HINTS & SOLUTIONS

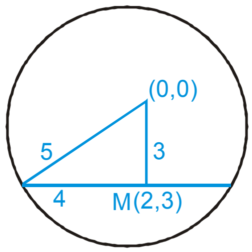
EXERCISE - 1

Single Choice

2. Let slope of required line is m

$y - 3 = m(x - 2)$

$\Rightarrow$ $mx - y + (3 - 2m) = 0$



length of $\perp$ from origin $= 3$

$\Rightarrow$ $9 + 4m^2 - 12m = 9 + 9m^2$

$\Rightarrow$ $5m^2 + 12m = 0$

$\Rightarrow$ $m = 0, -\dfrac{12}{5}$

Hence lines are $y - 3 = 0$

$\Rightarrow$ $y = 3$

$y - 3 = -\dfrac{12}{5}(x - 2)$

$\Rightarrow$ $5y - 15 = -12x + 24$

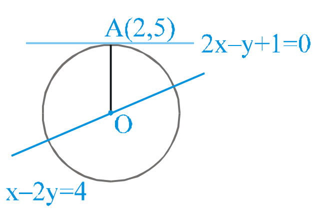
$\Rightarrow$ $12x + 5y = 39.$

3. $2x - y + 1 = 0$ is tangent

slope of line OA $= -\dfrac{1}{2}$

equation of OA, $(y - 5) = -\dfrac{1}{2}(x - 2)$

$2y - 10 = -x + 2$



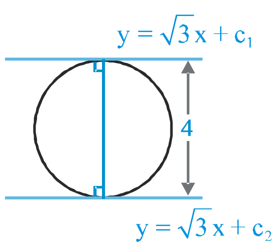
$x + 2y = 12$

$\therefore$ intersection with $x - 2y = 4$ will give coordinates of centre

solving we get (8, 2) distance OA $= \sqrt{(8-2)^2 + (2-5)^2}$

$= \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

4. Distance between both



lines is diameter of the circle $4 = \left|\dfrac{c_1 - c_2}{\sqrt{1+3}}\right|$

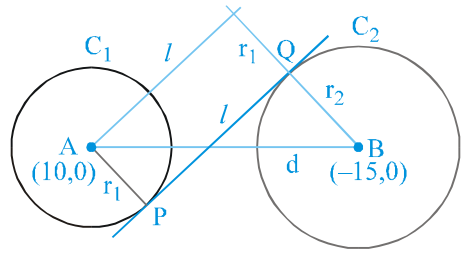
$|c_1 - c_2| = 8$

5. Centres are (10, 0) and (– 15, 0)

$r_1 = 6$; $r_2 = 9$

$d = 25$

$r_1 + r_2 < d$

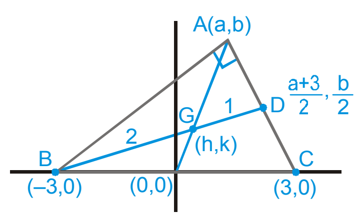


$\Rightarrow$ circles are separated

$PQ = l = \sqrt{d^2 - (r_1 + r_2)^2}$

$= \sqrt{625 - 225} = 20$

9.



$\angle BAC = 90°$

$\Rightarrow$ $\left(\dfrac{b}{a+3}\right)\left(\dfrac{b}{a-3}\right) = -1$

$\Rightarrow$ $b^2 = -(a^2 - 9)$ $\Rightarrow$ $a^2 + b^2 = 9$ ...... (i)

Now BG : GD = 2 : 1

$\Rightarrow$ $3h = \dfrac{2(a+3)}{2} + 1 \times -3$ $\Rightarrow$ $a = 3h$

& $3k = 2\left(\dfrac{b}{2}\right) + 1 \times 0$ $\Rightarrow$ $b = 3k$

substitute value of a & b in equation (i)

$9h^2 + 9k^2 = 9$

$\Rightarrow$ $x^2 + y^2 = 1$

10. $p = \left|\dfrac{(-g+g)\cos\theta + (-f+f)\sin\theta - k}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$

$= \sqrt{g^2 + f^2 - c}$

$\Rightarrow$ $g^2 + f^2 = c + k^2$

12. If three lines are given such that no two of them are parallel and they are not concurrent then a definite triangle is formed by them. There are four circles which touch sides of a triangle (3-excircles and 1-incircle).

