇒ $-10 m/s^2$

Area under v–t graph → displacement ⇒ 20 m

work = $\vec{f}.\vec{s}$ = 2 (10) (20) ⇒ –400 J

9. Total mass ; f ∝ 6m, f = $6m_C$ (20) = P

To Drive 12m : f ∝ 14m ⇒ f = $14\,m_C$

$(14\,m_C)\,v = 6(m_C)\,20$ ⇒ 8.57m/s

To drive 6 bogie : force ∝ 8m

force = $8m_C$ ⇒ P = $8\,m_C v$

$(8m_C)v = 120m_C$ ⇒ 15 m/s

10. Power = constant, Fv = C

$mvdv = Cdt \Rightarrow v^2 = \frac{2C}{m}t \Rightarrow v = \sqrt{\frac{2C}{m}t}$

as $v = \frac{dx}{dt} \Rightarrow \int dx = \sqrt{\frac{2C}{m}}\int\sqrt{t}dt$

$x = \sqrt{\frac{2C}{m}}\frac{t^{3/2}}{2/3} \Rightarrow x \propto t^{3/2}$

11. By applying work energy theorem

$\frac{1}{2}mv^2 - 0 = W_g + W_{fr}$

for the second half work energy theorem change in kinetic energy $= W_g + W_{fr}$

$0 = 100mg + W_{fr} = -100$ mg

As work done for the first half by the gravity is 100mg therefore work done by air resistance is less than 100 mg.

12. $x = 3t - 4t^2 + t^3$; $v = \frac{dx}{dt} = 3 - 8t + 3t^2$

$a = \frac{dv}{dt} = 0 - 8 + 6t$

$W = \int \vec{F}.\vec{dx} = \int_0^4 3(6t-8)(3-8t+3t^2)dt$

W = 528 mJ

OR

From work energy theorem

$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}(3 \times 10^{-3})$

$\left[(3 - 8(4) + 3(4)^2) - (3)^2\right] = 528$ mJ

13. P.E. → Maximum → Unstable equilibrium

P.E. → Minimum → Stable equilibrium

P.E. → Constant → Natural equilibrium

∴ None of these

14. $a_c = k^2 rt^2 \Rightarrow \frac{v^2}{r} = k^2rt^2$

$\Rightarrow \quad v^2 = k^2r^2t^2 \Rightarrow v = krt \quad \Rightarrow \quad a_T = \frac{dv}{dt} = kr$

$P = \int m\vec{a}_T.\vec{v} = m(kr).(krt) = mk^2r^2t$

15. By applying work energy theorem

ΔKE = Work done by all the forces

$0 = W_g + W_{spring} + W_{ext\ agent}$

$-W_g = (W_{spring} + W_{ext\ agent})$

$\Delta U = (W_{spring} + W_{ext\ agent})$ $[\because \Delta U = W_g]$

16. P.E. $\rightarrow$ Maximum $\rightarrow$ Unstable equilibrium

P.E. $\rightarrow$ Minimum $\rightarrow$ Stable equilibrium

P.E. $\rightarrow$ Constant $\rightarrow$ Natural equilibrium

Force $= -\frac{dU}{dx} \Rightarrow -(\text{slope})$

[slope is –ve from E to F]

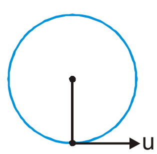
Force = +ve repulsion

Force = –ve attraction

17. $\Delta U = mgh$

height w.r.t. ground $= (\ell - h)$, $\Delta U = mg(\ell - h)$

18. At lowest point



$T - mg = \frac{mu^2}{\ell}$ ....(i)

at highest point T = 0

$mg = \frac{mv^2}{\ell}$, $v = \sqrt{g\ell}$ and $v^2 = u^2 + 2as$

$\left(\sqrt{g\ell}\right)^2 = u^2 + 2(-g) \times 2\ell$

$g\ell = u^2 - 4g\ell$

$u^2 = 5\,g\ell$

Put the value of u^2 in equation (i)

$T - mg = \frac{m(5g\ell)}{\ell} \Rightarrow T = 6\,mg$

19. By applying work energy theorem

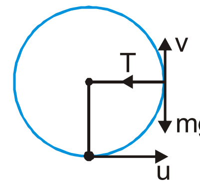
$\Delta K.E = W_S + W_{ext\ agent}$

$0 = -\frac{1}{2}Kx^2 + Fx \Rightarrow x = \frac{2F}{K}$

Work done $= \frac{2F^2}{K}$

20. In case of rod the minimum velocity of particle is zero at highest.

21. When the string is horizontal



$T = \frac{mv^2}{\ell}$ ...(i)

$v^2 = u^2 - 2g\ell$

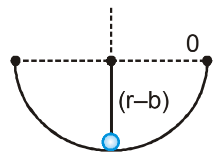
$v^2 = 5g\ell - 2g\ell = 3g\ell$

So $T = \frac{m \cdot 3g\ell}{\ell} = 3mg$

So net force

$= \sqrt{T^2 + (mg)^2} = \sqrt{(3mg)^2 + (mg)^2} = \sqrt{10}\ mg$

22. By applying work energy theorem $\Delta KE = W_g$



$\frac{1}{2}mv^2 = mg(r-b) \Rightarrow v = \sqrt{2g(r-b)}$

23. As velocity is vector quantity

$\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$ [as $\theta = 90°$]

$\Delta v = \sqrt{v_1^2 + v_2^2}$

By applying work energy theorem velocity at z

$\frac{1}{2}mv_2^2 - \frac{1}{2}mu^2 = -mgL$

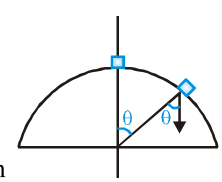
$v_2^2 = u^2 - 2gL \Rightarrow \Delta u = \sqrt{2(u^2 - gL)}$

24. $\Delta P = \sqrt{P_1^2 + P_2^2 - 2P_1P_2\cos\theta}$

for $\cos\theta$ = maximum $\Rightarrow$ ΔP minimum $\theta = 360°$

for $\cos\theta$ = minimum $\Rightarrow$ ΔP maximum $\theta = 180°$

25. Net force towards centre equal $= \frac{mv^2}{r}$



$mg\cos\theta - N = \frac{m_x v^2}{r}$

$v = \sqrt{rg\cos\theta}$

By applying work energy theorem

$\frac{1}{2}mrg\cos\theta - 0 = mgr(1 - \cos\theta) = \cos\theta = \frac{2}{3}$

26. Tension at any point $T = 3mg\cos\theta$

Given $3mg\cos\theta = 2mg$

EXERCISE - 2

Part # I : Multiple Choice

1. COME $\Rightarrow$ $K_1 + U_1 = K_2 + U_2$

$\Rightarrow 0 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}k_1\left(\frac{x}{2}\right)^2 + \frac{1}{2}k_2\left(\frac{x}{2}\right)^2$

$\Rightarrow \frac{1}{2}(k_1 + k_2)x^2 = \frac{1}{2}mv^2 + \frac{1}{8}(k_1 + k_2)x^2$

$\Rightarrow v = \sqrt{\frac{3}{4}\frac{(k_1 + k_2)x^2}{m}}$

2. For body B : $mg - T = m(2a)$

For body A : $2T - mg = ma \Rightarrow a = \frac{g}{5}$

$a_B = 2a_A$ and $a_A = a$

$\therefore$ Velocity of B after travelling distance ℓ

$= \sqrt{2as} = \sqrt{\frac{4g\ell}{5}}$

$\therefore$ Velocity of A : $v_A = \frac{v_B}{2} = \sqrt{\frac{g\ell}{5}}$

3. Work done against friction = mgh = loss in P.E.

$\therefore$ Work done by ext. agent

$= W_f + \Delta PE$

$= mgh + mgh = 2mgh$

4. COME : $K_1 + U_1 = K_2 + U_2$

$0 + mg(4R) = \frac{1}{2}mv^2 + mg(2R) \Rightarrow mv^2 = 4mgR$

Forces at position 2 :

$N = \frac{mv^2}{R} - mg = 4mg - mg = 3\,mg$

5. COME $\Rightarrow K_1 + U_1 = K_2 + U_2$

$0 + mg\ell(1 - \cos 60°) = \frac{1}{2}mv^2 + 0 \Rightarrow v = \sqrt{g\ell}$

6. $W_{man} = \Delta U = U_f - U_i = \left(\frac{m}{2}\right)g\left(\frac{\ell}{4}\right) - \frac{mg\ell}{2} = -\frac{3mg\ell}{8}$

7. COME : $K_B + U_B = K_C + U_C$

$\frac{1}{2}mv_0^2 + mgr = \frac{1}{2}mv_C^2 + mg\,r\cos\theta$ .....(i)

Force equation at C

$\Rightarrow N + \frac{mv_C^2}{r} = mg\cos\theta$ .....(ii)

at C, N = 0 $\Rightarrow \cos\theta = \frac{3}{4}$

8. $F_{ext} = m_2g - m_1g \quad \therefore P_{inst} = f_{ext}.v = (m_2 - m_1)gv$

9. At $x = -\sqrt{\frac{2E}{k}}$; $E_{total} = \frac{1}{2}kx^2 = U \quad \therefore KE = 0$

10. $W_f = \Delta KE \Rightarrow \int_r^\infty(-\mu.mg)\,dr = 0 - \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gA}$

11. Equation of motion :

$m_A g\sin 37° - T = m_Aa_A$ and $2T - m_Bg = m_Ba_B$

$a_A = 2a_B = 2 \times \frac{g}{12} = \frac{g}{6}$

$\therefore v_A = \sqrt{2a_A.s_A} = \sqrt{2 \times \frac{g}{6} \times 1} = \sqrt{\frac{g}{3}}$

$\therefore v_B = \frac{v_A}{2} = \frac{\sqrt{g}}{2\sqrt{3}}$

12. COME : $K_B + U_B = K_A + U_A$

$0 + \frac{1}{2}k(13-7)^2 = \frac{1}{2}mv_A^2 + 0$

$N_A = \frac{mv_A^2}{R} = \frac{k \times 6^2}{5} = 1440\,N$

13. COME : $K_A + U_A = K_B + U_B$

$0 + mg \times 25 = \frac{1}{2}mv_A^2 + mg \times 15 \Rightarrow mv_A^2 = 20mg$

Forces at B : $N = mg - \frac{mv_A^2}{R} = 0 \Rightarrow R = 20\,m$

14. Conservation of mechanical energy explains the K.E. at A & B are equal.

Acceleration for A = $g\sin\theta_1$

Acceleration for B = $g\sin\theta_2$

$\because \sin\theta_1 > \sin\theta_2 \quad \therefore a_1 > a_2$

F_{ext} and displacements are in opposite directions.

