SOLVED EXAMPLES

Ex. 1 Find the centre and the radius of the circles

(A) $3x^2 + 3y^2 - 8x - 10y + 3 = 0$ **(B)** $x^2 + y^2 + 2x\sin\theta + 2y\cos\theta - 8 = 0$

(C) $2x^2 + \lambda xy + 2y^2 + (\lambda - 4)x + 6y - 5 = 0$, for some λ.

Sol.

(A) We rewrite the given equation as

$$x^2 + y^2 - \frac{8}{3}x - \frac{10}{3}y + 1 = 0$$

$$\Rightarrow \quad g = -\frac{4}{3},\ f = -\frac{5}{3},\ c = 1$$

Hence the centre is $\left(\frac{4}{3}, \frac{5}{3}\right)$ and the radius is $\sqrt{\frac{16}{9} + \frac{25}{9} - 1} = \sqrt{\frac{32}{9}} = \frac{4\sqrt{2}}{3}$ units

(B) $x^2 + y^2 + 2x\sin\theta + 2y\cos\theta - 8 = 0.$

Centre of this circle is $(-\sin\theta, -\cos\theta)$

Radius $= \sqrt{\sin^2\theta + \cos^2\theta + 8} = \sqrt{1 + 8} = 3$ units

(C) $2x^2 + \lambda xy + 2y^2 + (\lambda - 4)x + 6y - 5 = 0$

We rewrite the equation as

$$x^2 + \frac{\lambda}{2}xy + y^2 + \left(\frac{\lambda - 4}{2}\right)x + 3y - \frac{5}{2} = 0 \qquad \text{...... (i)}$$

Since, there is no term of xy in the equation of circle

$$\Rightarrow \quad \frac{\lambda}{2} = 0 \quad \Rightarrow \quad \lambda = 0$$

So, equation **(i)** reduces to $x^2 + y^2 - 2x + 3y - \frac{5}{2} = 0$

$\therefore$ centre is $\left(1, -\frac{3}{2}\right)$

Radius $= \sqrt{1 + \frac{9}{4} + \frac{5}{2}} = \frac{\sqrt{23}}{2}$ units.

Ex. 2 Find the parametric equations of the circle $x^2 + y^2 - 4x - 2y + 1 = 0$

Sol. We have : $x^2 + y^2 - 4x - 2y + 1 = 0$

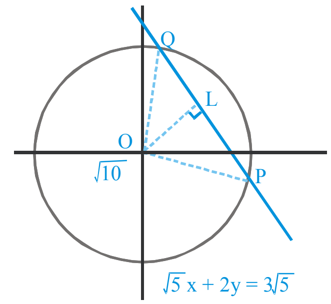
$\Rightarrow \quad (x^2 - 4x) + (y^2 - 2y) = -1 \qquad \Rightarrow \quad (x - 2)^2 + (y - 1)^2 = 2^2$

So, the parametric equations of this circle are

$x = 2 + 2\cos\theta$, $y = 1 + 2\sin\theta$.

Ex. 3 Find the area of the triangle formed by line joining the origin to the points of intersection(s) of the line $x\sqrt{5} + 2y = 3\sqrt{5}$ and circle $x^2 + y^2 = 10$.

Sol. Length of perpendicular from origin to the line $x\sqrt{5}+2y=3\sqrt{5}$ is



$$OL = \frac{3\sqrt{5}}{\sqrt{(\sqrt{5})^2+2^2}} = \frac{3\sqrt{5}}{\sqrt{9}} = \sqrt{5}$$

Radius of the given circle = $\sqrt{10}$ = OQ = OP

$PQ = 2QL = 2\sqrt{OQ^2 - OL^2} = 2\sqrt{10-5} = 2\sqrt{5}$

Thus area of $\Delta OPQ = \frac{1}{2} \times PQ \times OL = \frac{1}{2} \times 2\sqrt{5} \times \sqrt{5} = 5$

Ex. 4 Find the equations of the following curves in Cartesian form. Also, find the centre and radius of the circle $x = a + c\cos\theta,\ y = b + c\sin\theta$

Sol. We have : $x = a + c\cos\theta,\ y = b + c\sin\theta$ $\Rightarrow$ $\cos\theta = \frac{x-a}{c},\ \sin\theta = \frac{y-b}{c}$

$\Rightarrow$ $\left(\frac{x-a}{c}\right)^2 + \left(\frac{y-b}{c}\right)^2 = \cos^2\theta + \sin^2\theta$ $\Rightarrow$ $(x-a)^2 + (y-b)^2 = c^2$

Clearly, it is a circle with centre at (a, b) and radius c.

Ex. 5 If the straight line ax + by = 2; a, b $\neq$ 0 touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$, then find the values of a and b are respectively.

Sol. Given $x^2 + y^2 - 2x = 3$

$\therefore$ centre is (1, 0) and radius is 2

Given $x^2 + y^2 - 4y = 6$

$\therefore$ centre is (0, 2) and radius is $\sqrt{10}$. Since line ax + by = 2 touches the first circle

$\therefore$ $\frac{|a(1)+b(0)-2|}{\sqrt{a^2+b^2}} = 2$ or $|(a-2)| = [2\sqrt{a^2+b^2}]$ .... **(i)**

Also the given line is normal to the second circle. Hence it will pass through the centre of the second circle.

$\therefore$ $a(0) + b(2) = 2$ or $2b = 2$ or $b = 1$

Putting this value in equation **(i)** we get $|a-2| = 2\sqrt{a^2+1^2}$ or $(a-2)^2 = 4(a^2+1)$

or $a^2 + 4 - 4a = 4a^2 + 4$ or $3a^2 + 4a = 0$ or $a(3a+4) = 0$ or $a = 0, -\frac{4}{3}$ $(a \neq 0)$

$\therefore$ values of a and b are $\left(-\frac{4}{3}, 1\right)$.

Ex. 6 Find the equation of a circle having the lines $x^2 + 2xy + 3x + 6y = 0$ as its normal and having size just sufficient to contain the circle $x(x-4) + y(y-3) = 0$.

Sol. Pair of normals are $(x + 2y)(x + 3) = 0$

$\therefore$ Normals are $x + 2y = 0,\ x + 3 = 0$.

Point of intersection of normals is the centre of required circle i.e. $C_1(-3, 3/2)$ and centre of given circle is $C_2(2, 3/2)$

and radius $r_2 = \sqrt{4 + \frac{9}{4}} = \frac{5}{2}$

Let r_1 be the radius of required circle

$\Rightarrow \quad r_1 = C_1C_2 + r_2 = \sqrt{(-3-2)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} + \frac{5}{2} = \frac{15}{2}$

Hence equation of required circle is $x^2 + y^2 + 6x - 3y - 45 = 0$

Ex. 7 Let a circle be given by $2x(x - a) + y(2y - b) = 0 \qquad (a \neq 0, b \neq 0)$

Find the condition on a and b if two chords, each bisected by the x-axis, can be drawn to the circle from (a, b/2).

Sol. The given circle is $2x(x - a) + y(2y - b) = 0$

or $x^2 + y^2 - ax - by/2 = 0$

Let AB be the chord which is bisected by x-axis at a point M. Let its co-ordinates be M(h, 0).

and $S \equiv x^2 + y^2 - ax - by/2 = 0$

$\therefore$ Equation of chord AB is $T = S_1$

$$hx + 0 - \frac{a}{2}(x + h) - \frac{b}{4}(y + 0) = h^2 + 0 - ah - 0$$

Since its passes through (a, b/2) we have $ah - \frac{a}{2}(a + h) - \frac{b^2}{8} = h^2 - ah$

$$\Rightarrow \quad h^2 - \frac{3ah}{2} + \frac{a^2}{2} + \frac{b^2}{8} = 0$$

Now there are two chords bisected by the x-axis, so there must be two distinct real roots of h.

$\therefore \quad B^2 - 4AC > 0$

$$\Rightarrow \quad \left(\frac{-3a}{2}\right)^2 - 4.1.\left(\frac{a^2}{2} + \frac{b^2}{8}\right) > 0 \quad \Rightarrow \quad a^2 > 2b^2.$$

Ex. 8 Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other, if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

Sol. Given circles are $x^2 + y^2 + 2ax + c^2 = 0 \qquad$ ...... **(i)**

and $x^2 + y^2 + 2by + c^2 = 0 \qquad$ ...... **(ii)**

Let C_1 and C_2 be the centres of circles **(i)** and **(ii)**, respectively and r_1 and r_2 be their radii, then

$C_1 = (-a, 0), C_2 = (0, -b),\; r_1 = \sqrt{a^2 - c^2},\; r_2 = \sqrt{b^2 - c^2}$

Here we find the two circles touch each other internally or externally.

For touch, $|C_1C_2| = |r_1 \pm r_2|$

or $\quad \sqrt{(a^2 + b^2)} = \left|\sqrt{(a^2 - c^2)} \pm \sqrt{(b^2 - c^2)}\right|$

On squaring $a^2 + b^2 = a^2 - c^2 + b^2 - c^2 \pm 2\sqrt{(a^2 - c^2)}\sqrt{(b^2 - c^2)}$

or $c^2 = \pm \sqrt{a^2b^2 - c^2(a^2+b^2) + c^4}$

Again squaring, $c^4 = a^2b^2 - c^2(a^2 + b^2) + c^4$

or $c^2(a^2 + b^2) = a^2b^2$

or $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

Ex. 9 Find the equation of the circle through the points of intersection of $x^2 + y^2 - 1 = 0$, $x^2 + y^2 - 2x - 4y + 1 = 0$ and touching the line $x + 2y = 0$.

Sol. Family of circles is $x^2 + y^2 - 2x - 4y + 1 + \lambda(x^2 + y^2 - 1) = 0$

$(1 + \lambda)x^2 + (1 + \lambda)y^2 - 2x - 4y + (1 - \lambda) = 0$

$$x^2 + y^2 - \frac{2}{1+\lambda}x - \frac{4}{1+\lambda}y + \frac{1-\lambda}{1+\lambda} = 0$$

Centre is $\left(\frac{1}{1+\lambda}, \frac{2}{1+\lambda}\right)$ and radius $= \sqrt{\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2 - \frac{1-\lambda}{1+\lambda}} = \frac{\sqrt{4+\lambda^2}}{|1+\lambda|}$.

Since it touches the line $x + 2y = 0$, Hence Radius = Perpendicular distance from centre to the line.

i.e., $\left|\frac{\frac{1}{1+\lambda} + 2\frac{2}{1+\lambda}}{\sqrt{1^2+2^2}}\right| = \frac{\sqrt{4+\lambda^2}}{|1+\lambda|}$ $\Rightarrow$ $\sqrt{5} = \sqrt{4+\lambda^2}$ $\Rightarrow$ $\lambda = \pm 1$

$\lambda = -1$ cannot be possible in case of circle. So $\lambda = 1$.

Thus, we get the equation of circle.

Ex. 10 Find the pole of the line $3x + 5y + 17 = 0$ with respect to the circle $x^2 + y^2 + 4x + 6y + 9 = 0$

Sol. Given circle is $x^2 + y^2 + 4x + 6y + 9 = 0$

and given line is $3x + 5y + 17 = 0$

Let $P(\alpha, \beta)$ be the pole of line **(ii)** with respect to circle **(i)**

Now equation of polar of point $P(\alpha, \beta)$ with respect to circle **(i)** is

$x\alpha + y\beta + 2(x + \alpha) + 3(y + \beta) + 9 = 0$

or $(\alpha + 2)x + (\beta + 3)y + 2\alpha + 3\beta + 9 = 0$

Now lines **(ii)** and **(iii)** are same, therefore,

$$\underset{\textbf{(i)}}{\frac{\alpha+2}{3}} = \underset{\textbf{(ii)}}{\frac{\beta+3}{5}} = \underset{\textbf{(iii)}}{\frac{2\alpha+3\beta+9}{17}}$$

From **(i)** and **(ii)**, we get $5\alpha + 10 = 3\beta + 9$ or $5\alpha - 3\beta = -1$ ...... **(iv)**

From **(i)** and **(iii)**, we get $17\alpha + 34 = 6\alpha + 9\beta + 27$ or $11\alpha - 9\beta = -7$ ...... **(v)**

Solving **(iv)** & **(v)**, we get $\alpha = 1, \beta = 2$.