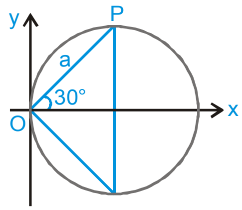
13. Coordinates of point P will be (acos30°, asin30°) P lies on the circle,

$\Rightarrow$ $a^2\cos^2 30° + a^2\sin^2 30°$

$= 2a\cos 30°$

$\Rightarrow$ $a^2 = 2a\cos 30°$

$\Rightarrow$ $a = \sqrt{3}$



Area $= \dfrac{\sqrt{3}a^2}{4} = \dfrac{3\sqrt{3}}{4}$

17. Let the centre of circle be $(-g, -f)$

Using condition of orthogonality :

$2(g_1g_2 + f_1f_2) = C_1 + C_2$

$2(2g - 3f) = 9 + C$ ...... **(i)**

$2\left(-\dfrac{5g}{2} + 2f\right) = -2 + C$ ...... **(ii)**

Subtract **(ii)** from **(i)**

$2\left[\dfrac{9g}{2} - 5f\right] = 11$ $\Rightarrow$ $9g - 10f = 11$

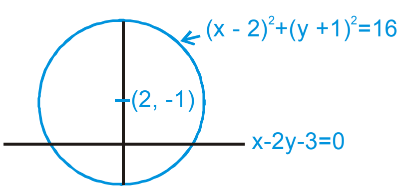
replacing $(-g)$ by h & $(-f)$ by k.

$-9h + 10k = 11$

$\Rightarrow$ $9x - 10y + 11 = 0$

19. Required diameter is $\perp$ to given line.

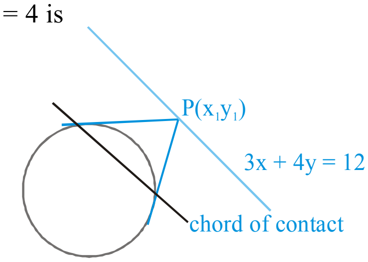
Hence $y + 1 = -2(x - 2)$



$\Rightarrow$ $2x + y - 3 = 0$

27. Let $P(x_1, y_1)$ be a point on the line $3x + 4y = 12$.

Equation of variable chord of contact of $P(x_1, y_1)$ wrt circle $x^2 + y^2 = 4$ is



$xx_1 + yy_1 - 4 = 0$ .....(1)

Also $3x_1 + 4y_1 - 12 = 0$

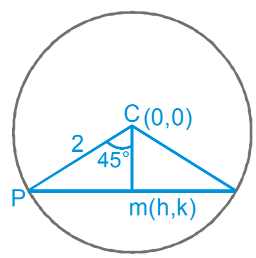
$x_1 + \dfrac{4}{3}y_1 - 4 = 0$ ...... **(ii)**

Comparing **(i)** & **(ii)**

$x = 1,\ y = \dfrac{4}{3}$

$\therefore$ variable chord of contact always passes through $(1, \dfrac{4}{3})$

28. $\cos 45° = \dfrac{cm}{cp} = \dfrac{\sqrt{h^2 + k^2}}{2}$



Hence locus $x^2 + y^2 = 2$

33. $S_1 - S_2 = 0$ $\Rightarrow$ $7x - 8y + 16 = 0$

$S_2 - S_3 = 0$ $\Rightarrow$ $2x - 4y + 20 = 0$

$S_3 - S_1 = 0$ $\Rightarrow$ $9x - 12y + 36 = 0$

On solving centre (8, 9)

Length of tangent

$= \sqrt{S_1} = \sqrt{64 + 81 - 16 + 27 - 7} = \sqrt{149}$

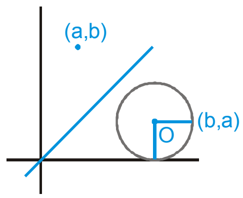
$= (x - 8)^2 + (y - 9)^2 = 149$

$= x^2 + y^2 - 16x - 18y - 4 = 0$

34. Reflection of point (a, b) on the line $y = x$ will be (b, a)

$(x - b)^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2bx - 2ay + b^2 = 0.$



36. $y^2 - 2xy + 4x - 2y = 0$

$y(y - 2x) - 2(y - 2x) = 0$

$\Rightarrow \quad y = 2$ and $y = 2x$ are the normals.

Now point of intersection of normals will give the centre of the circle i.e. (1, 2)

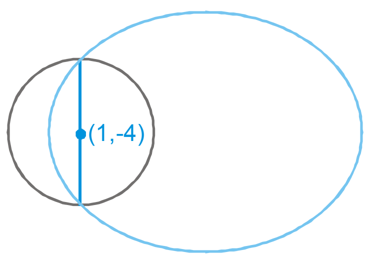
Radius of circle will be $\sqrt{2}$

$\therefore$ equation of circle : $(x - 1)^2 + (y - 2)^2 = 2$

37. Common chord of given circle

$6x + 4y + (p + q) = 0$

This is diameter of $x^2 + y^2 - 2x + 8y - q = 0$



centre (1, –4)

$6 - 16 + (p + q) = 0 \quad \Rightarrow \quad p + q = 10$

38. $C_1C_2 = r_1 \pm r_2$

$\Rightarrow \quad (g_1 - g_2)^2 + (f_1 - f_2)^2 = \left(\sqrt{g_1^2 + f_1^2} \pm \sqrt{g_2^2 + f_2^2}\right)^2$