15. Power = $\rho QgH = \rho Av.gH = \rho A\sqrt{2gH}.gH$

$$=10^3 \times \frac{\pi d^2}{4} \times \sqrt{2\times10\times40} \times 10 \times 40 \text{ (d=5 cm)} = 21.5 \text{ kW}$$

16. Area of graph

$$=\int P.dx = \int mv.a.dx = \int mv.\left(\frac{vdv}{dx}\right)dx$$

$$=\int_u^v mv^2 dv = \frac{m(v^3-u^3)}{3} = \frac{10.(v^3-1)}{7\times3}$$

$$=\frac{1}{2}(4+2)\times10 \Rightarrow v = 4 \text{ m/s}$$

17. For upward motion : $mgh + fh = \frac{1}{2}m\times16^2$

downward motion : $mgh - fh = \frac{1}{2}m\times 8^2 \Rightarrow h = 8m$

18. For equilibrium : $N\cos\theta = mg$ & $N\sin\theta = kx$

$\Rightarrow$ $kx = mg\tan\theta$ (N = normal between m & M)

$\therefore$ $U = \frac{1}{2}kx^2 = \frac{m^2g^2\tan^2\theta}{2k}$

19. $P = \frac{\Delta W}{\Delta t} = \frac{\vec{F}.\Delta\vec{S}}{\Delta t} = \frac{(3\hat{i}+4\hat{j})\cdot(8\hat{i}+6\hat{j})}{6} = 8W$

20. For motion $P\to0 \Rightarrow K_O + U_O = K_P + U_P$

For motion $Q\to0 \Rightarrow K'_O + U'_O = K_Q + U_Q$

$\Rightarrow K_O = U_P;\ \ K'_O = U_2 = 2U_P = 2K_O$

$$\Rightarrow t_{Q\to O} = \sqrt{\frac{2(2h/\sin\alpha)}{g\sin\alpha}} = t_1$$

$$\Rightarrow t_{P\to O} = \sqrt{\frac{2(h/\sin\alpha)}{g\sin\alpha}} = t_2 = \sqrt{2}\ t_1$$

21. $W_g + W_F = \Delta KE \Rightarrow -mgh - f.d = 0 - \frac{1}{2}mv^2$

$-mg\ 1.1 - \mu mg\,d = -\frac{1}{2}mv^2$ (μ= 0.6) $\Rightarrow$ d=1.17 m

22. Maximum elongation in spring $= \frac{2Mg}{K}$

Condition block 'm' to move is

$Kx \ge mg\sin37° + \mu mg\cos37°$ $\Rightarrow$ $M = \frac{3}{5}$

23. $v = a\sqrt{s} = \frac{ds}{dt} \Rightarrow s = \frac{a^2t^2}{4}$

$$\therefore W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}m\times a^2s = \frac{1}{2}ma^2\frac{(a^2t^2)}{4} = \frac{ma^4t^2}{8}$$

24. Conservative forces depends on the end points not on the path. Hence work done by it in a closed loop is zero.

25. COME : $K_1 + U_1 = K_2 + U_2$

$$\frac{1}{2}mv_0^2 + 0 = 0 + mg\ell(1-\cos60°) \Rightarrow v_0 = 7 \text{ m/s}$$

26. For equilibrium, F = 0 $\Rightarrow$ $x(3x-2)=0 \Rightarrow x=0 \Rightarrow x=\frac{2}{3}$

27. For velocity to maximum acceleration must be zero.

$\Rightarrow$ $mg - kx = ma = 0$

$\Rightarrow$ $x = \frac{mg}{k} = \frac{1\times10}{0.2} = 5\text{cm}$

$\therefore$ Height from table = 15 cm

28. $v^2 = v_0^2 + 2(-\mu g)L$

For v = 0, $v_0 = \sqrt{2\mu gL}$

29. Sum of KE and PE remains constant.

30. $W_N = \Delta KE = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = \frac{1}{2}\times1\times(10\sqrt{3})^2 = 150\text{ J}$

31. ΔK.E. = work done by all the forces

$\Delta K.E. = m\vec{a}.\vec{s}$

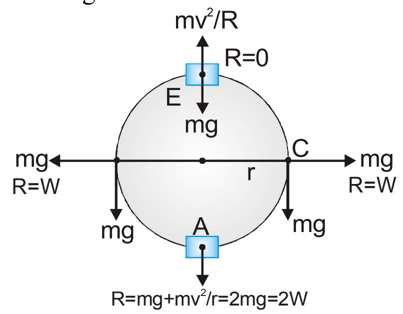
When acceleration is constant

$\Delta K.E. \propto t^2$ [as $s = \frac{1}{2}at^2$]

32. $\left(0-\frac{1}{2}kx^2\right) + (-\mu mgx) = 0 - \frac{1}{2}mv^2 \Rightarrow$ v=8 m/s

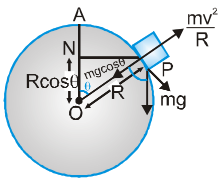
33. $\vec{F} = 3\hat{i} + 4\hat{j}$ is a conservative force ie therefore $W_1 = W_2$

34. Given $v = \sqrt{gr}$



35. To break off reaction becomes O,

i.e. $mg\cos\theta = \frac{mv^2}{R} \Rightarrow \cos\theta = \frac{v^2}{Rg}$ ....(1)



But from energy considerations

$mgR[1-\cos\theta] = \frac{1}{2}mv^2$

$\Rightarrow$ $v^2 = 2gR(1-\cos\theta)$ using it in (1)
$\cos\theta = 2(1-\cos\theta)$

$\Rightarrow$ $\cos\theta = 2 - 2\cos\theta \Rightarrow \cos\theta = \frac{2}{3}$

So $\sin\theta = \sqrt{1-\frac{4}{9}} = \frac{\sqrt{5}}{3}$

Now tangential acceleration $g\sin\theta = g\frac{\sqrt{5}}{3}$

36. Given $\frac{1}{2}mv^2 = as^2$ .....**(i)**

So $a_r = \frac{v^2}{R} = \frac{2as^2}{mR}$ .....**(ii)**

Also $a_t = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt} = v\frac{dv}{ds}$

But from equation (1) $v = s\sqrt{\frac{2a}{m}}$

put it above $a_t = s\sqrt{\frac{2a}{m}}\left(\sqrt{\frac{2a}{m}}\right) = \frac{2as}{m}$ .....**(iii)**

So that $a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{2as^2}{mR}\right)^2 + \left(\frac{2as}{m}\right)^2}$

i.e. $a = \frac{2as}{m}\sqrt{1+\left(\frac{s}{R}\right)^2}$

So force $F = ma = 2as\sqrt{1+\left(\frac{s}{R}\right)^2}$

37. In this case $T = \frac{2\pi r}{u}$ [for 1 resolution]

Also $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

But $t = nT \Rightarrow \sqrt{\frac{2h}{g}} = n\frac{2\pi r}{u} \Rightarrow n = \frac{u}{2\pi r}\sqrt{\frac{2h}{g}}$

38. Tension will be $mg\cos\theta$ at extremes but it becomes

$mg\cos\theta + \frac{mv^2}{\ell}$.

In the given situation by making diagram, we can shown that $T - Mg\cos\theta = \frac{Mv^2}{L}$ and tangential acceleration $= g\sin\theta$.

Part # II : Assertion & Reason

1. D **2.** A **3.** D **4.** B **5.** A **6.** C
7. A **8.** A

EXERCISE - 3

Part # I : Matrix Match Type

1. $f_{conservative} = -\frac{du}{dx} = 30\,N\hat{i}$

change in kinetic energy =2

[Area under (a–x) graph]

as mass is 1 kg $\Rightarrow [80+40] = 120$,

$KE_{initial} = \frac{1}{2}Mv^2 = 8$ J

(A) $KE_f = 128$ J

(B) $W_{can} = \vec{f}\times\vec{d} = 30\times 8 \Rightarrow 240$ J

(C) $W_{Net} = \Delta KE = 120$ J

(D) $W_{cons} + W_{ext} = 120$; $W_{ext} = -120$ J

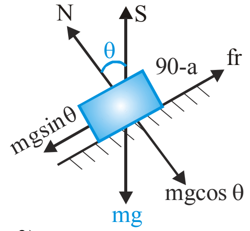
2. W_g = force × (displacement in the direction of force)

$W_g = [10\times\frac{1}{2}\times 2\times 16] = -160$ joule

$w_N = \vec{N}.\vec{s} = m(g+a)\cos\theta\left(\frac{1}{2}\times 2\times 16\right)\cos\theta$

$= (12)\times\frac{\sqrt{3}}{2}(16)\frac{\sqrt{3}}{2}$

$= 12\times 12 = 144$ J



$W_{fr} = \vec{f_r}.\vec{s}$

$= m(g+a)\sin\theta\,(16)\cos(90-\theta)$

$= (12)\times 16\times\frac{1}{4} = 48$ joule

$W_{net} = W_g + W_N + W_{fr} = 32$ joule

3. By applying conservation of momentum wedge will acquire some velocity $= -\frac{mv_x}{M+m}$ where v_x is velocity of block w.r.t wedge in negative x-direction.

(A) Work done by normal on block is

$$= -\frac{1}{2}M\left(\frac{mv_x}{M+m}\right)^2$$

(B) Work done by normal on wedge is

$$= \frac{1}{2}M\left(\frac{mv_x}{M+m}\right)^2 \text{ is positive.}$$

(C) Net work done by normal is = 0

(D) less than mgh as K.E. is $< \frac{1}{2}$ m2gh,

$KE_f >$ KE is positive.

4. For $v \geq \sqrt{5g\ell}$, the bob will complete a vertical circular path.

For $\sqrt{2g\ell} < v < \sqrt{5g\ell}$, the bob will execute projectile motion.

For $v < \sqrt{2g\ell}$, the bob oscillates.

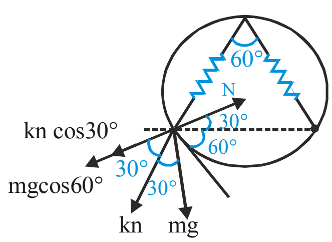
Part # II : Comprehension

Comprehension # 1

1. $N - Kx\cos 30° - mg\cos 60° = \frac{Mv^2}{R}$

As velocity of Ring = 0

$N = kx\cos 30° + mg\cos 60°$



$$= \frac{(2+\sqrt{3})mg}{\sqrt{3}R}(2-\sqrt{3})R\left(\frac{\sqrt{3}}{2}\right) + \frac{mg\sqrt{3}}{2}$$

$$= \frac{mg}{2} + \frac{mg}{2} = mg$$

2. $f_{net} = (k\cos 60°)\,x + mg\cos 30°$

$$= \frac{(2+\sqrt{3})mg}{\sqrt{3}R}(2-\sqrt{3})R\,\frac{1}{2} + \frac{mg\sqrt{3}}{2}$$

$$= \frac{mg}{2}\left[\frac{1}{\sqrt{3}} + \sqrt{3}\right] = \frac{2mg}{\sqrt{3}}$$

$$a_{rev} = 2a\cos 60 = a = \frac{2g}{\sqrt{3}} \text{ horizontal}$$

3. By applying work - energy theorem

$$\frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2;\ \frac{1}{2}mv^2 = \frac{1}{2}\frac{(2+\sqrt{3})mg}{\sqrt{3}g}(2-\sqrt{3})^2R^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{mg}{\sqrt{3}}(2-\sqrt{3})R \Rightarrow v = \sqrt{\frac{gR(2-\sqrt{3})}{\sqrt{3}}}$$

Comprehension # 2

1. $W = \vec{f}.d\vec{s} \Rightarrow W = -mg\left(\frac{1}{2}a_0t^2\right)$

2. For the motion of the block in vertical

$mg - N = ma_0,\ N = m(g - a_0)$

$$W_N = -\frac{Na_0t^2}{2} \Rightarrow -\frac{m(g-a_0)a_0t^2}{2}$$

3. For observer A pseudo force on the particle is zero W = 0

4. $W = \vec{f}_{net}.d\vec{s} \Rightarrow W = ma\,\frac{1}{2}at^2 \Rightarrow \frac{ma^2t^2}{2}$

5. For observer A the block appears to be stationary

$\therefore$ Displacement is zero hence w = 0

Comprehension # 3

1. By applying work energy theorem

$$\frac{1}{2}Mv^2 - 0 = W_g \Rightarrow \frac{1}{2}Mv^2 = mg\ell \Rightarrow v = \sqrt{2g\ell}$$

2. $\sqrt{2g\ell} = \sqrt{5g(\ell - x)}$

$$\Rightarrow 2g\ell = 5g(\ell - x) \Rightarrow 5x = 3\ell \Rightarrow x = \frac{3\ell}{5}$$

3. Net force towards the centre will provide the required centripetal force

$$kx - mg = \frac{mv^2}{R}$$

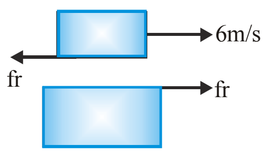
$$kx - mg = \frac{m2g\ell}{\ell}$$

$$\Rightarrow kx = 3mg \Rightarrow x = \frac{3mg}{k}$$

Comprehension # 4

1. From the F.B.D. of the blocks :

upper block is –ve and lower block is +ve as



v_{upper} = decreases, v_{lower} = Increases

2. By applying conservation of momentum

$1 \times 6 + 2 \times 3 = 3(v) \Rightarrow v = 4\text{m/s}$

By applying work energy theorem

$$-\frac{1}{2}(1)(36) + \frac{1}{2}(1)(16) = w_{fr}$$

$\Rightarrow$ $-18 + 8 = W_{fr}$ $\Rightarrow$ $W_{fr} = -10$ J

and Work done on the lower block +10j $\Rightarrow$ $W_{net} = 0$

Comprehension # 5

1. Particle will have some translatory kinetic energy as well as rotatory energy the whole of the K.E. is converted into potential energy h < 6

2. By applying conservation of mechanical energy

$$\Rightarrow \frac{1}{2}mu^2 = mg(h) \Rightarrow u^2 = 80$$

$$\Rightarrow \frac{1}{2}mu^2\sin^2 30 = mgh \Rightarrow h = 1\text{m}$$

Total height = 2 + 1 = 3m

Comprehension # 6

1. By applying work energy theorem change in kinetic energy $= w_S \Rightarrow 0 - \frac{1}{2}mv^2 = W_s$

2. As the kinetic energy of block is decreasing, therefore work done by the normal is $= -\frac{1}{2}mv^2$

3. $W_{net} = -\frac{1}{2}mv^2$

5. $W_{net} = 0$ as for the B change in velocity is zero.