Hence required pole is (1, 2).

Ex. 11 Find the equation of a circle which passes through the point (2, 0) and whose centre is the limit of the point of intersection of the lines $3x + 5y = 1$ and $(2 + c)x + 5c^2y = 1$ as $c \to 1$.

Sol. Solving the equations $(2 + c)x + 5c^2y = 1$ and $3x + 5y = 1$

then $(2+c)x + 5c^2\left(\frac{1-3x}{5}\right) = 1$ or $(2+c)x + c^2(1-3x) = 1$

$\therefore$ $x = \frac{1-c^2}{2+c-3c^2}$ or $x = \frac{(1+c)(1-c)}{(3c+2)(1-c)} = \frac{1+c}{3c+2}$

$\therefore$ $x = \lim_{c\to 1}\frac{1+c}{3c+2}$ or $x = \frac{2}{5}$

$\therefore$ $y = \frac{1-3x}{5} = \frac{1-\frac{6}{5}}{5} = -\frac{1}{25}$

Therefore the centre of the required circle is $\left(\frac{2}{5}, \frac{-1}{25}\right)$ but circle passes through (2, 0)

$\therefore$ Radius of the required circle $= \sqrt{\left(\frac{2}{5}-2\right)^2 + \left(-\frac{1}{25}-0\right)^2} = \sqrt{\frac{64}{25}+\frac{1}{625}} = \sqrt{\frac{1601}{625}}$

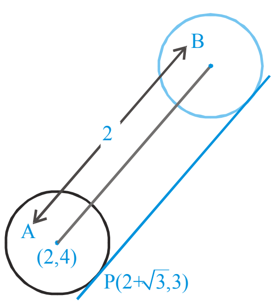
Hence the required equation of the circle is $\left(x-\frac{2}{5}\right)^2 + \left(y+\frac{1}{25}\right)^2 = \frac{1601}{625}$

or $25x^2 + 25y^2 - 20x + 2y - 60 = 0$

Ex. 12 The circle $x^2 + y^2 - 4x - 8y + 16 = 0$ rolls up the tangent to it at $(2 + \sqrt{3}, 3)$ by 2 units, find the equation of the circle in the new position.

Sol. Given circle is $x^2 + y^2 - 4x - 8y + 16 = 0$

let $P \equiv (2 + \sqrt{3}, 3)$



Equation of tangent to the circle at $P(2 + \sqrt{3}, 3)$ will be

$(2 + \sqrt{3})x + 3y - 2(x + 2 + \sqrt{3}) - 4(y + 3) + 16 = 0$

or $\sqrt{3}x - y - 2\sqrt{3} = 0$

slope $= \sqrt{3}$ $\Rightarrow$ $\tan\theta = \sqrt{3}$

$\theta = 60°$

line AB is parallel to the tangent at P

$\Rightarrow$ coordinates of point B $= (2 + 2\cos 60°, 4 + 2\sin 60°)$

thus B $= (3, 4 + \sqrt{3})$

radius of circle $= \sqrt{2^2 + 4^2 - 16} = 2$

$\therefore$ equation of required circle is $(x - 3)^2 + (y - 4 - \sqrt{3})^2 = 2^2$

Ex. 13 Find the equation of the circle through the points of intersection of the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$ and cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

Sol. The equation of the circle through the intersection of the given circles is

$$x^2 + y^2 - 4x - 6y - 12 + \lambda(-10x - 10y) = 0 \quad \text{...... (i)}$$

where $(-10x - 10y = 0)$ is the equation of radical axis for the circle

$$x^2 + y^2 - 4x - 6y - 12 = 0 \quad \text{and} \quad x^2 + y^2 + 6x + 4y - 12 = 0.$$

Equation **(i)** can be re-arranged as

$$x^2 + y^2 - x(10\lambda + 4) - y(10\lambda + 6) - 12 = 0$$

It cuts the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

Hence $2gg_1 + 2ff_1 = c + c_1$

$\Rightarrow \quad 2(5\lambda + 2)(1) + 2(5\lambda + 3)(0) = -12 - 4 \Rightarrow \lambda = -2$

Hence the required circle is $x^2 + y^2 - 4x - 6y - 12 - 2(-10x - 10y) = 0$

$\Rightarrow \quad x^2 + y^2 + 16x + 14y - 12 = 0$

Ex. 14 Find the equation of the image of the circle $x^2 + y^2 + 16x - 24y + 183 = 0$ by the line mirror $4x + 7y + 13 = 0$.

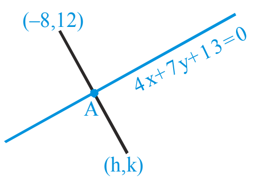
Sol. Centre of given circle = (–8, 12), radius = 5

the given line is $4x + 7y + 13 = 0$

let the centre of required circle is (h, k)

since radius will not change. so radius of required circle is 5.

Now (h, k) is the reflection of centre (–8, 12) in the line $4x + 7y + 13 = 0$



Co-ordinates of $A = \left(\dfrac{-8+h}{2}, \dfrac{12+k}{2}\right)$

$\Rightarrow \quad \dfrac{4(-8+h)}{2} + \dfrac{7(12+k)}{2} + 13 = 0$

$-32 + 4h + 84 + 7k + 26 = 0$

$4h + 7k + 78 = 0 \quad \text{.........(i)}$

Also $\quad \dfrac{k-12}{h+8} = \dfrac{7}{4}$

$4k - 48 = 7h + 56$

$4k = 7h + 104 \quad \text{.........(ii)}$

solving **(i)** & **(ii)**

$h = -16, k = -2$

$\therefore \quad$ required circle is $(x + 16)^2 + (y + 2)^2 = 5^2$

$x^2 + y^2 - 4x - 6y - 12 - 2(-10x - 10y) = 0$

i.e., $x^2 + y^2 + 16x + 14y - 12 = 0$

Ex. 15 Two straight lines rotate about two fixed points. If they start from their position of coincidence such that one rotates at the rate double that of the other. Prove that the locus of their point of intersection is a circle.

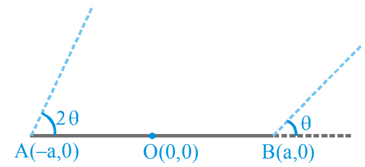
Sol. Let $A \equiv (-a, 0)$ and $B \equiv (a, 0)$ be two fixed points.

Let one line which rotates about B an angle θ with the x-axis at any time t and at that time the second line which rotates about A make an angle 2θ with x-axis.

Now equation of line through B and A are respectively

$y - 0 = \tan\theta(x - a) \quad \text{...... (i)}$

and $\quad y - 0 = \tan 2\theta(x + a) \quad \text{...... (ii)}$



From **(ii)**, $y = \dfrac{2\tan\theta}{1-\tan^2\theta}(x+a)$

$$= \left\{\frac{\frac{2y}{(x-a)}}{1-\frac{y^2}{(x-a)^2}}\right\}(x+a) \qquad \text{(from (i))}$$

$\Rightarrow \quad y = \dfrac{2y(x-a)(x+a)}{(x-a)^2-y^2} \qquad \Rightarrow \qquad (x-a)^2 - y^2 = 2(x^2-a^2)$

or $x^2 + y^2 + 2ax - 3a^2 = 0$ which is the required locus.

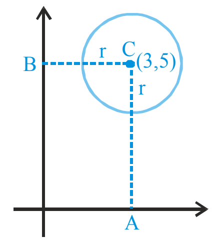
Ex. 16 The circle $x^2 + y^2 - 6x - 10y + k = 0$ does not touch or intersect the coordinate axes and the point (1, 4) is inside the circle. Find the range of the value of k.

Sol. Since (1, 4) lies inside the circle

$\Rightarrow \quad S_1 < 0$

$\Rightarrow \quad (1)^2 + (4)^2 - 6(1) - 10(4) + k < 0 \qquad \Rightarrow \qquad k < 29$

Also centre of given circle is (3, 5) and circle does not touch or intersect the coordinate axes



$\Rightarrow \quad r < CA \quad \& \quad r < CB$

$CA = 5$

$CB = 3$

$\Rightarrow \quad r < 5 \quad \& \quad r < 3$

$\Rightarrow \quad r < 3 \quad \text{or } r^2 < 9$

$r^2 = 9 + 25 - k$

$r^2 = 34 - k \qquad \Rightarrow \qquad 34 - k < 9$

$k > 25$

$\Rightarrow \quad k \in (25, 29)$

Ex. 17 Find the radical centre of circles $x^2 + y^2 + 3x + 2y + 1 = 0$, $x^2 + y^2 - x + 6y + 5 = 0$ and $x^2 + y^2 + 5x - 8y + 15 = 0$. Also find the equation of the circle cutting them orthogonally.

Sol. Given circles are $S_1 \equiv x^2 + y^2 + 3x + 2y + 1 = 0$

$S_2 \equiv x^2 + y^2 - x + 6y + 5 = 0$

$S_3 \equiv x^2 + y^2 + 5x - 8y + 15 = 0$

Equations of two radical axes are $S_1 - S_2 \equiv 4x - 4y - 4 = 0$

or $\quad x - y - 1 = 0$

and $\quad S_2 - S_3 \equiv -6x + 14y - 10 = 0$

or $\quad 3x - 7y + 5 = 0$

Solving them the radical centre is (3, 2). Also, if r is the length of the tangent drawn from the radical centre (3, 2) to any one of the given circles, say S_1, we have

$r = \sqrt{S_1} = \sqrt{3^2 + 2^2 + 3.3 + 2.2 + 1} = \sqrt{27}$

Hence (3, 2) is the centre and $\sqrt{27}$ is the radius of the circle intersecting them orthogonally.

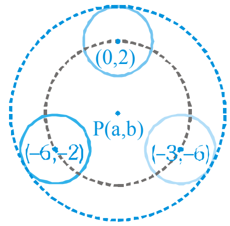
$\therefore \quad$ Its equation is $(x-3)^2 + (y-2)^2 = r^2 = 27 \qquad \Rightarrow \qquad x^2 + y^2 - 6x - 4y - 14 = 0$

Ex. 18 Find the equation of the circle of minimum radius which contains the three circles.

$S_1 \equiv x^2 + y^2 - 4y - 5 = 0$

$S_2 \equiv x^2 + y^2 + 12x + 4y + 31 = 0$

$S_3 \equiv x^2 + y^2 + 6x + 12y + 36 = 0$



Sol. For S_1, centre = (0, 2) and radius = 3

For S_2, centre = (–6, –2) and radius = 3

For S_3, centre = (–3, –6) and radius = 3

let P(a, b) be the centre of the circle passing through the centres of the three given circles, then

$(a - 0)^2 + (b - 2)^2 = (a + 6)^2 + (b + 2)^2$

$\Rightarrow$ $(a + 6)^2 - a^2 = (b - 2)^2 - (b + 2)^2$

$(2a + 6)6 = 2b(-4)$

$$b = \frac{2 \times 6(a+3)}{-8} = -\frac{3}{2}(a+3)$$

again $(a - 0)^2 + (b - 2)^2 = (a + 3)^2 + (b + 6)^2$

$\Rightarrow$ $(a + 3)^2 - a^2 = (b - 2)^2 - (b + 6)^2$

$(2a + 3)3 = (2b + 4)(-8)$

$$(2a+3)3 = -16\left[-\frac{3}{2}(a+3) + 2\right]$$

$6a + 9 = -8(-3a - 5)$

$6a + 9 = 24a + 40$

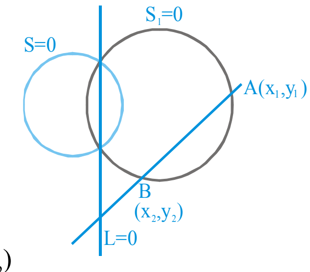
$18a = -31$

$$a = -\frac{31}{18},\ b = -\frac{23}{12}$$

radius of the required circle $= 3 + \sqrt{\left(-\frac{31}{18}\right)^2 + \left(-\frac{23}{12} - 2\right)^2} = 3 + \frac{5}{36}\sqrt{949}$

$\therefore$ equation of the required circle is $\left(x + \frac{31}{18}\right)^2 + \left(y + \frac{23}{12}\right)^2 = \left(3 + \frac{5}{36}\sqrt{949}\right)^2$

Ex. 19 A fixed circle is cut by a family of circles all of which, pass through two given points $A(x_1, y_1)$ and $B(x_2, y_2)$. Prove that the chord of intersection of the fixed circle with any circle of the family passes through a fixed point.