$\Rightarrow \quad -2g_1g_2 - 2f_1f_2 = \pm 2\sqrt{g_1^2 + f_1^2}\,.\sqrt{g_2^2 + f_2^2}$

$\Rightarrow \quad g_1f_2 - g_2f_1 = 0$

$\Rightarrow \quad \dfrac{g_1}{g_2} = \dfrac{f_1}{f_2}$

EXERCISE - 2

Part # I : Multiple Choice

2. Consider $a = \cos\theta$, $b = \sin\theta$

$m = \cos\phi$, $n = \sin\phi$

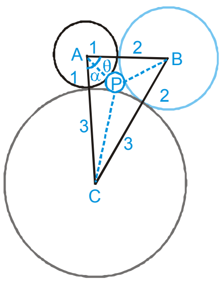
Now, $am \pm bn = \cos\theta\cos\phi \pm \sin\theta\sin\phi$

$am \pm bn = \cos(\theta \mp \phi)$

$\therefore |am \pm bn| \le 1$

6. Δ ABC is right angle

Applying cosine rule in ΔPAB



$$\cos\theta = \frac{3^2 + (1 + r)^2 - (2 + r)^2}{2.3(1 + r)}$$

$$= \frac{3 - r}{3(1 + r)}$$

Again applying cosine rule in ΔPAC

$$\cos\alpha = \frac{(1 + r)^2 + 4^2 - (3 + r)^2}{2.4(1 + r)} = \frac{2 - r}{2(1 + r)}$$

$\because \alpha + \theta = 90°$

$\alpha = 90° - \theta \quad \Rightarrow \quad \cos\alpha = \sin\theta$

$$\left(\frac{3 - r}{3(r + 1)}\right)^2 + \left(\frac{2 - r}{2(r + 1)}\right)^2 = 1$$

7. Let point on line be

$(h, 4 - 2h)$ (chord of contact)

$hx + y(4 - 2h) = 1$

$h(x - 2y) + 4y - 1 = 0$ Point $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$

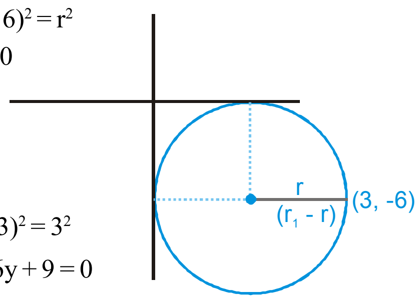
12. Now

$(r - 3)^2 + (-r + 6)^2 = r^2$

$r^2 - 18r + 45 = 0$

$\Rightarrow \quad r = 3, 15$

Hence circle



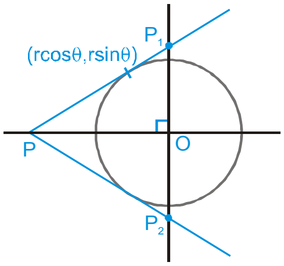
$(x - 3)^2 + (y + 3)^2 = 3^2$

$x^2 + y^2 - 6x + 6y + 9 = 0$

$(x - 15)^2 + (y + 15)^2 = (15)^2$

$\Rightarrow \quad x^2 + y^2 - 30x + 30y + 225 = 0$

13.



Were $r = 5\sqrt{2}$

Equation of PP_1 : $x\cos\theta + y\sin\theta = r$

point P will be : $(r\sec\theta, 0)$

point P_1 will be : $(0, r\operatorname{cosec}\theta)$

Area of $\Delta\ PP_1P_2$ will be $\left(\frac{1}{2} \times r\sec\theta \times r\operatorname{cosec}\theta\right) \times 2$

$$\Delta PP_1P_2 = \frac{2r^2}{\sin 2\theta}$$

Area of ΔPP_1P_2 will be minimum if $\sin 2\theta = 1$ or -1.

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$\Rightarrow \quad \theta = \frac{\pi}{4},\ \theta = \frac{3\pi}{4}$

$\Rightarrow$ P : $(5\sqrt{2} \times \sqrt{2}, 0)$ or $(5\sqrt{2}(-\sqrt{2}), 0)$

$(10, 0)$ or $(-10, 0)$

14. $\left|\frac{4C + 3C - 12}{5}\right| = C \Rightarrow C = 1, 6$

17. Let equation of required circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

it passes through $(1, -2)$ & $(3, -4)$

$2g - 4f + c = -5$

$6g - 8f + c = -25$

$4g - 8f + 2c = -10$

$6g - 8f + c = -25$

$-2g + c = 15$

circle touches x-axis $g^2 = c$

$\Rightarrow \quad g^2 - 2g - 15 = 0$

$g = 5, -3$

$g = 5, \quad c = 25, \quad f = 10$

$\Rightarrow \quad x^2 + y^2 + 10x + 20y + 25 = 0$

$g = -3, \quad c = 9, \quad f = 2$

$\Rightarrow \quad x^2 + y^2 - 6x + 4y + 9 = 0$

Part # II : Assertion & Reason

1. $x^2 + y^2 + 2x + 2y - 2 = 0$

$(x + 1)^2 + (y + 1)^2 = 4$

Director circle of the above circle is -

$(x + 1)^2 + (y + 1)^2 = 8$

$x^2 + y^2 + 2x + 2y - 6 = 0$

$\therefore$ Tangents drawn from any point on the second circle to the first circle are perpendicular.