6. As there is no change in kinetic energy stored is due to

Comprehension # 7

1. (A) $W_{CL} + W_f = \Delta KE$ $\quad \therefore W_{CL} = \Delta KE - W_f$

(a) During acceleration motion negative work is done against friction and there is also change is kinetic energy. Hence net work needed is positive.

(b) During uniform motion work is done against friction only and that is positive.

(c) During retarded motion, the load has to be stopped in exactly 50 metres. If only friction is considered then the load stops in 12.5 metres which is less than where it has to stop.

Hence the camel has to apply some force so that the load stops in 50 m (> 12.5 m). Therefore the work done in this case is also positive.

2. $W_{CL}|_{\text{accelerated motion}} = \Delta KE - W_{friction}$

where W_{CL} is work done by camel on load.

$$= \left[\frac{1}{2}mv^2 - 0\right] - [-\mu_k mg.50]$$

$$= \frac{1}{2} \times 1000 \times 5^2 + 0.1 \times 10 \times 1000 \times 50 = 1000\left[\frac{125}{2}\right]$$

similarly, $W_{CL}|\text{retardation} = \Delta KE - W_{friction}$

$$\left[0 - \frac{1}{2}mv^2\right] - [-\mu_k mg.50] = 1000\left[\frac{75}{2}\right]$$

$$\therefore \frac{W_{CL} \mid \text{accelerated motion}}{W_{CL} \mid \text{retarded motion}} = \frac{125}{75} = \frac{5}{3} \Rightarrow 5 : 3$$

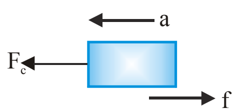
3. Maximum power = $F_{max} \times V$

Maximum force applied by camel is during the accelerated motion.

We have $V^2 - U^2 = 2as$, $25 = 0^2 + 2 \cdot a \cdot 50$

$a = 0.25 \text{ m/s}^2$

for accelerated motion



$\therefore F_c - f = ma$

$\therefore F_c = \mu mg + ma$

$= 0.1 \times 1000 \times 10 + 1000 \times 2.5$

$= 1000 + 250 = 1250$ N

This is the critical point just before the point where it attains maximum velocity of almost 5m/s .

Hence maximum power at this point is

$= 1250 \times 5 = 6250$ J/s.

4. We have $W = P\Delta T, P = 18 \times 10^3 V + 10^4$ J/s

$\therefore$ $P_5 = 18 \times 10^3 \times 5 + 10^4$ J/s and

$$\Delta T_5 = \frac{2000m}{5\,m/s} = 400s$$

$$P_{10} = 18 \times 10^3 \times 10^4 J/s$$

and $\Delta T_{10} = \dfrac{2000m}{10\,m/s} = 200s$

$$\therefore \frac{W_5}{W_{10}} = \frac{10^4(9+1)\times 400}{10^4(18+1)\times 200}$$

5. The time of travel in accelerated motion = time of travel in retarded motion.

$$T_{AB} = T_{CD} = \frac{V}{a} = \frac{5}{0.25} = 20 \text{ sec}$$

Now time for uniform motion = $T_{ac} = \dfrac{2000}{5} = 400$ s

$\therefore$ Total energy consumed $= \displaystyle\int_0^{440} P\,dt$

$$= \int_0^{20} [18.10^3 V + 10^4]dt + \int_{20}^{420} [18.10^3.5 + 10^4]dt$$

$$+ \int_{420}^{440} [18.10^3 V + 10^4]dt$$

$$= \int_0^{20} [18.10^3 V\,dt + \int_0^{20} 10^4\,dt + \left[10^5 t\right]_{20}^{420}$$

$$+ \int_{420}^{440} 18.10^3 V dt + \int_{420}^{440} 10^4 dt$$

Putting Vdt = dx and changing limits appropriately it becomes

$$\int_0^{60} 18.10^3 dx + \left[10^4 t\right]_0^{20} + 10^5[420-20]$$

$$+ \int_{2050}^{2100} 18.10^3 dx + \left[10^4\right]_{420}^{440}$$

$= 18.10^3.50 + 10^4[20] + 10^5 \cdot 400$

$\quad +18.10^3[50] + 10^4[20]$ Joules

$= 90 \times 10^4 + 20 \times 10^4 + 400 \times 10^5$

$+ 90 \times 10^4 + 20 \times 10^4$ J $= 4.22 \times 10^7$ J

Comprehension # 8

1. $u = \dfrac{A}{r^2} - \dfrac{B}{r} \Rightarrow \dfrac{du}{dr} = -\dfrac{2A}{r^3} + \dfrac{B}{r^2}$

$$f = -\frac{du}{dr} = \frac{2A}{r^3} - \frac{B}{r^2},\ F = 0 \Rightarrow r = \frac{2A}{B}$$

2. As potential is minimum at $r = r_0$ the equilibrium is stable.

3. Given that

$$U = \frac{A}{r^2} - \frac{B}{r} \text{ as } r = \frac{2A}{B};\ U_i = \frac{AB^2}{4A^2} - \frac{BB}{2A} = \frac{-B^2}{4A}$$

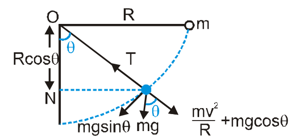
$$\Rightarrow U_f = 0,\ \Delta W = U_f - U_i \Rightarrow \frac{B^2}{4A}$$

4. K.E. + P.E. = T.E, $0 + \dfrac{A}{r^2} - \dfrac{B}{r} = \dfrac{-3B^2}{16A}$

By solving the above equation $r = \dfrac{2r_0}{3}$

Comprehension # 9

Balancing the forces $T = \dfrac{mv^2}{R} + mg\cos\theta$ ....(i)



From energy considerations

$$mg\,R\cos\theta = \frac{1}{2}mv^2 \Rightarrow v^2 = 2g\,R\cos\theta$$

putting this value in equation (i)

we get $\qquad T = 3mg\cos\theta$

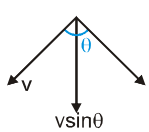
Also acceleration $a_{Total} = \sqrt{a_r^2 + a_t^2}$

$$= \sqrt{\left(\frac{v^2}{R}\right)^2 + (g\sin\theta)^2} = \sqrt{(2g\cos\theta)^2 + (g\sin\theta)^2}$$

$$= g\sqrt{4\cos^2\theta + \sin^2\theta} \Rightarrow a_{Total} = g\sqrt{1 + 3\cos^2\theta}$$

Now virtual component of sphere's velocity

$v_y = v\sin\theta = \sqrt{2gR}\sqrt{\cos\theta}\,\sin\theta$



Applying maxima-minima

$$\frac{dv_y}{d\theta} = \sqrt{2gR}\left[\frac{(-\sin\theta)\sin\theta}{2\sqrt{\cos\theta}} + \sqrt{\cos\theta}\cos\theta\right]$$

$$= \sqrt{2gR}\left[\frac{-\sin^2\theta}{2\sqrt{\cos\theta}} + \cos\theta\sqrt{\cos\theta}\right]$$

$\Rightarrow \dfrac{\sin^2\theta}{2} = \cos^2\theta \quad \Rightarrow \tan^2\theta = \sqrt{2}$

$\Rightarrow \theta = \tan^{-1}\sqrt{2} \quad \Rightarrow \tan\theta = \sqrt{2}$

So $\sin\theta = \dfrac{\sqrt{2}}{\sqrt{3}}$ and $\cos\theta = \dfrac{1}{\sqrt{3}}$

Thus tension $T = 3\,mg\cos\theta = 3mg \times \dfrac{1}{\sqrt{3}} = \sqrt{3}\,mg$

Comprehension 10

Using work energy theorem

$$\frac{m \times 2g}{9} \times R\sin\theta + mgR(1-\cos\theta) = \frac{1}{2}mv^2 \qquad \text{....(i)}$$

Also $mg\cos\theta = \dfrac{2mg}{9}\sin\theta + \dfrac{mv^2}{R}$

$$v^2 = gR\cos\theta - \frac{2g}{9}R\sin\theta \qquad \text{....(ii)}$$

From equation (i) & (ii)

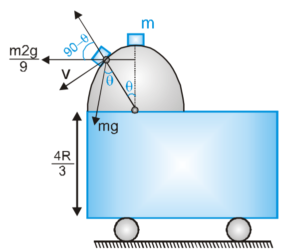
$$\frac{2mg}{9}R\sin\theta + mgR(1-\cos\theta) = \frac{m}{2}\left(gR\cos\theta - \frac{2g}{9}R\sin\theta\right)$$

$\Rightarrow 4\sin\theta + 18(1-\cos\theta) = 9\cos\theta - 2\sin\theta$

$\Rightarrow 6\sin\theta + 18 - 18\cos\theta = 9\cos\theta$

$\Rightarrow 6\sin\theta - 27\cos\theta + 18 = 0$

$\Rightarrow 2\sin\theta - 9\cos\theta + 6 = 0$



Now let $\sin\theta = x$ so $\cos\theta = \sqrt{1-x^2}$

Than $2x - 9\sqrt{1-x^2} + 6 = 0$

Solving $x = \dfrac{3}{5} = \sin\theta$ so $\cos\theta = \dfrac{4}{5}$; $\theta = 37°$

Now putting $\theta = 37°$

in $\mu = h + R\cos\theta = \dfrac{4R}{3} + R \times \dfrac{4}{5}$

$$= \frac{20R + 12R}{15} = \frac{32R}{15}$$

From equation (ii) $v^2 = gR\cos\theta\ \dfrac{2g}{9}R\sin\theta$

$$v^2 = gR \times \frac{4}{5} - \frac{2g}{9}R \times \frac{3}{5}$$

$$= gR\left[\frac{4}{5} - \frac{2}{15}\right] = \frac{10gR}{15} = \frac{2gR}{3}$$

Now using $S = ut + \dfrac{1}{2}gt^2$; $\dfrac{32R}{15} = \sqrt{\dfrac{2gR}{3}}\,t + \dfrac{1}{2}gt^2$

t can be obtained $t = \sqrt{\dfrac{2R}{g}}$

EXERCISE - 4

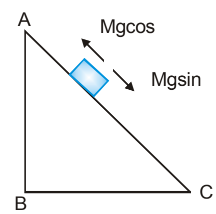
Subjective Type

1. Heat generated = work done against friction

$\Rightarrow (\mu mg)(vt) = (0.2 \times 2 \times 10) \times 2 \times 5 = 40\text{ J}$

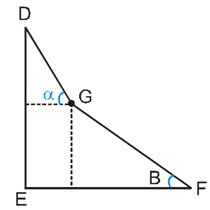
$$= \frac{40}{4.2}\text{ cal} = 9.52\text{ cal}$$

2.