Sol. Let S = 0 be the equation of fixed circle

let $S_1 = 0$ be the equation of any circle through A and B

which intersect S = 0 in two points.

$L \equiv S - S_1 = 0$ is the equation of the chord of intersection of S = 0 and $S_1 = 0$

let $L_1 = 0$ be the equation of line AB

let S_2 be the equation of the circle whose diametrical ends are $A(x_1, y_1)$ & $B(x_2, y_2)$

then $S_1 \equiv S_2 - \lambda L_1 = 0$

$\Rightarrow$ $L \equiv S - (S_2 - \lambda L_1) = 0$ or $L \equiv (S - S_2) + \lambda L_1 = 0$

or $L \equiv L' + \lambda L_1 = 0$ ....**(i)**

(i) Implies each chord of intersection passes through the fixed point, which is the point of intersection of lines $L' = 0$ & $L_1 = 0$.

Hence proved.

Ex. 20 Let P be any moving point on the circle $x^2 + y^2 - 2x = 1$, from this point chord of contact is drawn w.r.t. the circle $x^2 + y^2 - 2x = 0$. Find the locus of the circumcentre of the triangle CAB, C being centre of the circle and A, B are the points of contact.

Sol. The two circles are

$(x - 1)^2 + y^2 = 1$ ...... **(i)**

$(x - 1)^2 + y^2 = 2$ ...... **(ii)**

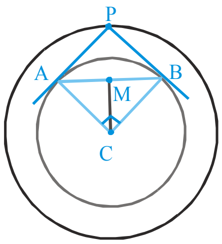
So the second circle is the director circle of the first. So $\angle APB = \pi/2$

Also $\angle ACB = \pi/2$

Now circumcentre of the right angled triangle CAB would lie on the mid point of AB

So let the point be $M \equiv (h, k)$

Now, $CM = CB\sin 45° = \dfrac{1}{\sqrt{2}}$



So, $(h - 1)^2 + k^2 = \left(\dfrac{1}{\sqrt{2}}\right)^2$

So, locus of M is $(x - 1)^2 + y^2 = \dfrac{1}{2}$.

Exercise # 1

[Single Correct Choice Type Questions]

1. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq. units. The equation of the circle is -

(A) $x^2 + y^2 - 2x - 2y = 47$ (B) $x^2 + y^2 - 2x - 2y = 62$
(C) $x^2 + y^2 - 2x + 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 62$

2. Two lines through (2, 3) from which the circle $x^2 + y^2 = 25$ intercepts chords of length 8 units have equations

(A) $2x + 3y = 13$, $x + 5y = 17$ (B) $y = 3$, $12x + 5y = 39$
(C) $x = 2$, $9x - 11y = 51$ (D) none of these

3. The line $2x - y + 1 = 0$ is tangent to the circle at the point (2, 5) and the centre of the circles lies on $x - 2y = 4$. The radius of the circle is

(A) $3\sqrt{5}$ (B) $5\sqrt{3}$ (C) $2\sqrt{5}$ (D) $5\sqrt{2}$

4. $y = \sqrt{3}x + c_1$ & $y = \sqrt{3}x + c_2$ are two parallel tangents of a circle of radius 2 units, then $|c_1 - c_2|$ is equal to -

(A) 8 (B) 4 (C) 2 (D) 1

5. Let C_1 and C_2 are circles defined by $x^2 + y^2 - 20x + 64 = 0$ and $x^2 + y^2 + 30x + 144 = 0$. The length of the shortest line segment PQ that is tangent to C_1 at P and to C_2 at Q is

(A) 15 (B) 18 (C) 20 (D) 24

6. The equation to the circle whose radius is 4 and which touches the negative x-axis at a distance 3 units from the origin is -

(A) $x^2 + y^2 - 6x + 8y - 9 = 0$ (B) $x^2 + y^2 \pm 6x - 8y + 9 = 0$
(C) $x^2 + y^2 + 6x \pm 8y + 9 = 0$ (D) $x^2 + y^2 \pm 6x - 8y - 9 = 0$

7. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) none

8. The centre of the smallest circle touching the circles $x^2 + y^2 - 2y - 3 = 0$ and $x^2 + y^2 - 8x - 18y + 93 = 0$ is

(A) (3, 2) (B) (4, 4) (C) (2, 7) (D) (2, 5)

9. B and C are fixed points having co-ordinates (3, 0) and (–3, 0) respectively. If the vertical angle BAC is 90°, then the locus of the centroid of the ΔABC has the equation -

(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = 2$ (C) $9(x^2 + y^2) = 1$ (D) $9(x^2 + y^2) = 4$

10. The condition so that the line $(x + g)\cos\theta + (y + f)\sin\theta = k$ is a tangent to $x^2 + y^2 + 2gx + 2fy + c = 0$ is

(A) $g^2 + f^2 = c + k^2$ (B) $g^2 + f^2 = c^2 + k$ (C) $g^2 + f^2 = c^2 + k^2$ (D) $g^2 + f^2 = c + k$

11. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the co-ordinates of the centre of C_2 are

(A) $\left(\pm\frac{9}{5}, \pm\frac{12}{5}\right)$ (B) $\left(\pm\frac{9}{5}, \mp\frac{12}{5}\right)$ (C) $\left(\pm\frac{12}{5}, \pm\frac{9}{5}\right)$ (D) $\left(\pm\frac{12}{5}, \mp\frac{9}{5}\right)$

12. Number of different circles that can be drawn touching 3 lines, no two of which are parallel and they are neither coincident nor concurrent, are -

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

13. The area of an equilateral triangle inscribed in the circle $x^2 + y^2 - 2x = 0$ is :

(A) $\frac{3\sqrt{3}}{2}$ **(B)** $\frac{3\sqrt{3}}{4}$ **(C)** $\frac{3\sqrt{3}}{8}$ **(D)** none

14. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is:

(A) $\sqrt{q-p}$ **(B)** $\sqrt{p-q}$ **(C)** $\sqrt{q+p}$ **(D)** none

15. The equations of the tangents drawn from the point (0,1) to the circle $x^2 + y^2 - 2x + 4y = 0$ are -

(A) $2x - y + 1 = 0, x + 2y - 2 = 0$ **(B)** $2x - y - 1 = 0, x + 2y - 2 = 0$

(C) $2x - y + 1 = 0, x + 2y + 2 = 0$ **(D)** $2x - y - 1 = 0, x + 2y + 2 = 0$

16. A circle of radius unity is centered at origin. Two particles start moving at the same time from the point (1, 0) and move around the circle in opposite direction. One of the particle moves counterclockwise with constant speed v and the other moves clockwise with constant speed 3v. After leaving (1, 0), the two particles meet first at a point P, and continue until they meet next at point Q. The coordinates of the point Q are

(A) (1, 0) **(B)** (0, 1) **(C)** (0, –1) **(D)** (–1, 0)

17. The locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is -

(A) $9x + 10y - 7 = 0$ **(B)** $x - y + 2 = 0$

(C) $9x - 10y + 11 = 0$ **(D)** $9x + 10y + 7 = 0$

18. The parametric coordinates of any point on the circle $x^2 + y^2 - 4x - 4y = 0$ are-

(A) $(-2 + 2\cos\alpha, -2 + 2\sin\alpha)$ **(B)** $(2 + 2\cos\alpha, 2 + 2\sin\alpha)$

(C) $(2 + 2\sqrt{2}\cos\alpha, 2 + 2\sqrt{2}\sin\alpha)$ **(D)** $(-2 + 2\sqrt{2}\cos\alpha, -2 + 2\sqrt{2}\sin\alpha)$

19. The equation of the diameter of the circle $(x - 2)^2 + (y + 1)^2 = 16$ which bisects the chord cut off by the circle on the line $x - 2y - 3 = 0$ is

(A) $x + 2y = 0$ **(B)** $2x + y - 3 = 0$ **(C)** $3x + 2y - 4 = 0$ **(D)** none

20. A line meets the co-ordinate axes in A and B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distances of the tangent to the circle at the origin O from the points A and B respectively, the diameter of the circle is

(A) $\frac{2d_1 + d_2}{2}$ **(B)** $\frac{d_1 + 2d_2}{2}$ **(C)** $d_1 + d_2$ **(D)** $\frac{d_1 d_2}{d_1 + d_2}$

21. The gradient of the tangent line at the point $(a\cos\alpha, a\sin\alpha)$ to the circle $x^2 + y^2 = a^2$, is -

(A) $\tan(\pi - \alpha)$ **(B)** $\tan\alpha$ **(C)** $\cot\alpha$ **(D)** $-\cot\alpha$

22. The equation of normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ which passes through (1, 1) is

(A) $3x + y - 4 = 0$ **(B)** $x - y = 0$ **(C)** $x + y = 0$ **(D)** none

23. The equation of the common tangent to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ at their point of contact is

(A) $12x + 5y + 19 = 0$ **(B)** $5x + 12y + 19 = 0$

(C) $5x - 12y + 19 = 0$ **(D)** $12x - 5y + 19 = 0$

24. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. The equation of the pair of tangents is -

(A) $x^2+y^2+5xy=0$ **(B)** $x^2+y^2+10xy=0$

(C) $2x^2+2y^2+5xy=0$ **(D)** $2x^2+2y^2-5xy=0$

25. In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to

(A) $\dfrac{AB \cdot AD}{\sqrt{AB^2 + AD^2}}$ **(B)** $\dfrac{AB \cdot AD}{AB + AD}$ **(C)** $\sqrt{AB \cdot AD}$ **(D)** $\dfrac{AB \cdot AD}{\sqrt{AB^2 - AD^2}}$

26. Tangents are drawn from (4, 4) to the circle $x^2 + y^2 - 2x - 2y - 7 = 0$ to meet the circle at A and B. The length of the chord AB is -

(A) $2\sqrt{3}$ **(B)** $3\sqrt{2}$ **(C)** $2\sqrt{6}$ **(D)** $6\sqrt{2}$

27. Pair of tangents are drawn from every point on the line $3x + 4y = 12$ on the circle $x^2 + y^2 = 4$. Their variable chord of contact always passes through a fixed point whose co-ordinates are -

(A) $\left(\frac{4}{3}, \frac{3}{4}\right)$ **(B)** $\left(\frac{3}{4}, \frac{3}{4}\right)$ **(C)** (1, 1) **(D)** $\left(1, \frac{4}{3}\right)$

28. The locus of the mid point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is:

(A) $x + y = 2$ **(B)** $x^2 + y^2 = 1$ **(C)** $x^2 + y^2 = 2$ **(D)** $x + y = 1$

29. The radical centre of three circles taken in pairs described on the sides of a triangle ABC as diameters is the :

(A) centroid of the ΔABC **(B)** incentre of the ΔABC

(C) circumcentre o the ΔABC **(D)** orthocentre of the ΔABC

30. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals -

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{4}$ **(D)** none

31. The number of common tangents of the circles $x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2y - 7 = 0$ -

(A) 1 **(B)** 3 **(C)** 2 **(D)** 4

32. Two circles are drawn through the points (1, 0) and (2, – 1) to touch the axis of y. They intersect at an angle

(A) $\cot^{-1}\frac{3}{4}$ **(B)** $\cos^{-1}\frac{4}{5}$ **(C)** $\frac{\pi}{2}$ **(D)** $\tan^{-1} 1$

33. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ and $x^2 + y^2 + 7x - 9y + 29 = 0$ is

(A) $x^2 + y^2 - 16x - 18y - 4 = 0$ **(B)** $x^2 + y^2 - 7x + 11y + 6 = 0$

(C) $x^2 + y^2 + 2x - 8y + 9 = 0$ **(D)** none of these

34. A circle is drawn touching the x-axis and centre at the point which is the reflection of (a, b) in the line $y - x = 0$. The equation of the circle is -

(A) $x^2 + y^2 - 2bx - 2ay + a^2 = 0$ **(B)** $x^2 + y^2 - 2bx - 2ay + b^2 = 0$

(C) $x^2 + y^2 - 2ax - 2by + b^2 = 0$ **(D)** $x^2 + y^2 - 2ax - 2by + a^2 = 0$

35. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles. The area of the rhombous is

(A) $8\sqrt{3}$ sq.units **(B)** $4\sqrt{3}$ sq.units **(C)** $16\sqrt{3}$ sq.units **(D)** none

36. The equation of the circle having the lines $y^2 - 2y + 4x - 2xy = 0$ as its normals & passing through the point (2,1) is -

(A) $x^2 + y^2 - 2x - 4y + 3 = 0$ **(B)** $x^2 + y^2 - 2x + 4y - 5 = 0$

(C) $x^2 + y^2 + 2x + 4y - 13 = 0$ **(D)** none

37. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then p + q is equal to:

(A) 25 **(B)** 100 **(C)** 10 **(D)** 48

38. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touches each other, then -

(A) $f_1g_1 = f_2g_2$ **(B)** $\frac{f_1}{g_1} = \frac{f_2}{g_2}$ **(C)** $f_1f_2 = g_1g_2$ **(D)** none

39. The equation of a line inclined at an angle $\frac{\pi}{4}$ to the axis X, such that the two circles $x^2 + y^2 = 4$, $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal lengths on it, is

(A) $2x - 2y - 3 = 0$ **(B)** $2x - 2y + 3 = 0$ **(C)** $x - y + 6 = 0$ **(D)** $x - y - 6 = 0$

40. The distance between the chords of contact of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and the point (g, f) is

(A) $\sqrt{g^2 + f^2}$ **(B)** $\frac{\sqrt{g^2 + f^2 - c}}{2}$ **(C)** $\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$ **(D)** $\frac{\sqrt{g^2 + f^2 + c}}{2\sqrt{g^2 + f^2}}$

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. Equation $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$, may represents -

(A) Equation of straight line, if θ is constant and r is variable.
(B) Equation of a circle, if r is constant & θ is variable.
(C) A straight line passing through a fixed point & having a known slope.
(D) A circle with a known centre and given radius.

2. If $a^2 + b^2 = 1$, $m^2 + n^2 = 1$, then which of the following is true for all values of m, n, a, b -
(A) $|am + bn| \le 1$ **(B)** $|am - bn| \ge 1$ **(C)** $|am + bn| \ge 1$ **(D)** $|am - bn| \le 1$

3. The circle $x^2 + y^2 - 2x - 3ky - 2 = 0$ passes through two fixed points, (k is the parameter)
(A) $(1 + \sqrt{3}, 0)$ **(B)** $(-1 + \sqrt{3}, 0)$ **(C)** $(-\sqrt{3} - 1, 0)$ **(D)** $(1 - \sqrt{3}, 0)$

4. Let L_1 be a line passing through the origin and L_2 be the line x + y = 1. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal then the equation of L_1 can be
(A) $x + y = 0$ **(B)** $x - y = 0$ **(C)** $x + 7y = 0$ **(D)** $x - 7y = 0$

5. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length 8 has a slope equal to $\frac{3}{4}$, then coordinates of centre of C_2 are -
(A) $\left(\frac{9}{5}, -\frac{12}{5}\right)$ **(B)** $\left(-\frac{9}{5}, \frac{12}{5}\right)$ **(C)** $\left(\frac{9}{5}, \frac{12}{5}\right)$ **(D)** $\left(-\frac{9}{5}, -\frac{12}{5}\right)$

6. 3 circle of radii 1, 2 and 3 and centres at A, B and C respectively, touch each other. Another circle whose centre is P touches all these 3 circles externally and has radius r. Also $\angle PAB = \theta$ & $\angle PAC = \alpha$ -
(A) $\cos\theta = \frac{3 - r}{3(1 + r)}$ **(B)** $\cos\alpha = \frac{2 - r}{2(1 + r)}$ **(C)** $r = \frac{6}{23}$ **(D)** $r = \frac{6}{\sqrt{23}}$

7. The chords of contact of the pair of tangents drawn from each point on the line 2x + y = 4 to the circle $x^2 + y^2 = 1$ pass through the point
(A) (1, 2) **(B)** $\left(\frac{1}{2}, \frac{1}{4}\right)$ **(C)** (2, 4) **(D)** none

8. Three distinct lines are drawn in a plane. Suppose there exist exactly n circles in the plane tangent to all the three lines, then the possible values of n is/are
(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

9. Circles are drawn touching the co-ordinate axis and having radius 2, then -
(A) centre of these circles lie on the pair of lines $y^2 - x^2 = 0$
(B) centre of these circles lie only on the line y = x
(C) Area of the quadrilateral whose vertices are centre of these circles is 16 sq.unit
(D) Area of the circle touching these four circles internally is $4\pi(3 + 2\sqrt{2})$

10. If r represent the distance of a point from origin & θ is the angle made by line joining origin to that point from line x-axis, then $r = |\cos\theta|$ represents -

(A) two circles of radii $\frac{1}{2}$ each. **(B)** two circles centered at $\left(\frac{1}{2}, 0\right)$ & $\left(-\frac{1}{2}, 0\right)$

(C) two circles touching each other at the origin. **(D)** pair of straight line

11. Consider the circles $S_1 : x^2 + y^2 = 4$ and $S_2 : x^2 + y^2 - 2x - 4y + 4 = 0$ which of the following statements are correct?

(A) Number of common tangents to these circles is 2.

(B) If the power of a variable point P w.r.t. these two circles is same then P moves on the line $x + 2y - 4 = 0$.

(C) Sum of the y-intercepts of both the circles is 6.

(D) The circles S_1 and S_2 are orthogonal.

12. The equation of circles passing through (3, –6) touching both the axes is

(A) $x^2 + y^2 - 6x + 6y + 9 = 0$ **(B)** $x^2 + y^2 + 6x - 6y + 9 = 0$

(C) $x^2 + y^2 + 30x - 30y + 225 = 0$ **(D)** $x^2 + y^2 - 30x + 30y + 225 = 0$

13. Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'P_1' and 'P_2'. Possible co-ordinates of 'P' so that area of triangle PP_1P_2 is minimum is/are -

(A) (10, 0) **(B)** $(10\sqrt{2}, 0)$ **(C)** (–10, 0) **(D)** $(-10\sqrt{2}, 0)$

14. The equation of the circle which touches both the axes and the line $\frac{x}{3} + \frac{y}{4} = 1$ and lies in the first quadrant is $(x - c)^2 + (y - c)^2 = c^2$ where c is

(A) 1 **(B)** 2 **(C)** 4 **(D)** 6

15. If y = c is a tangent to the circle $x^2 + y^2 - 2x + 2y - 2 = 0$, then the value of c can be -

(A) 1 **(B)** 3 **(C)** –1 **(D)** –3

16. A family of linear functions is given by $f(x) = 1 + c(x + 3)$ where $c \in R$. If a member of this family meets a unit circle centered at origin in two coincident points then 'c' can be equal to

(A) – 3/4 **(B)** 0 **(C)** 3/4 **(D)** 1

17. Equations of circles which pass through the points (1, –2) and (3, – 4) and touch the x-axis is

(A) $x^2 + y^2 + 6x + 2y + 9 = 0$ **(B)** $x^2 + y^2 + 10x + 20y + 25 = 0$

(C) $x^2 + y^2 - 6x + 4y + 9 = 0$ **(D)** none

18. The common chord of two intersecting circles C_1 and C_2 can be seen from their centres at the angles of 90° & 60° respectively. If the distance between their centres is equal to $\sqrt{3} + 1$ then the radii of C_1 and C_2 are -

(A) $\sqrt{3}$ and 3 **(B)** $\sqrt{2}$ and $2\sqrt{2}$ **(C)** $\sqrt{2}$ and 2 **(D)** $2\sqrt{2}$ and 4

19. If $al^2 - bm^2 + 2dl + 1 = 0$, where a, b, d are fixed real numbers such that $a + b = d^2$ then the line $lx + my + 1 = 0$ touches a fixed circle :

(A) which cuts the x–axis orthogonally

(B) with radius equal to b

(C) on which the length of the tangent from the origin is $\sqrt{d^2 - b}$

(D) none of these .

20. The equation(s) of the tangent at the point (0, 0) to the circle, making intercepts of length 2a and 2b units on the co-ordinate axes, is (are) -

(A) $ax + by = 0$ **(B)** $ax - by = 0$ **(C)** $x = y$ **(D)** $bx + ay = 0$

21. Consider the circles

$$S_1 : x^2 + y^2 + 2x + 4y + 1 = 0$$
$$S_2 : x^2 + y^2 - 4x + 3 = 0$$
$$S_3 : x^2 + y^2 + 6y + 5 = 0$$

Which of this following statements are correct?

(A) Radical centre of S_1, S_2 and S_3 lies in 1st quadrant.

(B) Radical centre of S_1, S_2 and S_3 lies in 4th quadrants.

(C) Radius of the circle intersecting S_1, S_2 and S_3 orthogonally is 1.

(D) Circle orthogonal to S_1, S_2 and S_3 has its x and y intercept equal to zero.

22. Locus of the intersection of the two straight lines passing through (1, 0) and (–1, 0) respectively and including an angle of 45° can be a circle with

(A) centre (1, 0) and radius $\sqrt{2}$. **(B)** centre (1, 0) and radius 2.

(C) centre (0, 1) and radius $\sqrt{2}$. **(D)** centre (0, – 1) and radius $\sqrt{2}$.

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. Consider two circles $C_1 \equiv x^2 + y^2 + 2x + 2y - 6 = 0$ & $C_2 \equiv x^2 + y^2 + 2x + 2y - 2 = 0$.

Statement-I : Two tangents are drawn from a point on the circle C_1 to the circle C_2, then tangents always perpendicular.

Statement-II : C_1 is the director circle of C_2.

2. Passing through a point A(6, 8) a variable secant line L is drawn to the circle $S : x^2 + y^2 - 6x - 8y + 5 = 0$. From the point of intersection of L with S, a pair of tangent lines are drawn which intersect at P.

Statement - I Locus of the point P has the equation $3x + 4y - 40 = 0$.

Statement - II Point A lies outside the circle.

3. **Statement - I** Only one normal can be drawn through the point P(2, –3) to the circle $x^2 + y^2 - 4x + 8y - 16 = 0$

Statement - II Passing through any point lying inside a given circle only one normal can be drawn.

4. **Statement - I** If three circles which are such that their centres are non-collinear, then exactly one circle exists which cuts the three circles orthogonally.

Statement - II Radical axis for two intersecting circles is the common chord.

5. Let C_1 denotes a family of circles with centre on x-axis and touching the y-axis at the origin.
and C_2 denotes a family of circles with centre on y-axis and touching the x-axis at the origin.

Statement - I Every member of C_1 intersects any member of C_2 at right angles at the point other than origin.

Statement - II If two circles intersect at 90° at one point of their intersection, then they must intersect at 90° on the other point of intersection also.

6. Let C be a circle with centre 'O' and HK is the chord of contact of pair of the tangents from point A. OA intersects the circle C at P and Q and B is the midpoint of HK, then

Statement - I AB is the harmonic mean of AP and AQ.

