Motion in a Straight Line

Motion

If an object changes its position with respect to its surroundings with time, then it is called in motion.

Rest

If an object does not change its position with respect to its surroundings with time, then it is called at rest.

Rest and motion are relative states. It means an object which is at rest in one frame of reference can be in motion in another frame of reference at the same time.

Point Object An object can be considered as a point object, if the distance travelled by it is very large in comparison to its dimensions.

Types of Motion

1. One Dimensional Motion

If only one out of three coordinates specifying the position of the object changes with respect to time, then the motion is called one dimensional motion or rectilinear motion.

For instance, motion of a block in a straight line, motion of a train along a straight track, a man walking on a level and narrow road and object falling under gravity etc.

2. Two Dimensional Motion

If only two out of three coordinates specifying the position of the object change with respect to time, then the motion is called two dimensional motion.

A circular motion is an instance of two dimensional motion.

3. **Three Dimensional Motion**

If all the three coordinates specifying the position of the object change with respect to time, then the motion is called three dimensional motion. A few instances of three dimension at motion are flying bird, a flying kite, the random motion of gas molecule etc.

Frame of Reference

The most convenient system is a rectangular coordinate system of three mutually perpendicular axes as X, Y, and Z. The point of intersection of these three axes is called origin (O) and considered as the reference point. The x, y, z-coordinates describe the position of the object w.r.t the coordinate system. This coordinate system along with a clock constitutes a frame of reference.

Distance or Path Length Covered

The length of the actual path covered by an object is called the distance.

It is a scalar quantity and it can never be zero or negative during the motion of an object. Its SI unit is metre.

Displacement

The shortest distance between the initial and final positions of any object during motion is called displacement. The displacement of an object in a given time can be positive, zero or negative.

$$\text{Displacement, } \Delta x = x_2 - x_1$$

where, x_1 and x_2 are the initial and final positions of object, respectively.

It is a vector quantity. Its SI unit is metre.

Speed

The time rate of change of position of the object in any direction is called speed of the object.

$$\text{Speed } (v) = \frac{\text{Distance travelled } (s)}{\text{Time taken } (t)}$$

Its SI unit is m/s.
It is a scalar quantity.
Its dimensional formula is $[M^0LT^{-1}]$.

Uniform Speed

If an object covers equal distances in equal intervals of time, then its speed is called uniform speed.

Non-uniform or Variable Speed

If an object covers unequal distances in equal intervals of time and *vice-versa* then its speed is called non-uniform or variable speed.

Average Speed

The ratio of the total distance travelled by the object to the total time taken is called average speed of the object.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

If a particle travels distances $s_1, s_2, s_3, \ldots$ with speeds $v_1, v_2, v_3, \ldots$, then

$$\text{Average speed} = \frac{s_1 + s_2 + s_3 + \ldots}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \ldots\right)}$$

If particle travels equal distances $(s_1 = s_2 = s)$ with velocities v_1 and v_2, then

$$\text{Average speed} = \frac{2v_1v_2}{(v_1 + v_2)}$$

If a particle travels with speeds $v_1, v_2, v_3, \ldots$ during time intervals $t_1, t_2, t_3, \ldots$, then

$$\text{Average speed} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

If particle travels with speeds v_1 and v_2 for equal time intervals, i.e. $t_1 = t_2 = t$, then

$$\text{Average speed} = \frac{v_1 + v_2}{2}$$

When a body travels equal distance with speeds v_1 and v_2, the average speed (v) is the harmonic mean of two speeds, *i.e.*

$$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$$

Instantaneous Speed

When an object is travelling with variable speed, then its speed at a given instant of time is called its instantaneous speed.

$$\text{Instantaneous speed} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Velocity

The time rate of change of displacement of an object in a particular direction is called its velocity.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

Its SI unit is m/s.

Its dimensional formula is $[M^0LT^{-1}]$.

It is a vector quantity, as it has both, the magnitude and direction.

The velocity of an object can be positive, zero or negative.

Uniform Velocity

If an object undergoes equal displacements in equal intervals of time, then it is said to be moving with a uniform velocity.

Non-uniform or Variable Velocity

If an object undergoes unequal displacements in equal intervals of time, then it is said to be moving with a non-uniform or variable velocity.