$W_{Mg} = Mg\sin\theta \times AC = Mg \times AB$

$W_f = \mu Mg\cos\theta \times AC \times \cos 180° = -\mu Mg \times (BC)$



$W_{Mg} = Mg(\sin\alpha \times DG + \sin\beta \times GF) = Mg \times DE$

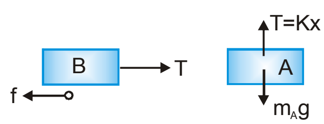
$W_f = -\mu Mg(DG\cos\alpha + GF\cos\beta) = -\mu Mg(EF)$

$= -\mu Mg \times BC \qquad (\because BC = EF)$

From WET, ΔKE will be same in both cases.

$\therefore v_C = v_F$

3. Blocks are moving with constant speed.



$\therefore m_A g = T = kx = f = \mu m_B g$

$\Rightarrow m_B = \dfrac{m_A}{\mu} = \dfrac{2}{0.2} = 10\text{ kg}$ and $x = \dfrac{2 \times 9.8}{1960}$

$\therefore$ Energy stored in spring $= \dfrac{1}{2}kx^2$

$$= \frac{1}{2} \times 1960 \times \left(\frac{19.6}{1960}\right)^2 = 0.098\text{ J}$$

4. COME : $K_1 + U_1 = K_2 + U_2$

$$\frac{3mgr}{2} = \frac{1}{2}mv^2 + \frac{1}{2}kr^2 \quad .....(i)$$

Force equation $kr = mg + \frac{mv^2}{r}$

Solving we get, $k = \frac{2mg}{r} = 500$ N/m

5. Work done by force $= \int Fdx$

$$W = \int_0^{1/2} \pi \sin \pi\, x dx = \pi \frac{[-\cos \pi x]_0^{1/2}}{\pi}$$

$$= -\cos\frac{\pi}{2} + \cos 0 = 1J$$

Work done by external agent = – 1 J

6. Potential energy $U = 1 \times \left(\frac{x^2}{2} - x\right) = \frac{x^2}{2} - x$

For minimum U,

$$\frac{dU}{dx} = \frac{2x}{2} - 1 = 0 \text{ and } \frac{d^2U}{dx^2} = 1 = \text{positive}$$

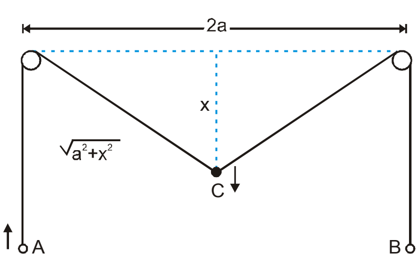
So at x = 1, U is minimum. Hence $U_{min} = -\frac{1}{2}$ J

Total mechanical energy = Max KE + Min PE

$$\Rightarrow \text{Max KE} = \frac{1}{2}mv^2_{max} = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$$

$$\Rightarrow v_{max} = \sqrt{\frac{2}{1} \times \frac{5}{2}} = \sqrt{5}\ ms^{-1}$$

7. As C falls down, A & B move up.
COME : $K_1 + U_1 = K_2 + U_2$



$$0 + mgx = 0 + 2mg\left(\sqrt{a^2 + x^2} - a\right) \Rightarrow x = \frac{4a}{3}$$

8. $a_n = bt^2 = \frac{v^2}{R} \Rightarrow v = \sqrt{bR}\ t \quad \Rightarrow a_t = \sqrt{bR}$

$\therefore$ P = FV = mbRt

$$<P> = \frac{\int_0^t Pdt}{\int_0^t dt} = \frac{mbR(t^2/2)}{t} = \frac{mbRt}{2}$$

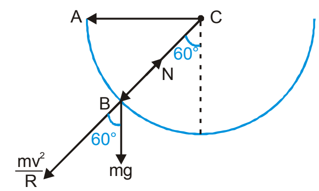
9. Let extension in spring be x_0 due to m_1

then $m_1 g x_0 = \frac{1}{2}kx_0^2 \quad \Rightarrow kx_0 = 2m_1 g$

but $kx_0 \geq mg$ so $2m_1 g \geq mg \Rightarrow m_1 \geq \frac{m}{2}$

therefore minimum value of $m_1 = \frac{m}{2}$

10. COME : $K_1 + U_1 = K_2 + U_2$



$$0 + MgR = \frac{1}{2}mv^2 + \frac{mgR}{2} \Rightarrow v = \sqrt{gR}$$

Forces at B $\Rightarrow N = mg\cos 60° + \frac{mv^2}{R} = \frac{15\sqrt{3}}{2}$

11. $\theta = 3(t + \sin t)$; $\omega = 3 + 3\cos t$; $\alpha = -3\sin t$

$$F = \sqrt{(m\omega^2 R)^2 + (m\alpha R)^2} \left(t = \frac{\pi}{2}\right) = 9\sqrt{10}N$$

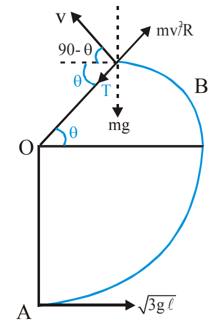
12. COME : $\frac{mv^2}{2} = mgh$

If resultant acceleration, a, makes angle θ with thread, then $a\sin\phi = g\sin\theta$

$$a\cos\phi = \frac{v^2}{\ell} = \frac{2gh}{\ell}$$

$$\therefore \tan\phi = \frac{\ell \sin\theta}{2h} \Rightarrow \phi = \tan^{-1}\left(\frac{\ell\sin\theta}{2h}\right)$$

13. Here the bob has velocity greater than $\sqrt{2g\ell}$ and smaller than $\sqrt{5g\ell}$. Hence the thread will slack after completing semicircle.



COME: $K_1 + U_1 = K_2 + U_2$

$\frac{1}{2}m(3g\ell)+0=\frac{1}{2}mv^2+mg(\ell+\ell\sin\theta)$ .....(i)

Force equation at B :

$T+mg\sin\theta=\frac{mv^2}{R}$ .....(ii)

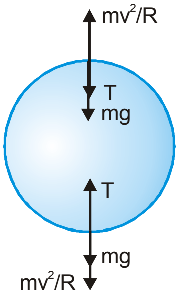
Solving for T=0, we get $\sin\theta=\frac{1}{3}$

$\therefore$ $v_B=\sqrt{g\ell\sin\theta}$

$\therefore$ The particle will execute projectile motion after tension become zero.

$\therefore$ $v_{min}=v\sin\theta=\sqrt{\frac{g\ell}{3}}\times\frac{1}{3}$

14. $T_{max}=mg+\frac{mv^2}{R}$, $T_{min}=\frac{mv^2}{R}-mg$



$\frac{T_{max}}{T_{min}}=\frac{mg+\frac{mv^2}{R}}{\frac{mv^2}{R}-mg}=\frac{5}{3}$ (R=2m)

$\Rightarrow$ $v=4\sqrt{5}$ m/s

15. For speed u_0, contact at top is lost.

$\Rightarrow$ $N+\frac{mu_0^2}{r}=mg$ $\Rightarrow$ (N=0) $u_0=\sqrt{gr}$

(a) For vertical motion; $t=\sqrt{\frac{2r}{g}}$

$\therefore$ Horizontal distance

$s=2u_0.t=2\sqrt{gr}\times\frac{\sqrt{2r}}{g}=2\sqrt{2}r$

(b) COME :

$\frac{1}{2}\frac{m(u_0)^2}{3}+mgr=\frac{1}{2}mv^2+mgr\cos\theta$ ......(i)

Force equation : $N+\frac{mv^2}{r}=mg\cos\theta$ ....(ii)

$\therefore$ $h=r\cos\theta=\frac{19}{27}r$

(c) $|\vec{a}_{net}|=|\vec{a}_r+\vec{a}_t|=\sqrt{(g\sin\theta)^2+(g\cos\theta)^2}=g$

16. COME : $K_A+U_A=K_B+U_B$

$0+mg(2R)+\frac{1}{2}kR^2=\frac{1}{2}mv^2+0+0(k=mg/R)$

$\Rightarrow$ $\frac{mv^2}{R}=5\,mg$ $\therefore$ Force equation at B

$\Rightarrow$ $T_B=mg+\frac{mv^2}{R}=6mg$

17. a : Natural length

a : Initial elongation

2a : additional elongation

COME : $\frac{1}{2}k(3a)^2=mgx$ $\Rightarrow$ $x=\frac{9a}{2}$

(above point of suspension)

18. Conservative force, $F=-\frac{dU}{dr}=-\frac{d(2r^3)}{dr}=-6r^2$

This force supplies the necessary centripetal acceleration.

$\frac{mv^2}{r}=6r^2$ $\Rightarrow$ $\frac{1}{2}mv^2=3r^3$

$E=K+U=5r^3=5\times5\times5\times5=625$ J

19. WET : $W_N+W_{Mg}+W_f+W_{sp}=\Delta KE$

$0+0-\mu_k.mg(2.14+x)+0-\frac{1}{2}kx^2=0-\frac{1}{2}mv^2$

$\Rightarrow$ $x=0.1$ m

At $x=1$m, $F_{spring}=kx=2\times0.1=0.2$ N

$F_{S.F.}=\mu_S.mg=0.22\times\frac{1}{2}\times10=1.1$ N

Hence the block stops after compressing the spring.

$\therefore$ Total distance travelled by block when it stops

$=2+2.14+0.1=4.24$ m

20. At position B;

$mg=T\cos\theta=k.\Delta\ell.\cos\theta$

$=\frac{2mg}{a}\left[a+\frac{a}{\sin\theta}-a\right]\cos\theta$

$=2mg\cot\theta$ $\Rightarrow$ $\cot\theta=\frac{1}{2}$

(a) $OB = a\cot\theta = \dfrac{a}{2}$

(b) COME : $K_C + U_C = K_O + U_O$

$$0 + mga + \frac{1}{2} \times \left(\frac{2mg}{a}\right)\left(\sqrt{2}a\right)^2 = \frac{1}{2}mv^2 + \frac{1}{2}ka^2$$

(i) $\Rightarrow v = 2\sqrt{ga}$

(ii) $K_C + U_C = K_P + U_P$

[P is the point of greatest depth]

$$\Rightarrow \quad mga + \frac{1}{2}\left(\frac{2mg}{a}\right)\left(\sqrt{2}a\right)^2$$

$$= -mgx + \frac{1}{2}\left(\frac{2mg}{a}\right)(a^2 + x^2) \Rightarrow x = 2a$$

21. For part AB : (R=4a)

$$\left(\frac{v_0}{4a}\right)t_1 = \frac{\pi}{2} \quad \Rightarrow \quad t_1 = 4\left(\frac{\pi a}{2v_0}\right)$$

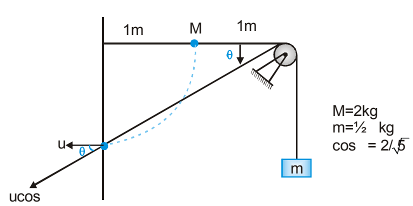
For part BC : (R=3a) $\Rightarrow t_2 = 3\left(\dfrac{\pi a}{2v_0}\right)$

For part CD : (R=2a) : $t_3 = 2\left(\dfrac{\pi a}{2v_0}\right)$

For part DA : (R=a) = : $t_4 = \left(\dfrac{\pi a}{2v_0}\right)$

$$\therefore \quad t = t_1 + t_2 + t_3 + t_4 = \frac{5\pi a}{v_0}$$

22. COME : $K_i + U_i = K_f + U_f$

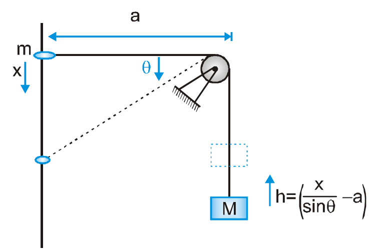


$$\Rightarrow \quad 0 + Mg \times R_1$$

$$= 0 + mg\left(\sqrt{5} - 1\right) + \frac{1}{2}Mu^2 + \frac{1}{2}m\left(u\cos\theta\right)^2$$

$\Rightarrow \quad u = 3029$ m/s

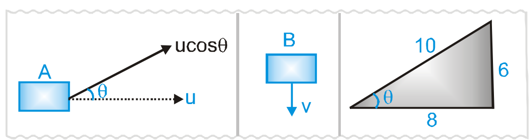
23. COME : $K_i + U_i = K_f + U_f$



$$\Rightarrow \quad 0 + mgx = 0 + Mg\left(\sqrt{a^2 + x^2} - a\right)$$

$$\Rightarrow \quad x = \frac{2mM}{M^2 - m^2}a$$

24.