Hence, statement-I is true and statement-II explains it.

4. **Statement-I** There is exactly one circle whose centre is the radical centre and the radius equal to the length of tangent drawn from the radical centre to any of the given circles.

Statement-II is True But does not explain Statement-I

6. $\frac{(AK)}{(OA)} = \cos\theta = \frac{AB}{AK}$

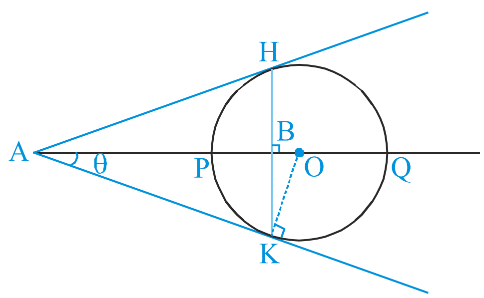
$\Rightarrow \quad (AK)^2 = (AB)(OA) = (AP)(AQ)$

$[AK^2 = AP \cdot AQ$ using power of point A]

Also $\quad OA = \frac{AP + AQ}{2}$

$[AQ - AO = r = AO - AP \Rightarrow 2AO = AQ + QP]$

$\Rightarrow \quad (AP)(AQ) = AB\left(\frac{AP + AQ}{2}\right)$



$\Rightarrow \quad AB = \frac{2(AP)(AQ)}{(AP + AQ)}$

7. Equation of director's circle is $(x - 3)^2 + (y + 4)^2 = 200$ and point $(13, 6)$ satisfies the given circle $(x - 3)^2 + (y + 4)^2 = 100$

8. Centre $(-2, -6)$. Substituting in L

$-2(k + 7) + 6(k - 1) - 4(k - 5)$

$= (-2k + 6k - 4k) - 14 - 6 + 20 = 0$

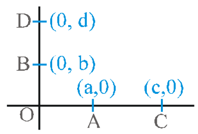
Hence every member of L passing through the centre of the circle $\Rightarrow$ cuts it at 90°.

Hence S-1 is true and S-2 is false.

11. Statement-1 is true and statement-2 is false as radius

$$= \frac{1}{2}\sqrt{\alpha^2+\beta^2}$$

12. If $OA \cdot OC = OB \cdot OD$ (Power of point) then points are concylic

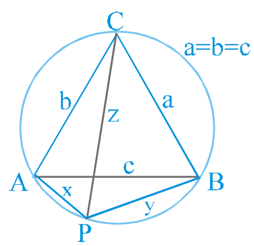


$\therefore$ $a \cdot c = bd$ (true)

13. Using Tolemy's theorem for a cyclic quadrilateral

$(z)(AB) = ax + by$

$z \cdot c = ax + by$



but $a = b = c$

hence $x + y = z$ is true always

$\Rightarrow$ S-1 is false and S-2 is true

EXERCISE - 3

Part # I : Matrix Match Type

2. (A) $S_1 - S_2 = 0$ is the required common chord i.e $2x=a$

Make homogeneous, we get $x^2 + y^2 - 8.4\dfrac{x^2}{a^2} = 0$

As pair of lines substending angle of 90° at origin

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore$ $a = \pm 4$

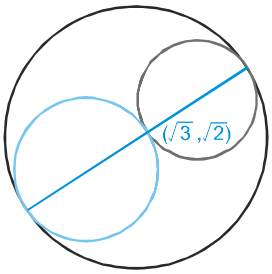
(B) $y = 22\sqrt{3}\,(x-1)$ passes through centre (1, 0) of circle

(C) Three lines are parallel

(D) $2(r_1 + r_2) = 4$

$r_1 + r_2 = 2$

$\dfrac{r_1 + r_2}{2} = 1$



5. (A) $x^2 + k^2x^2 - 20kx + 90 = 0$

$x^2(1+k^2) - 20kx + 90 = 0$

$D \le 0$

$400k^2 - 4 \times 90(1+k^2) \le 0$

$10k^2 - 9 - 9k^2 \le 0$

$k^2 - 9 \le 0 \quad \Rightarrow \quad k \in [-3, 3]$

(B) $2\left(\dfrac{p}{2}\times 5 + \dfrac{p}{2}\times p\right) = -6 \Rightarrow -5p + p^2 + 6 = 0$