Average Velocity

The ratio of the total displacement to the total time taken is called average velocity.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

Instantaneous Velocity

The velocity of a particle at any instant of time is known as instantaneous velocity.

$$\text{Instantaneous velocity} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Relative Velocity

Relative velocity of one object with respect to another object is the time rate of change of relative position of one object with respect to another object.

Relative velocity of object A with respect to object B

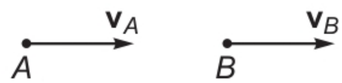
$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

If it is in one dimensional motion, we can treat vectors as scalars just by assigning the positive sign to one direction and negative to others.

When two objects are moving in the same direction, then

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

or $$v_{AB} = v_A - v_B$$



When two objects are moving in opposite direction, then

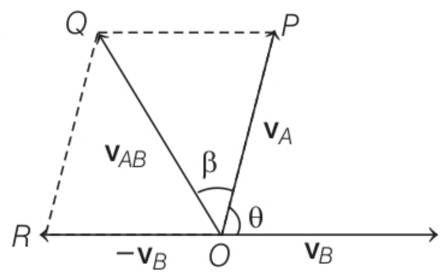
$$\mathbf{v}_{AB} = \mathbf{v}_A + \mathbf{v}_B$$

or $$v_{AB} = v_A + v_B$$

When two objects are moving at an angle θ, then

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

and $$\tan\beta = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$



Acceleration

The rate of change of velocity with time is called acceleration.

$$\text{Acceleration } (a) = \frac{\text{Change in velocity } (\Delta v)}{\text{Time interval } (\Delta t)}$$

Its SI unit is m/s^2.

Its dimensional formula is $[M^0 LT^{-2}]$.

It is a vector quantity.

Acceleration can be positive, zero or negative. **Positive acceleration** means velocity increasing with time, **zero acceleration** means velocity is uniform while **negative acceleration** (retardation) means velocity is decreasing with time.

Uniform Acceleration

If an object is moving with uniform acceleration, it means that the change in velocity is equal for equal interval of time.

Non-uniform Acceleration

If an object is moving with non-uniform acceleration, it means that the change in velocity is unequal for equal interval of time.

Average Acceleration

If a particle is accelerated for a time t_1 with acceleration a_1 and for a time t_2 with acceleration a_2, then average acceleration

$$a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$$

Instantaneous Acceleration

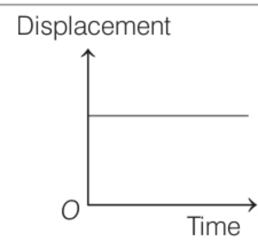
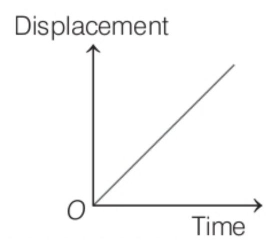
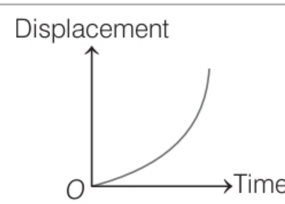
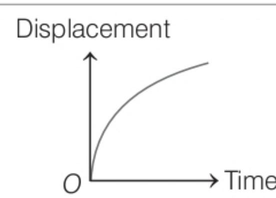
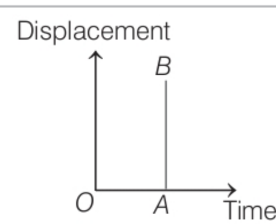
It is defined as the acceleration of object at any instant of time.

$$a_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Uniform Motion

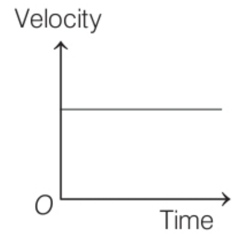
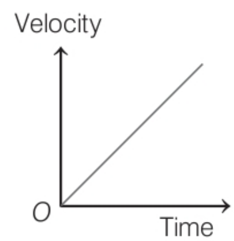
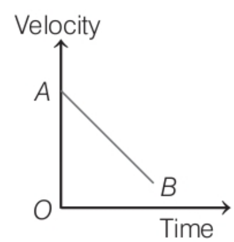
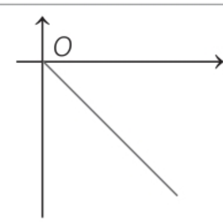
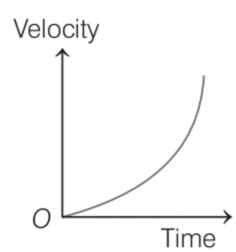
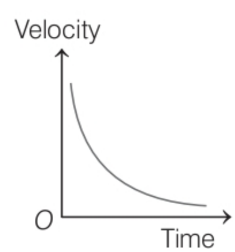
If an object is moving along the straight line covers equal distance in equal interval of time, it is said to be in uniform motion along a straight line.