For constant length of string =v = u cosθ

COME :

$$mg \times 5 = \frac{1}{2}mv^2 + \frac{1}{2}mu^2 \Rightarrow u = \frac{10}{\sqrt{1.64}}$$

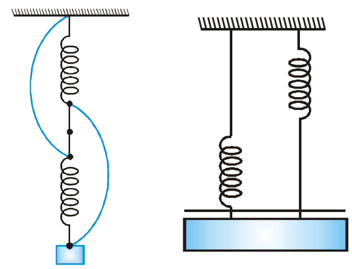
$$\therefore \quad v = u\cos\theta = \frac{40}{\sqrt{41}} \text{ m/s}$$

25. Initial elongation in each spring

$$= \frac{Mg}{2\left(\dfrac{kx_0}{2}\right)} = \frac{Mg}{kx_0} = 20\text{cm}$$

Total initial length of each spring

$= 50 + 20 = 70$ cm



Equilibrium position $= 2kx = mg$

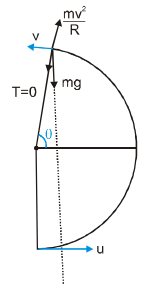
$$x = \frac{100}{2 \times 500} = 10 \text{ cm}$$

and due to inertia it goes

10 cm also up = 20 m

26. COME : $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgL(1+\sin\theta)$ .....(i)

For equation $\Rightarrow T + mg\sin\theta = \frac{mv^2}{L}$ ....(ii)



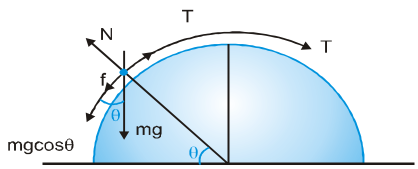
Since the particle crosses the $\frac{L}{8}$ line at its half of its range

$\therefore \frac{v^2\sin\theta.\cos\theta}{g} = L\cos\theta - \frac{L}{8}$ ....(iii)

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

From equation (i) $\Rightarrow u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$

27. WET $\Rightarrow W_{mg} + W_N + W_T + W_f = \Delta KE$



$-mgR + 0 + W_T + \int_0^{\pi/2}(\mu mg\sin\theta.Rd\theta)\cos180 = 0$

$\Rightarrow W_T = mgR(1+\mu)$

28. WET : $W_{SP} + W_{mg} + W_N + W_f = \Delta KE$

$\Rightarrow \left[0 - \frac{1}{2}k\left(\frac{h}{\sin\theta}\right)^2\right] + \left[mg\sin\theta \times \frac{h}{\sin\theta}\right] + 0$

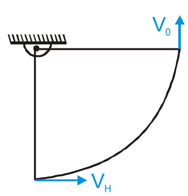
$-\mu mgh\cot\theta = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{2}{m}\left[mgh - \frac{1}{2}k\left(\frac{h}{\sin\theta}\right)^2 - \mu mgh\cot\theta\right]}$

29. The string can break at the lowest point

$\therefore T_{max} = mg + \frac{mv_H^2}{R}$

$\Rightarrow 45 = 5 + \frac{0.5 \times v^2}{0.5}$

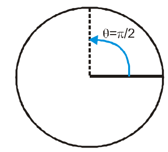


COME : $v_H^2 = v_0^2 + 2gR$

$v_0^2 = 40 - 2 \times 10 \times \frac{1}{2} = 30$

$\therefore H_{max} = \frac{v_0^2}{2g} = \frac{30}{2\times10} = \frac{3}{2} = 1.5$ m

30. $\frac{1}{2}\alpha t^2 = \frac{\pi}{2}$ $(\alpha = \frac{\pi}{4}) \Rightarrow t = 2$ sec



$\therefore$ Average velocity $= \frac{\sqrt{2}R}{t} = 1$ m/s

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Spring constant (k) = 800 $\frac{N}{m}$

Work done in extending a spring from

X_1 to $X_2 = U_f - U_i = \frac{1}{2}kX_2^2 - \frac{1}{2}kX_1^2$

$W = \frac{1}{2}k\left[X_2^2 - X_1^2\right] = \frac{1}{2} \times 800\,[0.15)^2 - (0.05)^2]$

$= 400\left[\left(\frac{15}{100}\right)^2 - \left(\frac{5}{100}\right)^2\right] = \frac{400}{10000}[225-25]$

$= \frac{400 \times 200}{10000} = 8J$

2. $k = 5 \times 10^3$ N/m

$W = \frac{1}{2}k\left[x_2^2 - x_1^2\right]$

$W = \frac{1}{2} \times 5 \times 10^3\left[\left(10 \times 10^{-2}\right) - \left(5 \times 10^{-2}\right)^2\right]$

$W = \frac{1}{2} \times 5 \times 10^3 \times 10^{-4}[100 - 25]$

$= \frac{75 \times 5 \times 10^{-1}}{2} = \frac{75}{4} = 18.75$ N-m

3. Power = FV = constant i.e., mav = k

$\Rightarrow av = k_1 \Rightarrow \left(\frac{dv}{dt}\right)v = k_1 \Rightarrow vdv = k_1dt$

On integrating both sides, we get

$\frac{v^2}{2} = k_1t \Rightarrow v^2 = 2k_1t \Rightarrow v = \sqrt{2k_1}t^{1/2}$

$\Rightarrow ds = k_2t^{1/2}dt \Rightarrow s = \left(\frac{k_2}{3/2}\right)t^{3/2} \Rightarrow s \propto t^{3/2}$

4. Here $F \propto x$, by using work energy theorem

$\Delta KE = \int F\,dx \Rightarrow \Delta KE \propto \int x\,dx \Rightarrow \Delta KE \propto x^2$

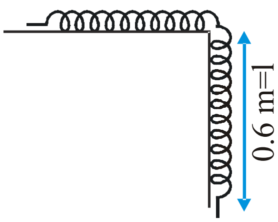
5. Given that acceleration $a = \frac{v_1}{t_1}$ ...(i)

Power = Fv P = (ma)v

$P = (ma^2 t)$ [∵ v=at]

$$P = \left(\frac{mv_1^2}{t_1^2}\right) t \left[\text{on replacing } a = \frac{v_1}{t_1}\right]$$

6. Work done in pulling the hanging part of the chain upon the table $= \frac{mg\ell}{2}$



where m = mass of the hanging part

l = hanging part of chain

$$W = \left(\frac{4}{3} \times 0.6\right) \times \frac{10 \times (0.6)}{2} = 3.6 \text{ J}$$

7. According to work-energy theorem,

$W = \Delta K$

Case I : $-F \times 3 = \frac{1}{2} m \left(\frac{v_0}{2}\right)^2 - \frac{1}{2} m v_0^2$

where F is resistive force and v_0 is initial speed.

Case II : Let, the further distance travelled by the bullet before coming to rest is s.

$$\therefore \; -F(3+s) = K_f - K_i = -\frac{1}{2} m v_0^2$$

$$\Rightarrow \; -\frac{1}{8} m v_0^2 (3+s) = -\frac{1}{2} m v_0^2$$

or $\frac{1}{4}(3+s) = 1$ or $\frac{3}{4} + \frac{s}{4} = 1$ or s = 1 cm

8. Momentum would be maximum when KE would be maximum and this is the case when total elastic PE is converted into KE.

According to conseration of energy

$$\frac{1}{2} kL^2 = \frac{1}{2} Mv^2$$

$\Rightarrow kL^2 = \frac{(Mv)^2}{M}$ or $MkL^2 = p^2$ (∵ p = Mv)

$\Rightarrow p = L\sqrt{Mk}$

9. Applying work-energy theorem at the lowest and highest point, we get

$$W_C + W_{NC} + W_{ext} = \Delta K$$

$$W_C + 0 + 0 = K_f - K_i$$

$$W_{C(Gravity)} = 0 - \frac{1}{2} \times 0.1 \times 25$$

$$W_{Gravity} = -1.25 \text{ J}$$

10. $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$

For minimum value of V,

$$\frac{dV}{dx} = 0 \Rightarrow \frac{4x^3}{4} - \frac{2x}{4} = 0 \Rightarrow x = 0, x = \pm 1$$

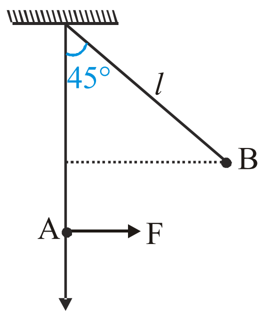
So, $V_{min}(x = \pm 1) = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$ J

Now, $K_{max} + V_{min}$ = Total mechanical energy

$\Rightarrow K_{max} = \left(\frac{1}{4}\right) + 2$ or $K_{max} = \frac{9}{4}$

or $\frac{mv^2}{2} = \frac{9}{4}$ or $v = \frac{3}{\sqrt{2}}$ ms^{-1}

11. Applying work-energy theorem,



Work done by F from A to B

= Work done by Mg from A to B

$\Rightarrow F(\ell \sin 45°) = Mg\ell [1 - \cos 45°]$

$\Rightarrow F = Mg(\sqrt{2} - 1)$

12. $a = \frac{F_k}{m} = \frac{15}{2} = 7.5 \text{ m/s}^2$.

Now, $ma = \frac{1}{2} kx^2 \Rightarrow 2 \times 7.5 = \frac{1}{2} \times 10000 \times x^2$

or $x^2 = 3 \times 10^{-3}$ or x = 0.055 m or x = 5.5 cm

13. Question is somewhat based on approximations. Let mass of athlete is 65 kg.
Approx velocity from the given data is 10 m/s

So, $KE = \frac{65 \times 100}{2} = 3250\ J$

So, option (d) is the most probable answer.