Statement - II AK is the Geometric mean of AB and AO and OA is the arithmetic mean of AP and AQ.

7. **Statement - I** Angle between the tangents drawn from the point P(13, 6) to the circle
$S : x^2 + y^2 - 6x + 8y - 75 = 0$ is 90°.

Statement - II Point P lies on the director circle of S.

8. Consider the lines

$$L : (k + 7)x - (k - 1)y - 4(k - 5) = 0 \text{ where k is a parameter}$$

and the circle

$$C : x^2 + y^2 + 4x + 12y - 60 = 0$$

Statement - I Every member of L intersects the circle 'C' at an angle of 90°

Statement - II Every member of L is tangent to the circle C.

9. **Statement - I** The line $(x - 3)\cos\theta + (y - 3)\sin\theta = 1$ touches a circle $(x - 3)^2 + (y - 3)^2 = 1$ for all values of θ.

Statement - II $x\cos\theta + y\sin\theta = a$ is a tangent of circle $x^2 + y^2 = a^2$ for all values of θ.

10. **Statement - I** The circle $C_1 : x^2 + y^2 - 6x - 4y + 9 = 0$ bisects the circumference of the circle
$C_2 : x^2 + y^2 - 8x - 6y + 23 = 0$.

Statement - II Centre of the circle C_1 lies on the circumference of C_2.

11. **Statement - I** The length of intercept made by the circle $x^2 + y^2 - 2x - 2y = 0$ on the x-axis is 2.

Statement - II $x^2 + y^2 - \alpha x - \beta y = 0$ is a circle which passes through origin with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ and radius $\sqrt{\frac{\alpha^2 + \beta^2}{2}}$.

12. Consider the circles, $S_1 : x^2 + y^2 + 2x - 4 = 0$ and $S_2 : x^2 + y^2 - y + 1 = 0$

Statement - I Tangents from the point P(0, 5) on S_1 and S_2 are equal.

Statement - II Point P(0, 5) lies on the radical axis of the two circles.

13. A circle is circumscribed about an equilateral triangle ABC and a point P on the minor arc joining A and B, is chosen. Let x = PA, y = PB and z = PC. (z is larger than both x and y.)

Statement - I Each of the possibilities (x + y) greater than z, equal to z or less than z, is possible for some P.

Statement - II In a triangle ABC, sum of the two sides of a triangle is greater than the third and the third side is greater than the difference of the two.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1.

Column-I	Column-II
(A) If point of intersection and number of common tangents of two circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$ are λ and μ respectively, then	**(p)** $\mu - \lambda = 3$
(B) If point of intersection and number of tangents of two circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 2x = 0$ are λ and μ respectively, then	**(q)** $\mu + \lambda = 5$
(C) If the straight line $y = mx \ \forall \ m \in I$ touches or lies outside the circle $x^2 + y^2 - 20y + 90 = 0$ and the maximum and minimum values of $\|m\|$ are μ & λ respectively then	**(r)** $\mu - \lambda = 4$
(D) If two circle $x^2 + y^2 + px + py - 7 = 0$ and $x^2 + y^2 - 10x + 2py + 1 = 0$ cut orthogonally and the value of p are λ & μ respectively then	**(s)** $\mu + \lambda = 4$

2.

Column – I	Column – II
(A) Number of values of a for which the common chord of the circles $x^2 + y^2 = 8$ and $(x - a)^2 + y^2 = 8$ subtends a right angle at the origin is	**(p)** 4
(B) A chord of the circle $(x - 1)^2 + y^2 = 4$ lies along the line $y = 22\sqrt{3}(x - 1)$. The length of the chord is equal to	**(q)** 2
(C) The number of circles touching all the three lines $3x + 7y = 2$, $21x + 49y = 5$ and $9x + 21y = 0$ are	**(r)** 0
(D) If radii of the smallest and largest circle passing through the point $\left(\sqrt{3}, \sqrt{2}\right)$ and touching the circle $x^2 + y^2 - 2\sqrt{2}y - 2 = 0$ are r_1 and r_2 respectively, then the mean of r_1 and r_2 is	**(s)** 1

3.

Column-I	Column-II
(A) Two intersecting circles	**(p)** have a common tangent
(B) Two circles touching each other	**(q)** have a common normal
(C) Two non concentric circles, one strictly inside the other	**(r)** do not have a common normal
(D) Two concentric circles of different radii	**(s)** do not have a radical axis.

4. **Column – I** | **Column – II**

	Column – I		Column – II
(A)	Number of common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 6x - 6y + 2 = 0$ is	**(p)**	1
(B)	Number of indirect common tangents of the circles $x^2 + y^2 - 4x - 10y + 4 = 0$ & $x^2 + y^2 - 6x - 12y - 55 = 0$ is	**(q)**	2
(C)	Number of common tangents of the circles $x^2 + y^2 - 2x - 4y = 0$ & $x^2 + y^2 - 8y - 4 = 0$ is	**(r)**	3
(D)	Number of direct common tangents of the circles $x^2 + y^2 + 2x - 8y + 13 = 0$ & $x^2 + y^2 - 6x - 2y + 6 = 0$ is	**(s)**	0

5.

	Column-I		Column-II
(A)	If the straight line $y = kx \ \forall \ K \in I$ touches or passes outside the circle $x^2 + y^2 - 20y + 90 = 0$ then $\|k\|$ can have the value	**(p)**	1
(B)	Two circles $x^2 + y^2 + px + py - 7 = 0$ and $x^2 + y^2 - 10x + 2py + 1 = 0$ intersect each other orthogonally then the value of p is	**(q)**	2
(C)	If the equation $x^2 + y^2 + 2\lambda x + 4 = 0$ and $x^2 + y^2 - 4\lambda y + 8 = 0$ represent real circles then the value of λ can be	**(r)**	3
(D)	Each side of a square is of length 4. The centre of the square is (3, 7). One diagonal of the square is parallel to $y = x$. The possible abscissas of the vertices of the square can be	**(s)**	5

Part # II [Comprehension Type Questions]

Comprehension # 1

Let $A \equiv (-3, 0)$ and $B \equiv (3, 0)$ be two fixed points and P moves on a plane such that $PA = nPB$ $(n > 0)$.
On the basis of above information, answer the following questions :

1. If $n \neq 1$, then locus of a point P is -
(A) a straight line **(B)** a circle **(C)** a parabola **(D)** an ellipse

2. If $n = 1$, then the locus of a point P is -
(A) a straight line **(B)** a circle **(C)** a parabola **(D)** a hyperbola

3. If $0 < n < 1$, then -
(A) A lies inside the circle and B lies outside the circle
(B) A lies outside the circle and B lies inside the circle
(C) both A and B lies on the circle **(D)** both A and B lies inside the circle

4. If $n > 1$, then -
(A) A lies inside the circle and B lies outside the circle **(B)** A lies outside the circle and B lies inside the circle
(C) both A and B lies on the circle **(D)** both A and B lies inside the circle

5. If locus of P is a circle, then the circle -
(A) passes through A and B **(B)** never passes through A and B
(C) passes through A but does not pass through B **(D)** passes through B but does not pass through A

Comprehension # 2

Two circles are $S_1 \equiv (x+3)^2 + y^2 = 9$ $S_2 \equiv (x-5)^2 + y^2 = 16$ with centres C_1 & C_2

1. A direct common tangent is drawn from a point P which touches S_1 & S_2 at Q & R, respectively. Find the ratio of area of ΔPQC_1 & ΔPRC_2.

(A) 3 : 4 **(B)** 9 : 16 **(C)** 16 : 9 **(D)** 4 : 3

2. From point 'A' on S_2 which is nearest to C_1, a variable chord is drawn to S_1. The locus of mid point of the chord.

(A) circle **(B)** Diameter of s_1

(C) Arc of a circle **(D)** chord of s_1 but not diameter

3. Locus of 7 cuts the circle S_1 at B & C, then line segment BC subtends an angle on the major arc of circle S_1 is

(A) $\cos^{-1}\frac{3}{4}$ **(B)** $\frac{\pi}{2} - \tan^{-1}\frac{4}{3}$

(C) $\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{3}{4}$ **(D)** $\frac{\pi}{2}\cot^{-1}\frac{4}{3}$

Comprehension # 3

Consider a line pair $ax^2 + 3xy - 2y^2 - 5x + 5y + c = 0$ representing perpendicular lines intersecting each other at C and forming a triangle ABC with the x-axis.

1. If x_1 and x_2 are intercepts on the x-axis and y_1 and y_2 are the intercepts on the y-axis then the sum $(x_1 + x_2 + y_1 + y_2)$ is equal to

(A) 6 **(B)** 5 **(C)** 4 **(D)** 3

2. Distance between the orthocentre and circumcentre of the triangle ABC is

(A) 2 **(B)** 3 **(C)** 7/4 **(D)** 9/4

3. If the circle $x^2 + y^2 - 4y + k = 0$ is orthogonal with the circumcircle of the triangle ABC then 'k' equals

(A) 1/2 **(B)** 1 **(C)** 2 **(D)** 3/2

Comprehension # 4

Let S_1, S_2, S_3 be the circles $x^2 + y^2 + 3x + 2y + 1 = 0$, $x^2 + y^2 - x + 6y + 5 = 0$ and $x^2 + y^2 + 5x - 8y + 15 = 0$, then

1. Point from which length of tangents to these three circles is same is

(A) (1, 0) **(B)** (3, 2) **(C)** (10, 5) **(D)** (–2, 1)

2. Equation of circle S_4 which cut orthogonally to all given circle is

(A) $x^2 + y^2 - 6x + 4y - 14 = 0$ **(B)** $x^2 + y_2 + 6x + 4y - 14 = 0$

(C) $x^2 + y^2 - 6x - 4y + 14 = 0$ **(D)** $x^2 + y^2 - 6x - 4y - 14 = 0$

3. Radical centre of circles S_1, S_2, & S_4 is

(A) $\left(-\frac{3}{5}, -\frac{8}{5}\right)$ **(B)** (3, 2) **(C)** (1, 0) **(D)** $\left(-\frac{4}{5}, -\frac{3}{2}\right)$

Comprehension # 5

P is a variable point of the line L = 0. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram PQSR is completed.

On the basis of above information, answer the following questions :

1. If $L \equiv 2x + y - 6 = 0$, then the locus of circumcetre of ΔPQR is -

(A) $2x - y = 4$ **(B)** $2x + y = 3$ **(C)** $x - 2y = 4$ **(D)** $x + 2y = 3$

2. If $P \equiv (6, 8)$, then the area of ΔQRS is -

(A) $\frac{(6)^{3/2}}{25}$ sq. units **(B)** $\frac{(24)^{3/2}}{25}$ sq. units **(C)** $\frac{48\sqrt{6}}{25}$ sq. units **(D)** $\frac{192\sqrt{6}}{25}$ sq. units

3. If $P \equiv (3, 4)$, then coordinate of S is -

(A) $\left(-\frac{46}{25}, -\frac{63}{25}\right)$ **(B)** $\left(-\frac{51}{25}, -\frac{68}{25}\right)$ **(C)** $\left(-\frac{46}{25}, -\frac{68}{25}\right)$ **(D)** $\left(-\frac{68}{25}, -\frac{51}{25}\right)$

Comprehension # 6

Let C be a circle of radius *r* with centre at O. Let P be a point outside C and D be a point on C. A line through P intersects C at Q and R, S is the midpoint of QR.