$\Rightarrow p^2 - 5p + 6 = 0 \Rightarrow p = 2$ or 3 **Ans.**

(C) $r_1^{\,2} = \lambda^2 - 4 \ge 0$

$\lambda \in (-\infty, -2] \cup [2, \infty)$ ....(1)

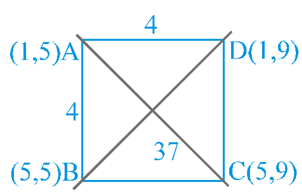
$r_2^{\,2} = 4\lambda^2 - 8 \ge 0$

$\lambda^2 - 2 \ge 0$

$\lambda \in (-\infty, -\sqrt{2}\,] \cup [\,\sqrt{2}, \infty)$ ....(2)

$(1) \cap (2)$ is $\lambda \in (-\infty, -2] \cup [2, \infty)$ **Ans.**

(D)



Ans. {1, 5}

Part # II : Comprehension

Comprehension-1

1. Let P be (h, k)

PA = nPB

$(h + 3)^2 + k^2 = n^2\,[(h - 3)^2 + k^2]$

$\therefore$ locus of P(h, k) is -

$x^2 + 6x + 9 + y^2 = n^2\,[x^2 - 6x + 9 + y^2]$

$x^2(1 - n^2) + y^2(1 - n^2) + 6x(1 + n^2) + 9(1 - n^2) = 0$

$$x^2 + y^2 + 6\,\frac{(1+n^2)}{1-n^2}\,x + 9 = 0 \quad \{\because\ n \neq 1\}$$

$\therefore$ Locus is a circle.

2. PA = PB when n = 1

$(h + 3)^2 + k^2 = (h - 3)^2 + k^2$

$h^2 + 6h + 9 + k^2 = h^2 - 6h + 9 + k^2$

$\therefore$ locus of P(h, k) is $x = 0$ $\therefore$ a straight line.

3. For $0 < n < 1$

locus is $(1 - n^2)(x^2 + y^2) + 6x(1 + n^2) + 9(1 - n^2) = 0$

putting A (–3, 0) in the above equation

$9(1 - n^2) - 18\,(1 + n^2) + 9(1 - n^2) = -36n^2 < 0$

$\therefore$ A lies inside the circle.

Similarly for B (3, 0)

$9(1 - n^2) + 18\,(1 + n^2) + 9(1 - n^2) = 36 > 0$

$\therefore$ B lies outside the circle.

4. for $n > 1$, locus is -

$(n^2 - 1)\,(x^2 + y^2) - 6x(1 + n^2) + 9(n^2 - 1) = 0$

putting A (– 3, 0) we get

$9(n^2 - 1) + 18(1 + n^2) + 9(n^2 - 1) = 36\,n^2 > 0$

& putting B(3, 0) we get

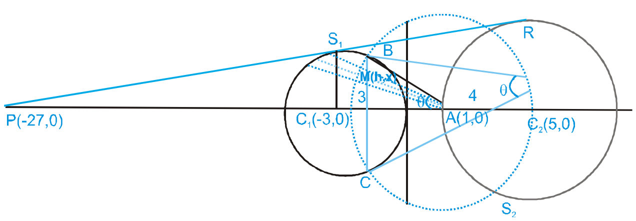
$9(n^2 - 1) - 18(1 + n^2) + 9(n^2 - 1) = -36 < 0$

$\therefore$ A lies out side and B lies inside the circle.

5. We have seen whenever locus of P is a circle it never passes through A and B.

Comprehension # 2

1. ΔPQC_1 and ΔPRC_2 are similar



$$\frac{\text{Area of } \Delta PQC_1}{\text{Area of } \Delta PRC_2} = \frac{r_1^2}{r_2^2} = \frac{9}{16}$$

2. Let mid point m(h, k). Now equation of chord

$T = S_1$

$hx + ky + 3(x + h) = h^2 + k^2 + 6h$

it passes through (1, 0)

$h + 3(1 + h) = h^2 + k^2 + 6h$

locus $x^2 + y^2 + 2x - 3 = 0$

But clear from Geometry it will be arc of BC

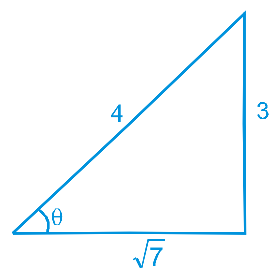
3. Common chord of S_1 & answer of 7

$4x + 3 = 0 \quad \Rightarrow \quad x = -3/4$

$$\text{at } x = -3/4 \quad \Rightarrow \quad \left(-\frac{3}{4} + 3\right)^2 + y^2 = 9$$

$$\Rightarrow \quad y^2 = 9 - \frac{81}{16}$$

$$y^2 = \frac{63}{16} \quad \Rightarrow \quad y = \pm\frac{3\sqrt{7}}{4}$$



$$\text{Hence } \tan\theta = \frac{\frac{3\sqrt{7}}{4}}{(1 + 3/4)} = \frac{3\sqrt{7}}{7} \Rightarrow \tan\theta = \frac{3}{\sqrt{7}}$$