Different Graphs of Motion Displacement-Time Graph

S. No.	Condition	Graph
(a)	For a stationary body	
(b)	Body moving with a constant velocity	
(c)	Body moving with a constant acceleration	
(d)	Body moving with a constant retardation	
(e)	Body moving with infinite velocity. But such motion of a body is never possible.	

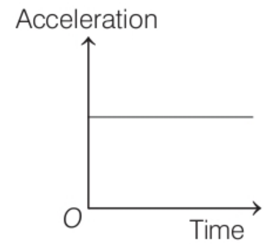
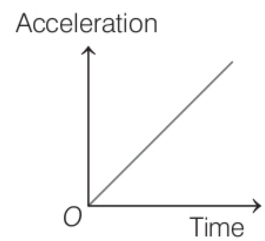
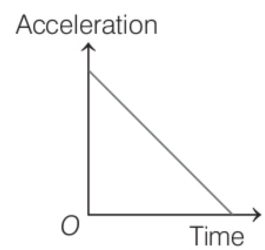
Note Slope of displacement-time graph gives average velocity.

Velocity-Time Graph

S. No.	Condition	Graph
(a)	Moving with a constant velocity	
(b)	Moving with a constant acceleration having zero initial velocity	
(c)	Body moving with a constant retardation and its initial velocity is not zero	
(d)	Moving with a constant retardation with zero initial velocity	
(e)	Moving with increasing acceleration	
(f)	Moving with decreasing acceleration	

Note Slope of velocity-time graph gives average acceleration.

Acceleration-Time Graph

S. No.	Condition	Graph
(a)	When object is moving with constant acceleration	
(b)	When object is moving with constant increasing acceleration	
(c)	When object is moving with constant decreasing acceleration	

Equations of Uniformly Accelerated Motion

If a body starts with velocity (u) and after time t its velocity changes to v, if the uniform acceleration is a and the distance travelled in time t is s, then the following relations are obtained, which are called equations of uniformly accelerated motion.

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

(iv) Distance travelled in nth second.

$$s_n = u + \frac{a}{2}(2n - 1)$$

If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then the velocity at the mid-point of its path

$$= \frac{\sqrt{u^2 + v^2}}{2}$$

Non-Uniformly Accelerated Motion

When acceleration of particle is not constant then motion is called non-uniformlly accelerated motion.

For one dimensional motion,

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = v \cdot \frac{dv}{ds} = \frac{d^2 s}{dt^2}$$

where, Δs is displacement in time Δt, Δv is velocity in time Δt and a is instantaneous acceleration.

In component form,

$$\mathbf{a} = a_x \cdot \hat{\mathbf{i}} + a_y \cdot \hat{\mathbf{j}} + a_z \cdot \hat{\mathbf{k}}$$

where, $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$ and $a_z = \frac{dv_z}{dt}$

Motion Under Gravity

If an object is falling freely $(u = 0)$ under gravity, then equations of motion becomes

(i) $v = u + gt$ (ii) $h = ut + \frac{1}{2} gt^2$ (iii) $v^2 = u^2 + 2gh$

Note If an object is thrown upward then g is replaced by $-g$ in above three equations.

It thus follows that:

(i) Time taken to reach maximum height, $t_A = \frac{u}{g} = \sqrt{\frac{2h}{g}}$

(ii) Maximum height reached by the body, $h_{max} = \frac{u^2}{2g}$

(iii) A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two ball are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively, then

$$t = \sqrt{t_1 t_2}$$

(iv) When a body is dropped freely from the top of the tower and another body is projected horizontally from the same point, both will reach the ground at the same time.