1. For different choices of line through P, the curve on which S lies, is

(A) a straight line **(B)** an arc of circle with P as centre

(C) an arc of circle with PS as diameter **(D)** an arc of circle with OP as diameter

2. Let P is situated at a distance 'd' from centre O, then which of the following does not equal the product (PQ) (PR)?

(A) $d^2 - r^2$ **(B)** PT^2, where T is a point on C and PT is tangent to C

(C) $(PS)^2 - (QS)(RS)$ **(D)** $(PS)^2$

3. Let XYZ be an equilateral triangle inscribed in C. If α, β, γ denote the distances of D from vertices X, Y, Z respectively, the value of product $(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)(\alpha + \beta - \gamma)$, is

(A) 0 **(B)** $\frac{\alpha\beta\gamma}{8}$ **(C)** $\frac{\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma}{6}$ **(D)** None of these

Comprehension # 7

Let A, B, C be three sets of real numbers (x, y) defined as

A : $\{(x, y) : y \geq 1\}$ B : $\{(x, y): x^2 + y^2 - 4x - 2y - 4 = 0\}$ C : $\{(x, y): x + y = \sqrt{2}\}$

1. Number of elements in the $A \cap B \cap C$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** infinite

2. $(x + 1)^2 + (y - 1)^2 + (x - 5)^2 + (y - 1)^2$ has the value equal to

(A) 16 **(B)** 25 **(C)** 36 **(D)** 49

3. If the locus of the point of intersection of the pair of perpendicular tangents to the circle B is the curve S then the area enclosed between B and S is

(A) 6π **(B)** 8π **(C)** 9π **(D)** 18π

Comprehension # 8

Consider a circle $x^2 + y^2 = 4$ and a point P(4, 2). θ denotes the angle enclosed by the tangents from P on the circle and A, B are the points of contact of the tangents from P on the circle.

1. The value of θ lies in the interval

(A) (0, 15°) **(B)** (15°, 30°) **(C)** 30°, 45°) **(D)** (45°, 60°)

2. The intercept made by a tangent on the x-axis is

(A) 9/4 **(B)** 10/4 **(C)** 11/4 **(D)** 12/4

3. Locus of the middle points of the portion of the tangent to the circle terminated by the coordinate axes is

(A) $x^{-2} + y^{-2} = 1^{-2}$ **(B)** $x^{-2} + y^{-2} = 2^{-2}$ **(C)** $x^{-2} + y^{-2} = 3^{-2}$ **(D)** $x^{-2} - y^{-2} = 4^{-2}$

Exercise # 4

[Subjective Type Questions]

1. Find the equation of the circle which cuts each of the circles $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$ & $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter.

2. Find the equation of the circle inscribed in a triangle formed by the lines $3x + 4y = 12$; $5x + 12y = 4$ & $8y = 15x + 10$ without finding the vertices of the triangle.

3. Find the equation of a circle which is co-axial with circles $2x^2 + 2y^2 - 2x + 6y - 3 = 0$ & $x^2 + y^2 + 4x + 2y + 1 = 0$. It is given that the centre of the circle to be determined lies on the radical axis of these two circles.

4. (x_1, y_1) & (x_2, y_2) are the ends of a diameter of a circle such that x_1 & x_2 are the roots of $ax^2 + bx + c = 0$ & y_1 & y_2 are roots of $py^2 + qy + r = 0$. Find the equation of the circle, its centre & radius.

5. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line is drawn from the point P intersects the curve at points Q & R. If the product $PQ \cdot PR$ is independent of the slope of the line, then show that the curve is a circle.

6. The line $\ell x + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the point P and Q. The circle on PQ as diameter passes through the origin. Prove that $n^2(a + b) = \ell^2 + m^2$.

7. Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.

8. A variable circle passes through the point A (a, b) & touches the x-axis ; show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.

9. If $3\ell^2 + 6\ell + 1 - 6m^2 = 0$, then find the equation of the circle for which $\ell x + my + 1 = 0$ is a tangent.

10. Prove that the length of the common chord of the two circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ is

$\frac{1}{c}\sqrt{(a+b+c)\,(a-b+c)\,(a+b-c)\,(-a+b+c)}$, where $a, b, c > 0$.

11. Obtain the equations of the straight lines passing through the point A(2, 0) & making 45° angle with the tangent at A to the circle $(x + 2)^2 + (y - 3)^2 = 25$. Find the equations of the circles each of radius 3 whose centres are on these straight lines at a distance of $5\sqrt{2}$ from A.

12. Show that the locus of the point the tangents from which to the circle $x^2 + y^2 - a^2 = 0$ include a constant angle α is $(x^2 + y^2 - 2a^2)^2 \tan^2\alpha = 4a^2(x^2 + y^2 - a^2)$.

13. The curves whose equations are

$S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$S' = a'x^2 + 2h'xy + b'y^2 + 2g'x + 2f'y + c' = 0$

intersect in four concyclic points then, what is the relation between a,b,h,a′b′h′

14. A circle touches the line y = x at a point P such that OP = $4\sqrt{2}$, where O is the origin. The circle contains the point (–10,2) in its interior and the length of its chord on the line x + y = 0 is $6\sqrt{2}$. Determine the equation of the circle.

15. If the line x sinα – y + a secα = 0 touches the circle with radius 'a' and centre at the origin then find the most general values of 'α' and sum of the values of 'α' lying in [0, 100π].

16. P is a variable point on the circle with centre at C . CA & CB are perpendiculars from C on x-axis & y-axis respectively. Show that the locus of the centroid of the triangle PAB is a circle with centre at the centroid of the triangle CAB & radius equal to one third of the radius of the given circle.

17. Show that the equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ represents, for different values of λ, a system of circles passing through two fixed points A, B on the x – axis, and find the equation of that circle of the system the tangents to which at A & B meet on the line x + 2y + 5 = 0.

18. A circle touches the line y = x at a point P such that OP = $4\sqrt{2}$ where O is the origin. The circle contains the point (–10, 2) in its interior and the length of its chord on the line x + y = 0 is $6\sqrt{2}$. Find the equation of the circle.

19. Find the intervals of values of 'a' for which the line y + x = 0 bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2 + 2y^2 - \left(1+\sqrt{2}a\right)x - \left(1-\sqrt{2}a\right)y = 0$.

20. Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centre of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ & $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally.

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. The square of the length of tangent from (3, –4) on the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ **[AIEEE-2002]**

(A) 20 **(B)** 30 **(C)** 40 **(D)** 50

2. Radical axis of the circles $x^2 + y^2 + 6x - 2y - 9 = 0$ and $x^2 + y^2 - 2x + 9y - 11 = 0$ is- **[AIEEE-2002]**

(A) $8x - 11y + 2 = 0$ **(B)** $8x + 11y + 2 = 0$ **(C)** $8x + 11y - 2 = 0$ **(D)** $8x - 11y - 2 = 0$

3. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then- **[AIEEE-2003]**

(A) $r > 2$ **(B)** $2 < r < 8$ **(C)** $r < 2$ **(D)** $r = 2$

4. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is- **[AIEEE-2003]**

(A) $x^2 + y^2 - 2x + 2y = 62$ **(B)** $x^2 + y^2 + 2x - 2y = 62$

(C) $x^2 + y^2 + 2x - 2y = 47$ **(D)** $x^2 + y^2 - 2x + 2y = 47$

5. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is- **[AIEEE-2004]**

(A) $2ax + 2by + (a^2 + b^2 + 4) = 0$ **(B)** $2ax + 2by - (a^2 + b^2 + 4) = 0$

(C) $2ax - 2by + (a^2 + b^2 + 4) = 0$ **(D)** $2ax - 2by - (a^2 + b^2 + 4) = 0$

6. A variable circle passes through the fixed point A(p, q) and touches x-axis. The locus of the other end of the diameter through A is- **[AIEEE-2004]**

(A) $(x - p)^2 = 4qy$ **(B)** $(x - q)^2 = 4py$ **(C)** $(y - p)^2 = 4qx$ **(D)** $(y - q)^2 = 4px$

7. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is- **[AIEEE-2004]**

(A) $x^2 + y^2 - 2x + 2y - 23 = 0$ **(B)** $x^2 + y^2 - 2x - 2y - 23 = 0$

(C) $x^2 + y^2 + 2x + 2y - 23 = 0$ **(D)** $x^2 + y^2 + 2x - 2y - 23 = 0$

8. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is- **[AIEEE-2004]**

(A) $x^2 + y^2 - x - y = 0$ **(B)** $x^2 + y^2 - x + y = 0$ **(C)** $x^2 + y^2 + x + y = 0$ **(D)** $x^2 + y^2 + x - y = 0$

9. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct point P and Q then the line $5x + by - a = 0$ passes through P and Q for- **[AIEEE-2005]**

(A) exactly one value of a **(B)** no value of a

(C) infinitely many values of a **(D)** exactly two values of a

10. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is- **[AIEEE-2005]**

(A) an ellipse **(B)** a circle **(C)** a hyperbola **(D)** a parabola

11. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is- **[AIEEE-2005]**

(A) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$ **(B)** $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(C) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$ **(D)** $2ax + 2by - (a^2 + b^2 + p^2) = 0$

12. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then- **[AIEEE-2005]**

(A) $3a^2 - 10ab + 3b^2 = 0$ **(B)** $3a^2 - 2ab + 3b^2 = 0$
(C) $3a^2 + 10ab + 3b^2 = 0$ **(D)** $3a^2 + 2ab + 3b^2 = 0$

13. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is- **[AIEEE-2006]**

(A) $x^2 + y^2 + 2x - 2y - 62 = 0$ **(B)** $x^2 + y^2 - 2x + 2y - 62 = 0$
(C) $x^2 + y^2 - 2x + 2y - 47 = 0$ **(D)** $x^2 + y^2 + 2x - 2y - 47 = 0$

14. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is - **[AIEEE-2006, IIT-1996]**

(A) $x^2 + y^2 = 1$ **(B)** $x^2 + y^2 = \frac{27}{4}$ **(C)** $x^2 + y^2 = \frac{9}{4}$ **(D)** $x^2 + y^2 = \frac{3}{2}$

15. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval- **AIEEE-2007]**

(A) $0 < k < 1/2$ **(B)** $k \geq 1/2$ **(C)** $-1/2 \leq k \leq 1/2$ **(D)** $k \leq 1/2$

16. The point diametrically opposite to the point (1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is- **[AIEEE-2008]**

(A) (3, –4) **(B)** (–3, 4) **(C)** (–3, –4) **(D)** (3, 4)

17. Three distinct points A, B and C are given in the 2–dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point :- **[AIEEE-2009]**

(A) $\left(\frac{5}{2}, 0\right)$ **(B)** $\left(\frac{5}{3}, 0\right)$ **(C)** (0, 0) **(D)** $\left(\frac{5}{4}, 0\right)$

18. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for :- **[AIEEE-2009]**

(A) All except two values of p **(B)** Exactly one value of p
(C) All values of p **(D)** All except one value of p

19. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is :- **[AIEEE-2010]**

(A) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ **(B)** There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(C) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$ **(D)** There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

20. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if :- **[AIEEE-2010]**

(A) $-85 < m < -35$ **(B)** $-35 < m < 15$ **(C)** $15 < m < 65$ **(D)** $35 < m < 85$

21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c>0)$ touch each other if :- **[AIEEE-2011]**

(A) $a = 2c$ (B) $|a| = 2c$ (C) $2|a| = c$ (D) $|a| = c$

22. The equation of the circle passing through the points (1, 0) and (0, 1) and having the smallest radius is:

(A) $x^2 + y^2 + x + y - 2 = 0$ (B) $x^2 + y^2 - 2x - 2y + 1 = 0$ **[AIEEE-2011]**
(C) $x^2 + y^2 - x - y = 0$ (C) $x^2 + y^2 + 2x + 2y - 7 = 0$

23. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is : **[AIEEE-2012]**

(A) 5/3 (B) 10/3 (C) 3/5 (D) 6/5

24. The circle passing through (1, – 2) and touching the axis of x at (3, 0) also passes through the point :
(A) (–5, 2) (B) (2, –5) (C) (5, –2) (D) (–2, 5) **[JEE(Main)-2013]**

25. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centered at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to : **[JEE Main 2014]**

(A) $\frac{\sqrt{3}}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

26. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a : **[JEE Main 2015]**

(A) circle of radius $\sqrt{2}$ (B) circle of radius $\sqrt{3}$
(C) straight line parallel to x-axis (D) straight line parallel to y-axis

27. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :
(A) 3 (B) 4 (C) 1 (D) 2 **[JEE Main 2015]**

28. The centres of those circle which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on : **[JEE Main 2016]**

(A) an ellipse which is not a circle (B) a hyperbola
(C) a parabola (D) a circle

29. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is : **[JEE Main 2016]**

(A) $5\sqrt{3}$ (B) 5 (C) 10 (D) $5\sqrt{2}$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then 2r equals **[JEE 2001]**

(A) $\sqrt{PQ \cdot RS}$ (B) $\frac{PQ + RS}{2}$ (C) $\frac{2PQ \cdot RS}{PQ + RS}$ (D) $\sqrt{\frac{(PQ)^2 + (RS)^2}{2}}$

2. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. **[JEE 2001]**

3. Find the equation of the circle which passes through the points of intersection of circles $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ and intersects the circle $x^2 + y^2 + 4x + 6y + 4 = 0$ orthogonally. **[JEE 2001]**

4. Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of centre of the circumcircle of triangle TPQ. **[JEE 2001]**

5. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is **[JEE 2002]**

(A) 4 **(B)** $2\sqrt{5}$ **(C)** 5 **(D)** $3\sqrt{5}$

6. If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is **[JEE 2002]**

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ **(B)** $\frac{\sqrt{a^2-4b^2}}{2b}$ **(C)** $\frac{2b}{a-2b}$ **(D)** $\frac{b}{a-2b}$

7. The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ **[JEE 2004]**

(A) 1 **(B)** 2 **(C)** 3 **(D)** $\sqrt{3}$

8. Line $2x + 3y + 1 = 0$ is a tangent to a circle at (1, –1). This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points (0, –1) and (– 2, 3). Find equation of circle. **[JEE 2004]**

9. A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is **[JEE 2005]**

(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ **(B)** $\{(x, y) : x^2 + (y-1)^2 = 4\} \cup \{x, y) : y \le 0\}$

(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ **(D)** $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

10. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is **[JEE 2007]**

(A) 3 **(B)** 2 **(C)** 3/2 **(D)** 1

11. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

Statement-1 : The tangents are mutually perpendicular.

Statement-2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true. **[JEE 2007]**

12. Consider the two curves $C_1 : y^2 = 4x$; $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other **[JEE 2008]**

13. Consider, $L_1 : 2x + 3y + p - 3 = 0$; $L_2 : 2x + 3y + p + 3 = 0$,

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$.

Statement-1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

Statement-2 : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement-1 is True, Statement-2 is True; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True; statement-2 is **NOT** a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True **[JEE 2008]**

14. **Comprehension**

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}\,x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$.
Further, it is given that the origin and the centre of C are on the same side of the line PQ.

(i) The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ **(B)** $(x - 2\sqrt{3})^2 + (y + \frac{1}{2})^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ **(D)** $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

(ii) Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$ **(B)** $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\sqrt{3}, 0\right)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **(D)** $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(iii) Equations of the sides RP, RQ are

(A) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$ **(B)** $y = \frac{1}{\sqrt{3}}x, y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$ **(D)** $y = \sqrt{3}\,x, y = 0$ **[JEE 2008]**

15. Tangents drawn from the point P(1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ **(B)** $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ **(D)** $x^2 + y^2 - 6x - 4y + 19 = 0$ **[JEE 2009]**

16. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is **[JEE 2009]**

17. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is **[JEE 2010]**

[**Note :** [k] denotes the largest integer less than or equal to k]

18. The circle passing through the point (–1,0) and touching the y-axis at (0, 2) also passes through the point -

(A) $\left(-\frac{3}{2},0\right)$ **(B)** $\left(-\frac{5}{2},2\right)$ **(C)** $\left(-\frac{3}{2},\frac{5}{2}\right)$ **(D)** (–4,0) **[JEE 2011]**

19. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is **[JEE 2011]**

20. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is- **[JEE 2012]**

(A) $20(x^2 + y^2) - 36x + 45y = 0$ **(B)** $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ **(D)** $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question 21 and 22

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P\left(\sqrt{3},1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

21. A common tangent of the two circles is **[JEE 2012]**

(A) $x = 4$ **(B)** $y = 2$ **(C)** $x + \sqrt{3}y = 4$ **(D)** $x + 2\sqrt{2}y = 6$

22. A possible equation of L is **[JEE 2012]**

(A) $x - \sqrt{3}y = 1$ **(B)** $x + \sqrt{3}y = 1$ **(C)** $x - \sqrt{3}y = -1$ **(D)** $x + \sqrt{3}y = 5$

23. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ or y-axis is (are) **[JEE Ad. 2013]**

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ **(B)** $x^2 + y^2 - 6x + 7y + 9 = 0$

(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ **(D)** $x^2 + y^2 - 6x - 7y + 9 = 0$

24. A circle *S* passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then,

(A) radius of *S* is 8 **(B)** radius of *S* is 7 **[JEE Ad. 2014]**

(C) centre of *S* is (–7, 1) **(D)** centre of *S* is (–8, 1)

25. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meets at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) **[JEE Ad. 2016]**

(A) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$ **(B)** $\left(\frac{1}{4},\frac{1}{2}\right)$ **(C)** $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$ **(D)** $\left(\frac{1}{4},-\frac{1}{2}\right)$

26. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then **[JEE Ad. 2016]**

(A) $Q_2Q_3 = 12$ **(B)** $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ **(D)** area of the triangle PQ_2Q_3 is $4\sqrt{2}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. The axes are translated so that the new equation of the circle $x^2 + y^2 - 5x + 2y - 5 = 0$ has no first degree terms. Then the new equation is :

(A) $x^2 + y^2 = 9$ **(B)** $x^2 + y^2 = \frac{49}{4}$ **(C)** $x^2 + y^2 = \frac{81}{16}$ **(D)** none of these

2. S(x, y) = 0 represents a circle. The equation S(x, 2) = 0 gives two identical solutions x = 1 and the equation S(1, y) = 0 gives two distinct solutions y = 0, 2. Find the equation of the circle.

(A) $x^2 + y^2 + 2x - 2y + 1 = 0$ **(B)** $x^2 + y^2 - 2x + 2y + 1 = 0$

(C) $x^2 + y^2 - 2x - 2y - 1 = 0$ **(D)** $x^2 + y^2 - 2x - 2y + 1 = 0$

3. A line meets the co–ordinate axes in A and B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distances of the tangent to the circle at the origin O from the points A and B respectively, then diameter of the circle is:

(A) $\frac{2d_1 + d_2}{2}$ **(B)** $\frac{d_1 + 2d_2}{2}$ **(C)** $d_1 + d_2$ **(D)** $\frac{d_1 d_2}{d_1 + d_2}$

4. Consider a family of circles passing through two fixed points A (3,7) & B(6,5). Find the point of concurrency of the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family :

(A) $\left(\frac{11}{17}, \frac{3}{7}\right)$ **(B)** $\left(2, \frac{23}{3}\right)$ **(C)** $(-4, 3)$ **(D)** chords are not concurrent

5. A circle is inscribed (i.e. touches all four sides) into a rhombous ABCD with one angle 60°. The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point of the circle, then $|PA|^2 + |PB|^2 + |PC|^2 + |PD|^2$ is equal to:

(A) 12 **(B)** 11 **(C)** 10 **(D)** 13

6. If the radius of the circumcircle of the triangle TPQ, where PQ is chord of contact corresponding to point T with respect to circle $x^2 + y^2 - 2x + 4y - 11 = 0$, is 6 units, then minimum distance of T from the director circle of the given circle is:

(A) 6 **(B)** 12 **(C)** $6\sqrt{2}$ **(D)** $12 - 4\sqrt{2}$

7. Consider points $A(\sqrt{13}, 0)$ and $B(2\sqrt{13}, 0)$ lying on x-axis. These points are rotated in an-anticlockwise direction about the origin through an angle of $\tan^{-1}\left(\frac{2}{3}\right)$. Let the new position of A and B be A′ and B′ respectively. With A′ as centre and radius $\frac{2\sqrt{13}}{3}$ a circle C_1 is drawn and with B′ as a centre and radius $\frac{\sqrt{13}}{3}$ circle C_2 is drawn. The radical axis of C_1 and C_2 is :

(A) $9x + 6y = 65$ **(B)** $3x + 3y = 10$ **(C)** $3x + 2y = 20$ **(D)** none of these

8. A circle touches the lines $y = \frac{x}{\sqrt{3}}$, $y = x\sqrt{3}$ and has unit radius. If the centre of this circle lies in the first quadrant, then one possible equation of this circle is -

(A) $x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 8 + 4\sqrt{3} = 0$

(B) $x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 5 + 4\sqrt{3} = 0$

(C) $x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 7 + 4\sqrt{3} = 0$

(D) $x^2 + y^2 - 2x(\sqrt{3}+1) - 2y(\sqrt{3}+1) + 6 + 4\sqrt{3} = 0$

9. A circle of constant radius 'r' passes through origin O and cuts the axes of coordinates in points P and Q, then the equation of the locus of the foot of perpendicular from O to PQ is :

(A) $(x^2 + y^2)(x^{-2} + y^{-2}) = 4r^2$
(B) $(x^2 + y^2)^2(x^{-2} + y^{-2}) = r^2$
(C) $(x^2 + y^2)^2(x^{-2} + y^{-2}) = 4r^2$
(D) $(x^2 + y^2)(x^{-2} + y^{-2}) = r^2$

10. **S_1 :** If the length of tangent drawn from an external point P to the circle of radius r is ℓ, then area of triangle formed by pair of tangents and its chord of contact is $\frac{r\ell^3}{r^2 + \ell^2}$.

S_2 : If the points where the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ meet the co-ordinate axes are concyclic, then $a_1c_1 = a_2c_2$

S_3 : A circle is inscribed in an equilateral triangle of side a, the area of any square inscribed in the circle is $\frac{a^2}{8}$

S_4 : The equation of the circle with origin as centre passing the vertices of an equilateral triangle whose median is of length 3a is $x^2 + y^2 = 4a^2$

(A) F F T T
(B) T T T F
(C) T F F T
(D) T T T T

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. Consider the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. Let O be the centre of the circle and tangent at A(7, 3) and B(5, 1) meet at C. Let S = 0 represents family of circles passing through A and B, then -

(A) area of quadrilateral OACB = 4

(B) the radical axis for the family of circles S = 0 is x + y = 10

(C) the smallest possible circle of the family S = 0 is $x^2 + y^2 - 12x - 4y + 38 = 0$

(D) the coordinates of point C are (7, 1)

12. The centre of a circle passing through the points (0, 0), (1, 0) & touching the circle $x^2 + y^2 = 9$ is :

(A) $\left(\frac{3}{2}, \frac{1}{2}\right)$
(B) $\left(\frac{1}{2}, \sqrt{2}\right)$
(C) $\left(\frac{1}{2}, \frac{1}{2}\right)$
(D) $\left(\frac{1}{2}, -\sqrt{2}\right)$

13. Point M moved on the circle $(x-4)^2+(y-8)^2=20$. Then it broke away from it and moving along a tangent to the circle, cuts the x–axis at the point $(-2, 0)$. The co–ordinates of a point on the circle at which the moving point broke away is

(A) $\left(-\frac{3}{5}, \frac{46}{5}\right)$ **(B)** $\left(-\frac{2}{5}, \frac{44}{5}\right)$ **(C)** (6, 4) **(D)** (3, 5)

14. If $a\ell^2 - bm^2 + 2d\ell + 1 = 0$, where a, b, d are fixed real numbers such that $a + b = d^2$, then the line $\ell x + my + 1 = 0$ touches a fixed circle

(A) which cuts the x–axis orthogonally
(B) with radius equal to b
(C) on which the length of the tangent from the origin is $\sqrt{d^2 - b}$
(D) none of these.