Comprehension # 3

1. As lines are perpendicular

$\therefore \quad a - 2 = 0$

$\Rightarrow \quad a = 2 \quad$ (coefficient of x^2 + coefficient of $y^2 = 0$)

using $\Delta = 0$

$\Rightarrow \quad c = -3 \quad (D \equiv abc + 2fgh - af^2 - bg^2 - ch^2)$

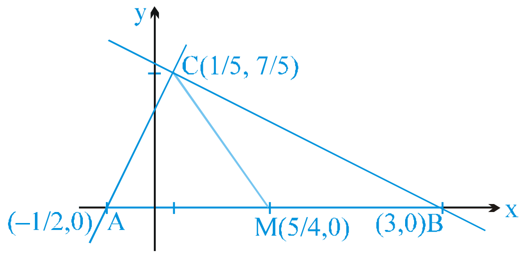
hence the two lines are

$x + 2y - 3 = 0$ and $2x - y + 1 = 0$

$$\left.\begin{array}{ll} x\text{-intercepts} & x_1 = 3;\ \ x_2 = -1/2 \\ & \\ y\text{-intercepts} & y_1 = 3/2;\ \ y_2 = 1 \end{array}\right] \Rightarrow$$

$x_1 + x_2 + y_1 + y_2 = 5$ **Ans.**

2. $(CM)^2 = \left(\frac{5}{4}-\frac{1}{5}\right)^2 + \frac{49}{25} = \left(\frac{25-4}{20}\right)^2 + \frac{49}{25}$



$= \frac{441}{400} + \frac{49}{25}$

$= \frac{441+784}{400} = \frac{1225}{400} = \frac{49}{16}$

$\Rightarrow$ $CM = \frac{7}{4}$ **Ans.**

3. Circumcircle of ABC

$\left(x+\frac{1}{2}\right)(x-3)+y^2=0$

$\Rightarrow$ $(2x+1)(x-3)+2y^2=0$

$\Rightarrow$ $2(x^2+y^2)-5x-3=0$

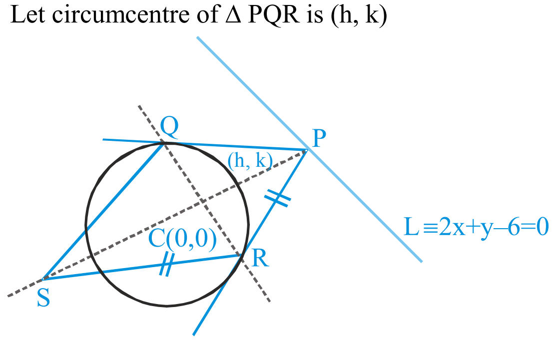
$\Rightarrow$ $x^2+y^2-\frac{5}{2}x-\frac{3}{2}=0$ ....**(1)**

given $x^2+y^2-4y+k=0$ which is orthogonal to (1)
using the condition of orthogonality

we get, $0+0=k-\frac{3}{2}$ $\Rightarrow$ $k=\frac{3}{2}$

Comprehension # 5

1. Parallelogram PQSR is a rhombus
Let circumcentre of Δ PQR is (h, k)



which is the middle point of CP

$\therefore$ P becomes (2h, 2k) which satisfies the line $2x+y-6=0$

$\therefore$ $2(2h)+2k-6=0$

$\therefore$ locus is $2x+y-3=0$

2. If P(6, 8) then

Area (Δ PQR) = Area (Δ QRS)

$\therefore$ Area (Δ PQR) $= \frac{RL^3}{R^2+L^2}$

$= \frac{2.64.6\sqrt{6}}{100} = \frac{192\sqrt{6}}{25}$ $\{R = 2, L = 4\sqrt{6}\}$

3. If P(3, 4) then
equation of chord of contact is

$3x+4y-4=0$ ...... **(i)**

Straight line perpendicular to (1) & passing through centre of the circle is -

$4x-3y=0$ ...... **(ii)**

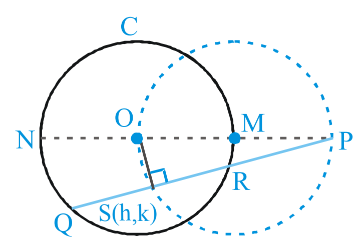
point of intersection of (1) & (2) is $\left(\frac{12}{25},\frac{16}{25}\right)$