14. $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

$F = -\frac{dU}{dx} = +12\frac{a}{x^{13}} - \frac{6b}{x^7} = 0 \Rightarrow x = \left(\frac{2a}{b}\right)^{1/6}$

$U(x=\infty) = 0$

$U_{equilibrium} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a}$

$\therefore\ U(x=\infty) - U_{equilibrium} = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$

15. $\frac{1}{2} m\upsilon^2 \propto t$

$\upsilon \propto \sqrt{t} \Rightarrow \frac{d\upsilon}{dt} \propto t^{-\frac{1}{2}} \Rightarrow F = ma \propto t^{-\frac{1}{2}} \Rightarrow \propto \frac{1}{\sqrt{t}}$

16. Given same force $F = k_1x_1 = k_2x_2 \Rightarrow \frac{k_1}{k_2} = \frac{x_2}{x_1}$

$W_1 = \frac{1}{2}k_1x_1^2$ & $W_2 = \frac{1}{2}k_2x_2^2$

As $\frac{W_1}{W_2} > 1$ so $\frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} > 1$

$\Rightarrow \frac{F x_1}{F x_2} > 1 \qquad \Rightarrow \frac{k_2}{k_1} > 1$

$\therefore\ k_2 > k_1$ statement 2 is true

OR

if $x_1 = x_2 = x$

$\frac{W_1}{W_2} = \frac{\frac{1}{2}K_1x^2}{\frac{1}{2}K_2x^2} = \frac{K_1}{K_2}$

$\therefore\ \frac{W_1}{W_2} = \frac{K_1}{K_2} < 1$

$\therefore\ W_1 < W_2$

statement 1 is false

17. $m \times 3.8 \times 10^7 \times 0.2 = W$

$= (10 \times 9.8 \times 1) \times 1000$

$m = 12.89 \times 10^{-3}$ kg

Part # II : IIT-JEE ADVANCED

1. Force $= v \times \frac{dm}{dt} = v \times \frac{d}{dt}$ (volume × density)

$= v\frac{d}{dt}(Ax \times \rho) = v \times A\rho\frac{dx}{dt} = A\rho v^2$

$\therefore$ Power = Force × velocity

$= (A\rho v^2)(v) = A\rho v^3 \quad \therefore$ Power $\propto v^3$

2. $F = -\frac{dU}{dx} \quad \therefore dU = -F dx$

$\int dU = -\int_0^x \left(-kx + ax^3\right)dx$ or $U(x) = \frac{kx^2}{2} - \frac{ax^4}{4}$

Let potential energy $U(x) = 0$

$\therefore\ 0 = \frac{x^2}{2}\left(k - \frac{ax^2}{2}\right)$

x has two roots viz $x = 0$ and $x = \sqrt{\frac{2k}{a}}$.

If $k < \frac{ax^2}{2}$, P.E. will be – ve or

when $x > \sqrt{\frac{2k}{a}}$, P.E. will be negative.

$\because\ F = -kx + ax^3 \qquad \therefore$ At $x = 0, F = 0,$

Slope of U – x graph is zero at x = 0.

Thus P.E. is zero at x = 0 and at $x = \sqrt{\frac{2k}{a}}$

Slope of U – x graph, at x = 0, is zero.

3. Mechanical energy is conserved in the process.
Let x=Maximum extension of the spring.

$\therefore$ Increase in elastic potential energy $= \frac{1}{2}kx^2$

Loss of gravitational potential energy = Mgx

$\therefore\ Mgx = \frac{1}{2}kx^2$ or $x = \frac{2Mg}{k}$

4. The gravitational field is a conservative field. In a conservative field, the work done W does not depend on the path (from A to B). It depends on initial and final points.

$\therefore\ W_1 = W_2 = W_3$

5. For conservative forces,

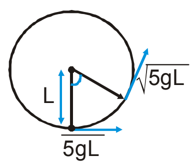
$$\Delta U = -\int_0^x F dx = -\int_0^x kx\ dx \text{ or } U(x) - U(0) = -\frac{kx^2}{2}$$

But U(0) = 0, as given in the question,

$$\therefore \quad U(x) = \frac{-kx^2}{2} \text{ or } x^2 = \frac{-2U(x)}{k}$$

It represents a parabola, below x–axis, symmetrical about U–axis, passing through origin.

6. Energy conservation gives



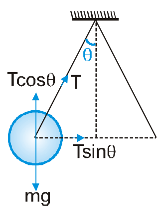
$v^2 = u^2 - 2g(L - L\cos\theta)$

or $\frac{5gL}{4} = 5gL - 2gL\,(1 - \cos\theta)$

or $5 = 20 - 8 + 8\cos\theta$ or $\cos\theta$

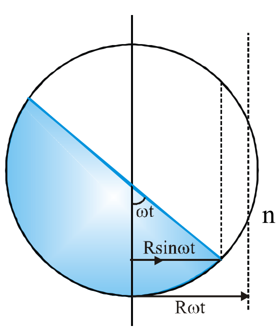
$$= -\frac{7}{8} \Rightarrow \frac{3\pi}{4} < \theta < \pi$$

7. $T\sin\theta = m\omega^2 (L\sin\theta) \Rightarrow T = m\omega^2 L$



$$\omega_{max} = \sqrt{\frac{T_{max}}{mL}} = \sqrt{\frac{324}{0.5 \times 0.5}} = 36 \text{ rad/s}$$

8. According to problem particle is to land on disc.



If we consider a time 't' then x component of displacement is Rωt

$$R\sin\omega t < R\omega t$$

Thus particle P lands in unshaded region.

For Q, x-component is very small and y-component equal to P it will also land in unshaded region.

9. B

10. It is a case of uniform circular motion.

Velocity and acceleration keep on changing their directions. Their magnitudes remain constants. Kinetic energy remains constant.

12. **(i)** For circular motion of the ball, the centripetal force is provided by $(mg\cos\theta - N)$

$$\therefore \quad mg\cos\theta - N = \frac{mv^2}{\left(R + \frac{d}{2}\right)} \qquad .....(i)$$

By geometry, $h = \left(R + \frac{d}{2}\right)(1 - \cos\theta)$

By conservation of energy,

Kinetic energy = potential energy

$$\frac{1}{2}mv^2 - mg\left(R + \frac{d}{2}\right)(1 - \cos\theta) \text{ or}$$

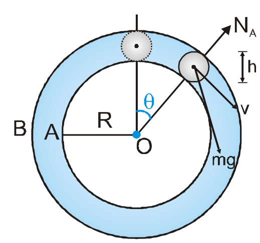
$$v^2 = 2\left(R + \frac{d}{2}\right)(1 - \cos\theta)g \qquad ..... (ii)$$

From (i) & (ii), we get total normal reaction force N.

$$N = mg(3\cos\theta - 2) \qquad ..... (iii)$$

(ii) To find N_A and N_B

For graphs :



From **(iii)**, at A,

$N_A = mg\,(3\cos\theta - 2)$ .... **(iv)**

(i) If $N_A = 0$,

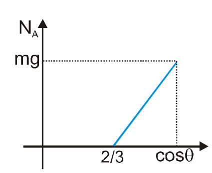
i.e. At A, N = 0,

$0 = mg\,(3\cos\theta - 2)$

or $3\cos\theta = 2$ or $\cos\theta = \frac{2}{3}$

When N_A becomes zero, the ball will lose contact with inner sphere A. After this, it makes contact with outer sphere B. When $\theta - 0$, $N_A = mg$

The N_A versus cos θ graph is a straight line as shown in the figure.



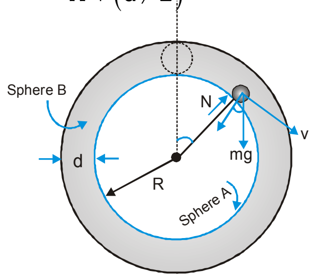
(ii) To find N_B :

Consider : $\cos\theta > \frac{2}{3}$

The ball makes contact with B.

$$N_B - (-mg\cos\theta) = \frac{mv^2}{R + \frac{d}{2}} \quad \text{or}$$

$$N_B + mg\cos\theta = \frac{mv^2}{R + (d/2)} \quad \text{... (v)}$$



By energy conservation,

$$\frac{1}{2}mv^2 = mg\left[\left(R + \frac{d}{2}\right) - \left(R + \frac{d}{2}\right)\cos\theta\right]$$

$$\text{or} \quad \frac{mv^2}{R + \frac{d}{2}} = 2mg\,(1 - \cos\theta) \quad \text{..... (vi)}$$

From (iv) and (v)

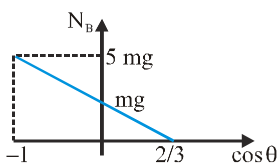
$$N_B + mg\cos\theta = 2mg - 2mg\cos\theta$$

$$N_B = mg(2 - 3\cos\theta) \quad \text{..... (vii)}$$

When $\cos\theta = \frac{2}{3}$, $N_B = 0$

When $\cos\theta = -1$, $N_B = 5$ mg.

Thus the $N_B - \cos\theta$ graph is as shown in the figure.



13. $m_1g - T = m_1a$ .....(i)

$T - m_2g = m_2a$ .....(ii)

$(m_1 = 0.72\,kg\,;\, m_2 = 0.36\,kg)$

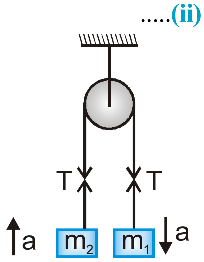
From (i) and (ii) $a = \frac{10}{3}\,m/s^2$

$$d = \frac{1}{2} \times \frac{10}{3} \times 1^2 = \frac{5}{3}\ m$$

$$v = 0 + \frac{10}{3} \times 1 = \frac{10}{3}\ m/s$$

$$W_T = 0.36 \times 10 \times \frac{5}{3} + \frac{1}{2} \times 0.36 \times \frac{100}{9}$$

$$W_T = 8\ J$$



14. By using work energy theorem ($W = \Delta KE$)

$$-\mu mgx - \frac{1}{2}kx^2 = 0 - \frac{1}{2}mV^2$$

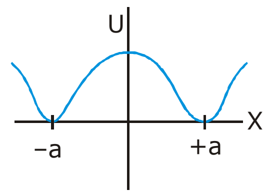
$$\Rightarrow V^2 = \frac{1.44}{9} \Rightarrow V = \frac{1.2}{3} = 0.4 = \frac{4}{10} \quad \Rightarrow N = 4$$

15. 5

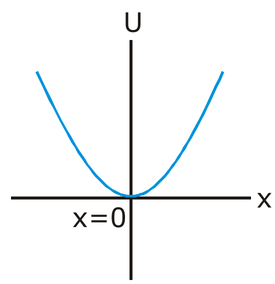
16. **(A)** $U_1 = \frac{U_0}{2}\left[1 - \left(\frac{x}{a}\right)^2\right]^2$

U_{min} at $1 - \left(\frac{x}{a}\right)^2 = 0$

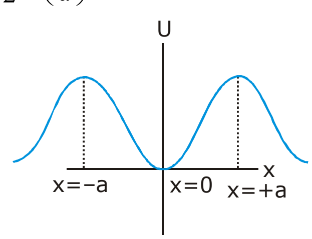
$x = \pm a$, $F = 0$ at $x = \pm a$



(B) $U_2 = \frac{U_0}{2}\left(\frac{X^2}{A^2}\right)$



(C) $U_3 = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 e^{\frac{x^2}{a^2}}$



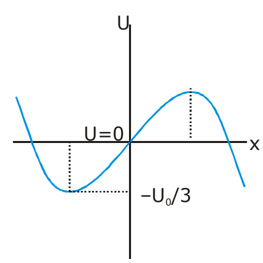
(D) $U_4 = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right] = \frac{U}{3}$

AT $x = -a$

$$U_4 = \frac{4}{3}\frac{U_0}{2} = -\frac{U_0}{3}$$

At $x = a$,

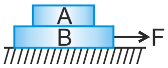
$$U_4 = \frac{2}{3} \times \frac{U_0}{2} = \frac{U_0}{3}$$



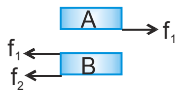
$$\frac{1}{a} - \frac{x^3}{a^3} = 0$$

$$\frac{1}{a} = \frac{x^2}{a^3} = x = \pm a$$

MOCK TEST

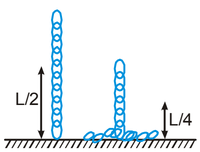
1. 

Consider the blocks shown in the figure to be moving together due to friction between them.
The free body diagrams of both the blocks are shown below.



Work done by static friction on A is positive and on B is negative.