15. If the area of the quadrilateral formed by the tangents from the origin to the circle $x^2 + y^2 + 6x - 10y + c = 0$ and the radii corresponding to the points of contact is 15, then values of c is/are

(A) 9 **(B)** 4 **(C)** 5 **(D)** 25

SECTION - III : ASSERTION AND REASON TYPE

16. Consider the lines

$$L : (k+7)x - (k-1)y - 4(k-5) = 0 \text{ where k is a parameter}$$

and the circle

$$C : x^2 + y^2 + 4x + 12y - 60 = 0$$

Statement-I : Every member of L intersects the circle 'C' at an angle of 90°

Statement-II : Every member of L is tangent to the circle C.

(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

17. Let C be a circle with centre 'O' and HK is the chord of contact of pair of the tangents from point A. OA intersects the circle C at P and Q and B is the midpoint of HK, then

Statement-I : AB is the harmonic mean of AP and AQ.

Statement-II : AK is the Geometric mean of AB and AO and OA is the arithmetic mean of AP and AQ.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

18. **Statement-I** : Let $S_1 : x^2 + y^2 - 10x - 12y - 39 = 0$ $S_2 : x^2 + y^2 - 2x - 4y + 1 = 0$
and $S_3 : 2x^2 + 2y^2 - 20x - 24y + 78 = 0$

The radical centre of these circles taken pairwise is (–2, –3)

Statement-II : Point of intersection of three radical axis of three circles taken in pairs is known as radical centre

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

19. **Statement-I :** Angle between the tangents drawn from the point P(13, 6) to the circle S : $x^2 + y^2 - 6x + 8y - 75 = 0$ is 90°.

Statement-II : Point P lies on the director circle of S.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

20. **Statement-I :** Only one normal can be drawn through the point P(2, –3) to the circle $x^2 + y^2 - 4x + 8y - 16 = 0$

Statement-II : Passing through any point lying inside a given circle only one normal can be drawn.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

SECTION - IV : MATRIX - MATCH TYPE

21.

	Column-I		Column-II
(A)	If the straight line $y = kx \ \forall\ K \in I$ touches or passes outside the circle $x^2 + y^2 - 20y + 90 = 0$ then $\lvert k \rvert$ can have the value	(p)	1
(B)	Two circles $x^2 + y^2 + px + py - 7 = 0$ and $x^2 + y^2 - 10x + 2py + 1 = 0$ intersect each other orthogonally then the value of p is	(q)	2
(C)	If the equation $x^2 + y^2 + 2\lambda x + 4 = 0$ and $x^2 + y^2 - 4\lambda y + 8 = 0$ represent real circles then the value of λ can be	(r)	3
(D)	Each side of a square is of length 4. The centre of the square is (3, 7). One diagonal of the square is parallel to y = x. The possible abscissae of the vertices of the square can be	(s)	5

22.

	Column – I		Column – II
(A)	If $ax + by - 5 = 0$ is the equation of the chord of the circle $(x - 3)^2 + (y - 4)^2 = 4$, which passes through (2, 3) and at the greatest distance from the centre of the circle, then $\lvert a + b \rvert$ is equal to -	(p)	6
(B)	Let O be the origin and P be a variable point on the circle $x^2 + y^2 + 2x + 2y = 0$. If the locus of mid-point of OP is $x^2 + y^2 + 2gx + 2fy = 0$, then the value of (g + f) is equal to -	(q)	3
(C)	The x-coordinates of the centre of the smallest circle which cuts the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ and $x^2 + y^2 - 10x + 12y + 52 = 0$ orthogonally, is -	(r)	2
(D)	If θ be the angle between two tangents which are drawn to the circle $x^2 + y^2 - 6\sqrt{3}\,x - 6y + 27 = 0$ from the origin, then $2\sqrt{3}\tan\theta$ equals to -	(s)	1

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

Let C be a circle of radius *r* with centre at O. Let P be a point outside C and D be a point on C. A line through P intersects C at Q and R, S is the midpoint of QR.

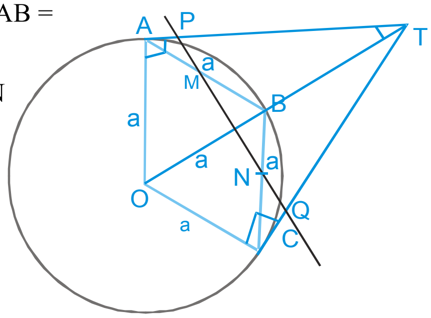
1. For different choices of line through P, the curve on which S lies, is
   (A) a straight line (B) an arc of circle with P as centre
   (C) an arc of circle with PS as diameter (D) an arc of circle with OP as diameter

2. Let P is situated at a distance 'd' from centre O, then which of the following does not equal the product (PQ) (PR) ?
   (A) $d^2 - r^2$ (B) PT^2, where T is a point on C and PT is tangent to C
   (C) $(PS)^2 - (QS)(RS)$ (D) $(PS)^2$

3. Let XYZ be an equilateral triangle inscribed in C. If α, β, γ denote the distances of D from vertices X, Y, Z respectively, the value of product $(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)(\alpha + \beta - \gamma)$, is
   (A) 0 (B) $\frac{\alpha\beta\gamma}{8}$ (C) $\frac{\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma}{6}$ (D) None of these

24. Read the following comprehension carefully and answer the questions.

Two variable chords AB and BC of a circle $x^2 + y^2 = a^2$ are such that AB = BC = a, and M and N are
The mid points of AB and BC respectively such that line joining MN intersect the circle at P and Q where P is closer to AB and O is the centre of the circle



1. ∠OAB is -
   (A) 30° (B) 60° (C) 45° (D) 15°

2. Angle between tangents at A and C is -
   (A) 90° (B) 120° (C) 60° (D) 150°

3. Locus of point of intersection of tangents at A and C is
   (A) $x^2 + y^2 = a^2$ (B) $x^2 + y^2 = 2a^2$ (C) $x^2 + y^2 = 4a^2$ (D) $x^2 + y^2 = 8a^2$

25. Read the following comprehension carefully and answer the questions.

Consider the two quadratic polynomials

$$C_a : y = \frac{x^2}{4} - ax + a^2 + a - 2 \quad \text{and} \quad C : y = 2 - \frac{x^2}{4}$$

1. If the origin lies between the zeroes of the polynomial C_a then the number of integral value(s) of 'a' is
   (A) 1 (B) 2 (C) 3 (D) more than 3

2. If 'a' varies then the equation of the locus of the vertex of C_a , is

(A) $x - 2y - 4 = 0$ **(B)** $2x - y - 4 = 0$ **(C)** $x - 2y + 4 = 0$ **(D)** $2x + y - 4 = 0$

3. For a = 3, if the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are common tangents to the graph of C_a and C then the value of $(m_1 + m_2)$ is equal to

(A) – 6 **(B)** – 3 **(C)** 1/2 **(D)** none

SECTION - VI : INTEGER TYPE

26. The lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6 unit. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts of intercepts of length 8 on these lines.

27. If $C_1 : x^2 + y^2 = (3 + 2\sqrt{2})^2$ be a circle and PA and PB are pair of tangents on C_1 where P is any point on the director circle of C_1, then find the radius of smallest circle which touches C_1 externally and also the two tangents PA and PB.

28. A ball moving around the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ in anti–clockwise direction leaves it tangentially at the point P(–2, –2). After getting reflected from a straight line it passes through the centre of the circle. Find the equation of this straight line if its perpendicular distance from P is $\frac{5}{2}$. You can assume that the angle of incidence is equal to the angle of reflection.

29. S is a circle having centre at (0, a) and radius b(b < a). A variable circle centred at $(\alpha, 0)$ and touching circle S, meets the X-axis at M and N. A point $P \equiv \left(0, \pm\lambda\sqrt{a^2 - b^2}\right)$ on the Y-axis, such that $\angle MPN$ is a constant for any choice of α, then find λ.

30. The ends A , B of a fixed straight line of length ‘a’ and ends A′ and B′ of another fixed straight line of length ‘b’ slide upon the axis of X & the axis of Y (one end on axis of X & the other on axis of Y). Find the locus of the centre of the circle passing through A, B, A′ and B′.

ANSWER KEY

EXERCISE - 1

1. C **2.** B **3.** A **4.** A **5.** C **6.** C **7.** C **8.** D **9.** A **10.** A **11.** B **12.** D **13.** B
14. A **15.** A **16.** D **17.** C **18.** C **19.** B **20.** C **21.** D **22.** A **23.** B **24.** C **25.** D **26.** B
27. D **28.** C **29.** D **30.** A **31.** A **32.** A **33.** A **34.** B **35.** A **36.** A **37.** C **38.** B **39.** A
40. C

EXERCISE - 2 : PART # I

1. ABCD **2.** AD **3.** AD **4.** BC **5.** AB **6.** ABC **7.** B **8.** ACD
9. ACD **10.** ABC **11.** ABD **12.** AD **13.** AC **14.** AD **15.** AD **16.** AB
17. BC **18.** C **19.** AC **20.** AB **21.** BCD **22.** CD

PART - II

1. A **2.** D **3.** C **4.** D **5.** A **6.** A **7.** A **8.** C **9.** A **10.** B **11.** C **12.** A **13.** D

EXERCISE - 3 : PART # I

1. $A \to r,s$ $B \to s$ $C \to p$ $D \to q$ **2.** $A \to q$ $B \to p$ $C \to r$ $D \to s$ **3.** $A \to p,q$ $B \to p,q$ $C \to q$ $D \to q,s$
4. $A \to r$ $B \to s$ $C \to p$ $D \to q$ **5.** $A \to p,q,r$ $B \to q,r$ $C \to q,r,s$ $D \to p,s$

PART - II

Comprehension # 1 : 1. B **2.** A **3.** A **4.** B **5.** B
Comprehension # 2 : 1. B **2.** C **3.** A
Comprehension # 3 : 1. B **2.** C **3.** D
Comprehension # 4 : 1. B **2.** D **3.** A
Comprehension # 5 : 1. B **2.** D **3.** B
Comprehension # 6 : 1. D **2.** D **3.** A
Comprehension # 7 : 1. B **2.** C **3.** C
Comprehension # 8 : 1. D **2.** B **3.** A

EXERCISE - 5 : PART # I

1. C **2.** A **3.** B **4.** D **5.** B **6.** A **7.** A **8.** A **9.** B **10.** D **11.** D **12.** D **13.** C
14. C **15.** B **16.** C **17.** D **18.** D **19.** C **20.** B **21.** D **22.** C **23.** B **24.** C **25.** D **26.** A
27. A **28.** C **29.** A

PART - II

1. A **2.** $OA = 3(3+\sqrt{10})$ **3.** $x^2 + y^2 + 14x - 6y + 6 = 0$; **4.** $2px + 2qy = r$ **5.** C **6.** A **7.** C
8. $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ **9.** D **10.** B **11.** A **12.** B **13.** C **14.** **(i)** D, **(ii)** A, **(iii)** D
15. B **16.** 8 **17.** 3 **18.** D **19.** 2 **20.** A **21.** D **22.** A **24.** B,C **25.** A,C **26.** A,B,C

MOCK TEST

1. B **2.** D **3.** C **4.** B **5.** B **6.** D **7.** A **8.** C **9.** C **10.** C **11.** ACD **12.** B
13. BC **14.** AC **15.** AD **16.** C **17.** A **18.** D **19.** A **20.** C
21. $A \to p,q,r$ $B \to q,r$ $C \to q,r,s$ $D \to p,s$ **22.** $A \to r$ $B \to s$ $C \to q$ $D \to p$
23. **1.** D **2.** D **3.** A **24.** **1.** B **2.** C **3.** C **25.** **1.** B **2.** A **3.** B
26. $x^2 + y^2 - 10x - 4y + 4 = 0$ **27.** **1** **28.** $(4\sqrt{3} - 3)x - (4 + 3\sqrt{3})y - (39 - 2\sqrt{3}) = 0$
29. 1 **30.** $(2ax - 2by)^2 + (2bx - 2ay)^2 = (a^2 - b^2)^2$