which is the middle point of PS

$\therefore$ coordinate of S are $\left(\frac{-51}{25},\frac{-68}{25}\right)$

Comprehension # 6

1. Locus of S is a part of circle with OP as diameter passing inside the circle 'C'



2. $(PR)(PQ) = PT^2 = (PN)(PM) = (d-r)(d+r) = d^2-r^2$

$= (PS-SR)(PS+SQ) = PS^2 - SQ^2$

$(\therefore$ $SQ = SR)$

$= PS^2 - (SQ)(SR)$

$\therefore$ $(PQ)(PR) \neq (PS)^2$

3. Using Ptolemy's theorem,

$(YD)(XZ) = (XY)(ZD) + (YZ)(XD)$

$= XZ(ZD + XD)$

$\{\because (XY = YZ = ZX)\}$

$\Rightarrow$ $\beta = \gamma + \alpha$ $\Rightarrow$ **(A)**

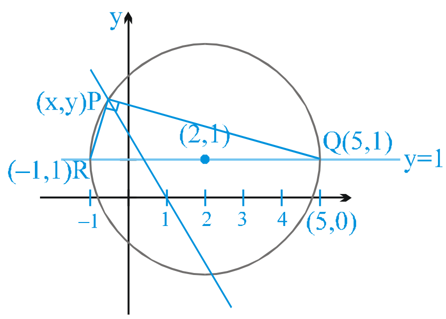
Comprehension # 7

1. refer figure

2. when $y = 1$

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0$



$x = -1$ or $x = 5$

$(x + 1)^2 + (y - 1)^2 + (x - 5)^2 + (y - 1)^2 = (QR)^2 = 36$ Ans.

3. equation of director circle is

$$(x-2)^2 + (y-1)^2 = \left(3\sqrt{2}\right)^2 = 18$$

$$\text{Area} = \pi\left[r_1^2 - r_2^2\right] = \pi[18 - 9] = 9\pi$$

EXERCISE - 4
Subjective Type

5. Let P be (x_1, y_1)

Coordinates of any point on the curve at a distance r from P are $(x_1 + r\cos\theta, y_1 + r\sin\theta)$

$$a(x_1 + r\cos\theta)^2 + 2h(x_1 + r\cos\theta)(y_1 + r\sin\theta) + b(y_1 + r\sin\theta)^2 = 1$$

$$\Rightarrow r^2(a\cos^2\theta + 2h\sin\theta\cos\theta + b\sin^2\theta) + 2r(ax_1\cos\theta + hx_1\sin\theta + hy_1\cos\theta + by_1\sin\theta) + ax_1^2 + 2hx_1y_1 + by_1^2 - 1 = 0$$

which is quadratic in 'r'

$$\therefore\ r_1 r_2 = \frac{ax_1^2 + 2hx_1y_1 + by_1^2 - 1}{a\cos^2\theta + h\sin 2\theta + b\sin^2\theta}$$

$$PQ\,.\,PR = \frac{ax_1^2 + 2hx_1y_1 + by_1^2 - 1}{a + (b-a)\sin^2\theta + h\sin 2\theta}$$

PQ . PR will be independent of θ if

$b - a = 0$ & $h = 0$

$\Rightarrow a = b$ & $h = 0$

Hence, in this condition curve becomes a circle.

7. Let mid-point be (h, k)

$hx + ky = h^2 + k^2$

subtend right angle

$$x^2 - 2(x + y)\left(\frac{hx + ky}{h^2 + k^2}\right) = 0$$

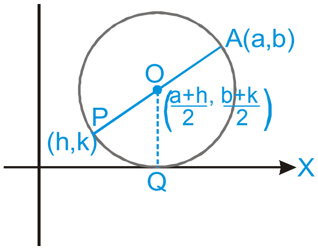
$(h^2 + k^2)x^2 - 2(x + y)(hx + ky) = 0$

As angle 90°, Coefficient of x^2 + Coefficient of $y^2 = 0$

$h^2 + k^2 - 2h - 2k = 0$

Locus $x^2 + y^2 - 2x - 2y = 0$

8.



AP = 2.OQ

$$\sqrt{(h-a)^2+(k-b)^2} = 2 \cdot \frac{b+k}{2}$$

$(h-a)^2 = (k+b)^2 - (k-b)^2$

$(h-a)^2 = 4bk$

$\therefore$ locus of P(h, k) is $(x-a)^2 = 4by$

13. Equation of any curve passing through the four points of intersects of $S = 0$ and $S' = 0$ is $S + \lambda S' = 0$. For this to be a circle, we must have coefficient of x^2 = coefficient of y^2 & coefficient of $xy = 0$.