2. The work done by man is negative of magnitude of decrease in potential energy of chain



$$\Delta U = mg\,\frac{L}{2} - \frac{m}{2}\,g\,\frac{L}{4} = 3\,mg\,\frac{L}{8}$$

$\therefore$ $W = -\dfrac{3mg\ell}{8}$

3. From conservation of energy

K.E. + P.E. = E or $K.E. = E - \frac{1}{2}kx^2$

$\therefore$ K.E. at $x = -\sqrt{\dfrac{2E}{k}}$ is $\quad x = -\sqrt{\dfrac{2E}{k}}$

$$E - \frac{1}{2}k\left(\frac{2E}{k}\right) = 0$$

$\therefore$ The speed of particle at $x = -\sqrt{\dfrac{2E}{k}}$ is zero.

4. If A moves down the incline by 1 metre, B shall move up by $\frac{1}{2}$ metre. If the speed of B is v then the speed of A will be 2v.
From conservation of energy:
Gain in K.E. = loss in P.E.

$$\frac{1}{2}m_A(2v)^2 + \frac{1}{2}m_B v^2 = m_A g \times \frac{3}{5} - m_B g \times \frac{1}{2}$$

Solving we get

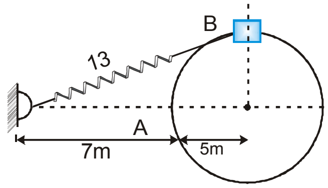
$$v = \frac{1}{2}\sqrt{\frac{g}{3}}$$ **Ans.**

5. Internal forces can not change acceleration of centre of mass. Thus internal forces have no effect on velocity of centre of mass.
The kinetic energy of system of two particles of mass m_1 and m_2 having velocities $\vec{v}_1$ and $\vec{v}_2$, in centre of mass frame is:

$$k = \frac{1}{2}m_1(\vec{v}_1 - \vec{v}_{cm})\cdot(\vec{v}_1 - \vec{v}_{cm}) + \frac{1}{2}m_2(\vec{v}_2 - \vec{v}_{cm})\cdot(\vec{v}_2 - \vec{v}_{cm})$$

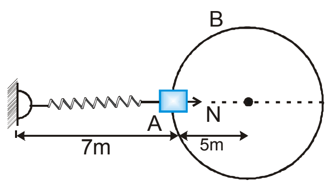
Internal forces change velocities $\vec{v}_1$ and $\vec{v}_2$ and hence kinetic energies of constituent particles of the system. Thus internal forces change kinetic energy of the system in centre of mass frame.
$\therefore$ only (i) is correct.

6. Initial extension will be equal to 6 m.



$\therefore$ Initial energy $= \frac{1}{2}(200)(6)^2 = 3600$ J.

Reaching A : $\frac{1}{2}mv^2 = 3600$ J



$\Rightarrow$ $mv^2 = 7200$ J
From F.B.D. at A :

$$N = \frac{mv^2}{R} = \frac{7200}{5} = 1440\text{ N}$$

7. From given graphs :

$$a_x = \frac{3}{4}t \text{ and } a_y = -\left(\frac{3}{4}t + 1\right) \Rightarrow v_x = \frac{3}{8}t^2 + C$$

At t = 0 : $v_x = -3 \Rightarrow C = -3$

$\therefore v_x = \frac{3}{8}t^2 - 3$

$$\Rightarrow dx = \left(\frac{3}{8}t^2 - 3\right)dt \quad \text{.... (1)}$$

Similarly ; $dy = \left(-\frac{3}{8}t^2 - t + 4\right)dt$ .... (2)

As $dw = \vec{F}.\vec{ds} = \vec{F}.(dx\,\hat{i} + dy\,\hat{j})$

$\therefore \int_0^W dw = \int_0^4 \left[\frac{3}{4}t\hat{i} - \left(\frac{3}{4}t+1\right)\hat{j}\right].\left[\left(\frac{3}{8}t^2-3\right)\hat{i} + \left(-\frac{3}{8}t^2 - t + 4\right)\hat{j}\right]dt$

$\therefore$ **W = 10 J**

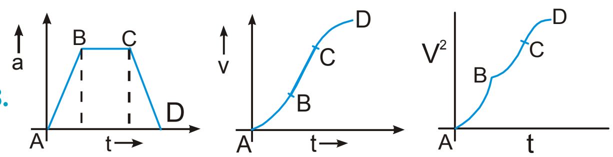
Alternate Solution :

Area of the graph ;

$\int a_x\,dt = 6 = V_{(x)f} - (-3) \Rightarrow V_{(x)f} = 3.$

and $\int a_y\,dt = -10 = V_{(y)f} - (4) \Rightarrow V_{(y)f} = -6.$

Now work done $= \Delta KE = 10$ J

8. 

The above graphs show v – t graph from a – t graph & Then $v^2 - t$ graph, which are self explanatory.

9. $\vec{f} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} = -[6\,\hat{i}] + [8]\,\hat{j} = -6\hat{i} + 8\hat{j}$

$\therefore \vec{a} = -3\hat{i} + 4\hat{j}$ has same direction as that of

$\vec{u} = \frac{-3\hat{i}+4\hat{j}}{2} = \left(\frac{\vec{a}}{2}\right)$

$|\vec{a}| = 5 \Rightarrow |\vec{u}| = 5/2$

Since $\vec{u}$ and $\vec{a}$ are in same direction, particle will move along a straight line

$\therefore S = \frac{5}{2} \times 2 + \frac{1}{2} \times 5 \times 2^2 = 5 + 10 = 15$ m. **15 m. Ans**

10. **Statement I :** Work done by gravity is same for motion from A to J and B to M for equal mass. So K.E. will be equal.

Statement II : Acceleration = g sin θ

$\sin\theta_A > \sin\theta_B$

$\frac{h}{\ell} > \frac{h}{2\ell}$

Statement III :

$W_g + W_{ext} = 0$ (Because moved slowly) $W_{ext} = -W_g$
from B to O : W_g is positive so $W_{ext} < 0$

11. Let at any time the speed of the block along the incline upwards be v.
Then from Newton's second law

$\frac{P}{v} - mg\sin\theta - \mu mg\cos\theta = \frac{mdv}{dt}$

the speed is maximum when $\frac{dv}{dt} = 0$

$\therefore v_{max} = \frac{P}{mg\sin\theta + \mu mg\cos\theta}$

12. $x = x_1$ and $x = x_3$ are not equilibrium positions because $\frac{du}{dx} \neq 0$ at these points.
$x = x_2$ is unstable, as U is maximum at this point.

13. Let v be the speed of B at lowermost position, the speed of A at lowermost position is 2v.
From conservation of energy

$\frac{1}{2}m(2v)^2 + \frac{1}{2}mv^2 = mg(2\ell) + mg\ell.$

Solving we get $v = \sqrt{\frac{6}{5}g\ell}$.

14. At equilibrium position $x = \frac{mg}{k}$

$U_{spring} = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right).x = \frac{mgx}{2}$

$= \frac{1}{2}$(loss in G.P.E.) $\Rightarrow$ G = 2S

15. $dU = -\vec{F}.\vec{ds} = -\vec{F}.(dx\,\hat{i} + dy\,\hat{j})$

Also by reverse method using $F_x = -\frac{\partial U}{\partial X}$ and $F_y = -\frac{\partial U}{\partial Y}$,

only (B) option satisfies the criteria.

16. As long as the block of mass m remains stationary, the block of mass M released from rest comes down by $\frac{2Mg}{K}$ (before coming it rest momently again).
Thus the maximum extension in spring is

$x = \frac{2Mg}{K}$ .... (1)

for block of mass m to just move up the incline
$kx = mg\sin\theta + \mu\,mg\cos\theta$ .... (2)

$2Mg = mg \times \frac{3}{5} + \frac{3}{4}mg \times \frac{4}{5}$ or $M = \frac{3}{5}m$ **Ans.**

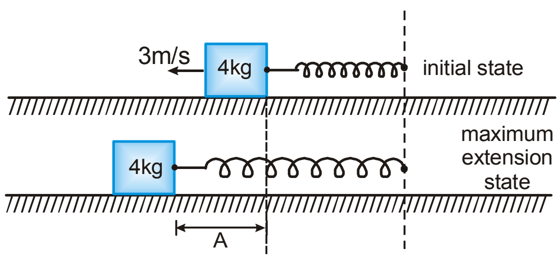
17. $F_x = -\frac{\partial U}{\partial x} = \sin(x+y)$ $F_y = -\frac{\partial U}{\partial y} = \sin(x+y)$

$$F_x = \sin(x+y)\Big]_{(0,\pi/4)} = \frac{1}{\sqrt{2}}$$

$$F_y = \sin(x+y)\Big]_{(0,\pi/4)} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad F = \frac{1}{\sqrt{2}}\left[\hat{i}+\hat{j}\right]$$

18. In the frame (inertial w.r.t earth) of free end of spring, the initial velocity of block is 3 m/s to left and the spring unstretched .



Applying conservation of energy between initial and maximum extension state.

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2 \text{ or } A = \sqrt{\frac{m}{k}}\,v = \sqrt{\frac{4}{10{,}000}} \times 3 = 6\text{cm}.$$

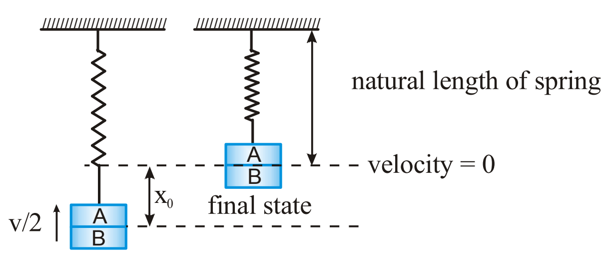
19. The force is constant and hence conservative

$\therefore \quad W_1 = W_2$

20. The initial extension in spring is $x_0 = \frac{mg}{k}$

Just after collision of B with A the speed of combined mass is $\frac{v}{2}$.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$(sec fig.) and comes to rest.



Applying conservation of energy between initial and final states

$$\frac{1}{2}2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get $v = \sqrt{\frac{6mg^2}{k}}$

Alternative solution by SHM

$$\frac{v}{2} = \sqrt{\frac{k}{2m}}\sqrt{\left(\frac{2mg}{k}\right)^2 - \left(\frac{mg}{k}\right)^2}\ ;$$

$$v = \sqrt{\frac{2k}{m}}\sqrt{\frac{3m^2g^2}{k^2}} = \sqrt{\frac{6mg^2}{k}}\ ]$$

21. **(A)** Area under P–x graph $= \int p\,dx = \int\left(m\frac{dv}{dt}\right)v\,dx$

$$= \int_1^v mv^2\,dV = \left[\frac{mv^3}{3}\right]_1^v = \frac{10}{7\times 3}(v^3-1)$$

from graph ; area $= \frac{1}{2}(2+4)\times 10 = 30$

$$\therefore \quad \frac{10}{7\times 3}(v^3-1) = 30$$

$\therefore \quad v = 4$ m/s

Aliter :

from graph

$P = 0.2\,x + 2$

or $mv\frac{dv}{dx}v = 0.2\,x + 2$

or $mv^2\,dv = (0.2\,x+2)\,dx$

Now integrate both sides,

$$\int_1^v mv^2 dv = \int_1^{10}(0.2x+2)dx \Rightarrow v = 4 \text{ m/s}.$$

22. The speed of the water leaving the hose must be $\sqrt{2gh}$ if it is to reach a height h when directed vertically upward. If the diameter is d, the volume of water ejected at this speed is

$$(A\,.v) = \frac{1}{4}\pi d^2 \times \sqrt{2gh}\ \frac{m^3}{s}$$

$\Rightarrow$ Mass ejected is $\frac{1}{4}\pi d^2 \times \sqrt{2gh} \times \rho\ \frac{kg}{s}$.

The kinetic energy of this water leaving the hose

$$= \frac{1}{2}mv^2 = \frac{1}{8}\pi d^2 \times (2gh)^{3/2} \times \rho = 21.5\text{ kW}$$

23. From work energy theorem

for upward motion

$$\frac{1}{2}\,m\,(16)^2 = mgh + W \text{ (work by air resistance)}$$

for downward motion

$$\frac{1}{2}\,m\,(8)^2 = mgh - W$$

$$\frac{1}{2}\,[(16)^2 + (8)^2] = 2\,gh \quad \text{or} \quad h = 8\text{ m}$$

24. When 4 coaches (m each) are attached with engine (2m)

according to question $P = K\,6mgv$ ....(1)

(constant power), (K being proportionality constant)

Since resistive force is proportional to weight

Now if 12 coaches are attached

$P = K.14mg.v_1$ ....(2)

Since engine power is constant

So by equation (1) and (2)

$$6Kmgv = 14Kmgv_1 \quad \Rightarrow v_1 = \frac{6}{14} \times v$$

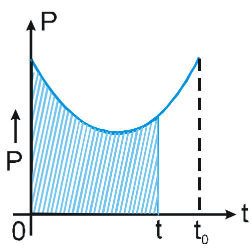
$$= \frac{6}{14} \times 20 = \frac{6 \times 10}{7} = \frac{60}{7} = v_1 = 8.5\text{ m/sec}$$

Similarly for 6 coaches $\Rightarrow$ $K6mgv = K8mgv_2$

$$\Rightarrow v_2 = \frac{6}{8} \times 20 = \frac{3}{4} \times 20 = 15\text{ m/sec}$$

25. The work done by force from time t = 0 to t = t sec. is given by shaded area in graph below.

Hence as t increases, this area increases.



$\therefore$ Work done by force keeps on increasing.

26. Increase in KE = work done

$$\frac{1}{2}\,m\,v_2^{\,2} - \frac{1}{2}\,m\,x\left(\frac{2F_0 x_0}{m}\right) = \frac{1}{2}\,(2F_0 + F_0)\,3x_0$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{11F_0 x_0}{m}}\ .$$

27. $mgl = \frac{1}{2}\,mu^2 \Rightarrow u^2 = 2g$ ....(1)

$$v^2 = u^2 + 2as \Rightarrow 0 = 2g - 2a(3)$$

$$\Rightarrow a = \frac{g}{3} \Rightarrow \because \mu_k\,g = a \;\therefore\; \mu_k\,g = \frac{g}{3} \quad \therefore\; u_K = \frac{1}{3}$$

28. Let m be minimum mass of ball.

Let mass A moves downwards by x.

From conservation of energy,

$$mgx = \frac{1}{2}\,kx^2$$

$$x = \left(\frac{2mg}{k}\right)$$

For mass M to leave contact with ground,

$kx = Mg$

$$K\left(\frac{2mg}{k}\right) = Mg \Rightarrow m = \frac{M}{2}\,.$$

29. $W_{spring} + W_{100\,N} = \Delta k \text{ (on A)}$

$$W_{spring} + (100)\left(\frac{10}{100}\right) = \frac{1}{2}\,(2)\,(2)^2$$

$$W_{spring} = 4 - 10 = -6\text{ J}$$

30. Since ; $W = \int \vec{F}.\vec{dr}$

Clearly for forces (A) and (B) the integration do not require any information of the path taken.

For (C) : $W_c = \int \frac{3(x\hat{i} + y\hat{j})}{(x^2 + y^2)^{3/2}}.(dx\,\hat{i} + dy\,\hat{j})$

$$= 3\int \frac{x\,dx + y\,dy}{(x^2 + y^2)^{3/2}}$$

Taking : $x^2 + y^2 = t$

$2xdx + 2y\,dy = dt$

$$\Rightarrow xdx + ydy = \frac{dt}{2} \quad \Rightarrow W_c = 3\int \frac{dt/2}{t^{3/2}} = \frac{3}{2}\int \frac{dt}{t^{3/2}}$$

which is solvable.

Hence (A), (B) and (C) are conservative forces.

But (D) requires some more information on path. Hence non-conservative.

31. Free body diagram of block is as shown in figure.

From work-energy theorem

$W_{net} = \Delta KE$

or $(40 - 20)s = 40$

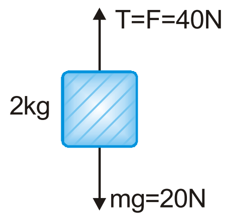
$\therefore$ $s = 2m$

Work done by gravity is

$-20 \times 2 = -40$ J

and work done by tension is

$40 \times 2 = 80$ J



32. If the springs are compressed to same amount :

$W_A = \frac{1}{2} K_A x^2$; $W_B = \frac{1}{2} K_B x^2$

$\because$ $K_A > K_B$ $\Rightarrow$ $W_A > W_B$

If the springs are compressed by same force.

$F = K_A x_A = K_B x_B$; $x_A = \frac{F}{K_A}$; $x_B = \frac{F}{K_B}$; $\frac{W_A}{W_B}$

$$= \frac{\frac{1}{2}K_A.\frac{F^2}{K_A^2}}{\frac{1}{2}K_B\frac{F^2}{K_B^2}} = \frac{K_B}{K_A}$$

Hence, $W_A < W_B$

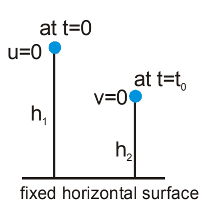
33. **(A)** If velocity and acceleration are not in same direction, work done by force perpendicular to acceleration will not be zero.

(B) If the object is at rest no force can do work.

(C) If force is perpendicular to velocity work done will be zero.

(D) If the point on the body has velocity component in direction of application of force work done will be non-zero.

34. From the figure-1 work done by gravity from t = 0 to $t = t_0$ is $W = mg(h_1 - h_2)$



Since initial and final velocity of ball is zero its average acceleration will be zero.

Since net work done is zero from time interval t = 0 to $t = t_0$. Hence work done by forces except gravity is $mg(h_2 - h_1)$.

35. $U = 3x + 4y$

$$a_y = \frac{F_y}{m} = \frac{-(\partial U / \partial x)}{m} = -3$$

$$a_x = \frac{F_y}{m} = \frac{-(\partial U / \partial y)}{m} = -4 \Rightarrow |\vec{a}| = \mathbf{5\ m/s^2}$$

Let at time 't' particle crosses y-axis

then $-6 = \frac{1}{2}(-3)t^2 \Rightarrow t = 2$ sec.

Along y-direction :

$\Delta y = \frac{1}{2}(-4)(2)^2 = -8$

$\Rightarrow$ particle crosses y-axis at $\mathbf{y = -4}$

At $(6, 4)$: $U = 34$ & $KE = 0$

At $(0, -4)$: $U = -16 \Rightarrow KE = 50$

or $\frac{1}{2}mv^2 = 50 \Rightarrow$ **v = 10 m/s** while crossing y-axis

36. Maximum extension will be at the moment when both masses stop momentarily after going down. Applying W-E theorem from starting to that instant.

$k_f - k_i = W_{gr.} + W_{sp} + W_{ten}$.

$0 - 0 = 2\,M.g.x + \left(-\frac{1}{2}Kx^2\right) + 0$

$x = \frac{4Mg}{K}$

System will have maximum KE when net force on the system becomes zero. Therefore

$2\,Mg = T$ and $T = kx$ $\Rightarrow$ $x = \frac{2Mg}{K}$

Hence KE will be maximum when 2M mass has gone down by $\frac{2Mg}{K}$.

Applying W/E theorem

$k_f - 0 = 2Mg.\frac{2Mg}{K} - \frac{1}{2}K.\frac{4M^2g^2}{K^2}$

$k_f = \frac{2M^2g^2}{K^2}$

Maximum energy of spring $= \frac{1}{2}K.\left(\frac{4Mg}{K}\right)^2 = \frac{8M^2g^2}{K}$

Therefore Maximum spring energy = 4 × maximum K.E.

When K.E. is maximum $x = \frac{2Mg}{K}$.

Spring energy $= \frac{1}{2}.K.\frac{4M^2g^2}{K^2} = \frac{2M^2g^2}{K^2}$

i.e. (D) is wrong.

37. The maximum extension is non-zero, while the spring never undergoes compression.
Hence statement-1 is false.

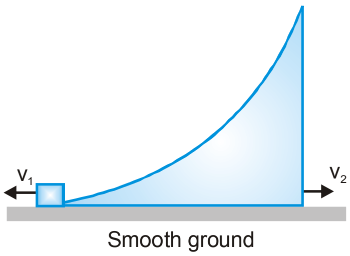
38. When frictional force is opposite to velocity, kinetic energy will decrease.

39. Both the statements are true. The work done by all forces on a system is equal to change in its kinetic energy, irrespective of fact whether work done by internal forces is positive, is zero or is negative.

40. Linear momentum is conserved only in horizontal direction.

41. Net F_{ext} on system is zero in horizontal direction therefore linear momentum is conserved only in horizontal direction.

42.



$mv_1 = mv_2$ ....(i)

$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = mgh$ ....(ii)

From (i) & (ii),
$v_2 = 10ms^{-1}$.

43. Applying W-E theorem on the block for any compression x :

$W_{ext} + W_g + W_{spring} = \Delta KE \Rightarrow Fx + 0 - \frac{1}{2}Kx^2 = \frac{1}{2}mv^2$.

$\Rightarrow$ KE vs x is inverted parabola.

44. $W_{ext} = F \cdot x \Rightarrow$ linear variation

45. From beginning to end of motion : $\Delta KE = 0$
$\Rightarrow x = 2F/K$. (from W-E theorem)
$\therefore$ first half corresponds to $0 \le x \le (F/K)$.

46. **(A) – p ; (B) – p ; (C) – s ; (D) – q**

47. **(Easy)** Point J ⟶ No equilibrium
K ⟶ Unstable equilibrium
L ⟶ Stable equilibrium
M ⟶ Neutral equilibrium

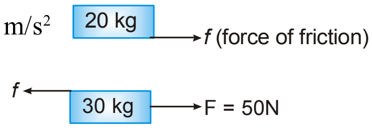
48. Change in velocity $= \frac{\text{area under F} - \text{T graph}}{\text{mass}}$

$= \frac{40 + (-10)}{5} = 6$ m/s

$W_F = \Delta K.E. = \frac{1}{2}(5)\,6^2 = 90$ J

49. Assume 20 kg and 30 kg block move together

$\therefore\; a = \frac{50}{50} = 1$ m/s^2



$\therefore$ frictional force on 20 kg block is
$f = 20 \times 1 = 20$ N
The maximum value of frictional force is

$f_{max} = \frac{1}{2} \times 200 = 100$ N

Hence no slipping is occurring.
$\therefore$ The value of frictional force is f = 20 N.
Distance travelled in t = 2 seconds.

$S = \frac{1}{2} \times 1 \times 4 = 2$m.

Work done by frictional force on upper block is
$W_{fri} = 20 \times 2 = 40$ J
Work done by frictional force on lower block is
$= -20 \times 2 = -40$ J.

50. (40)

51. Work done by force F

$W = \int \vec{F}.dx = (y\hat{i} - x\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) = (ydx\hat{i} - xdy\hat{j})$ .....(1)

$\because x^2 + y^2 = a^2 \qquad \therefore xdx + y\,dy = 0$

$\Rightarrow \; W = \int \left(y\left(\frac{-ydy}{x}\right) - xdy \right) = -\int \frac{(x^2 + y^2)}{x}\,dy$

$= -\int_0^a \frac{a^2}{\sqrt{a^2 - y^2}}\,dy = -\frac{\pi a^2}{2}$

52. For the block of mass m_2, not to move, the maximum compression in the spring x_o should be such that
$kx_o = \mu m_2 g$ .... (1)
Applying work energy theorem to block of mass m_1 we get

$\frac{1}{2}m_1 u^2 = \frac{1}{2}k x_o^2 + \mu m_1 g x_o$ .... (2)

From equation (1) and (2) we get

$\frac{1}{2}m_1 u^2 = \frac{1}{2}\frac{\mu^2 m_2^2 g^2}{K} + \frac{\mu^2 m_1 m_2 g^2}{K}$

putting the appropriate value we get u=10m/s.