$\Rightarrow$ $a + \lambda a' = b + \lambda b'$

$a - b = -\lambda(a' - b')$ ...... **(i)**

and $2h + \lambda 2h' = 0$ $\Rightarrow$ $\lambda = -\dfrac{h}{\lambda'}$ ...... **(ii)**

$\Rightarrow$ from (i) and (ii)

$$a - b = \frac{h}{h'}(a' - b') \quad \text{or} \quad \frac{a-b}{h} = \frac{a'-b'}{h'}$$

14. The parametric form of OP is $\dfrac{x-0}{\cos 45°} = \dfrac{y-0}{\sin 45°}$

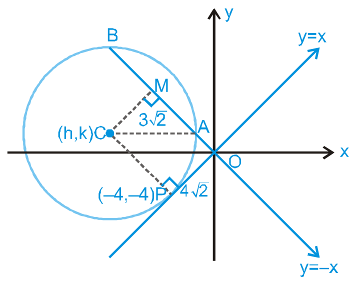
Since, $OP = 4\sqrt{2}$

So, the coordinates of P are given by

$$\frac{x-0}{\cos 45°} = \frac{y-0}{\sin 45°} = -4\sqrt{2}$$

So, P(–4, –4)

Let, C(h, k) be the centre of circle and r be its radius, Now, CP ⊥ OP



$\Rightarrow$ $\dfrac{k+4}{h+4}.(1) = -1$

$\Rightarrow$ $h + k = -8$ ...... **(i)**

also, $CP^2 = (h+4)^2 + (k+4)^2$

$\Rightarrow$ $(h+4)^2 + (k+4)^2 = r^2$ ...... **(ii)**

In ΔACM, we have $AC^2 = (3\sqrt{2})^2 + \left(\dfrac{h+k}{\sqrt{2}}\right)^2$

$\Rightarrow$ $r^2 = 18 + 32$

$\Rightarrow$ $r = 5\sqrt{2}$ ...... **(iii)**

also, CP = r

$\Rightarrow$ $\left|\dfrac{h-k}{\sqrt{2}}\right| = r$ $\Rightarrow$ $h - k = \pm 10$ ...... **(iv)**

From **(i)** and **(iv)**, we get

(h = –9, k = 1) or (h = 1, k = –9)

Thus, the equation of the circles are

$$(x+9)^2 + (y-1)^2 = (5\sqrt{2})^2$$

and $(x-1)^2 + (y+9)^2 = (5\sqrt{2})^2$

or $x^2 + y^2 + 18x - 2y + 32 = 0$

and $x^2 + y^2 - 2x + 18y + 32 = 0$

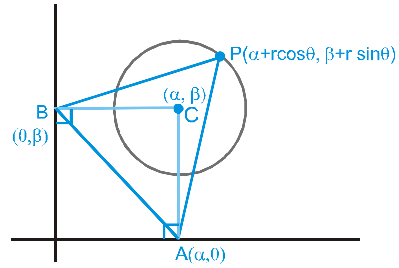
Clearly, (–10, 2) lies interior of

$x^2 + y^2 + 18x - 2y + 32 = 0$

Hence, the required equation of circle is

$x^2 + y^2 + 18x - 2y + 32 = 0$

16. Let the equation of the circle be $(x-\alpha)^2 + (y-\beta)^2 = r^2$



coordinates of P are

$\therefore$ $(\alpha + r\cos\theta, \beta + r\sin\theta)$

Let centroid of Δ PAB be (h, k)

$3h = \alpha + \alpha + r\cos\theta$ $\Rightarrow$ $r\cos\theta = 3h - 2\alpha$

$3k = \beta + \beta + r\sin\theta$ $\Rightarrow$ $r\sin\theta = 3k - 2\beta$

squaring and adding

$(3h - 2\alpha)^2 + (3k - 2\beta)^2 = r^2$

$\therefore$ locus of (h, k) is

$$\left(x - \frac{2\alpha}{3}\right)^2 + \left(y - \frac{2\beta}{3}\right)^2 = \frac{r^2}{9}$$

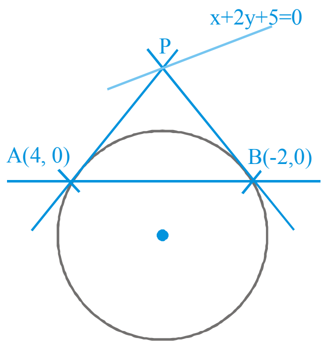
17. $x^2 + y^2 - 2x - 8 - 2\lambda y = 0 \Rightarrow S + \lambda L = 0$

$S : x^2 + y^2 - 2x - 8 = 0$

$L : y = 0$

Points of intersection of S = 0 & L = 0 are -

(4, 0) & (– 2, 0)



Let P be (h, k)

equation of chord of contact of P wrt given circle is

$hx + ky - 1\,(x + h) - \lambda(y + k) - 8 = 0$

$(h - 1)x + (k - \lambda)y - h - \lambda k - 8 = 0$

comparing with the line y = 0.

$$\frac{h-1}{0} = \frac{k-\lambda}{1} = \frac{-h-\lambda k-8}{0}$$

$h - 1 = 0 \qquad \Rightarrow \quad h = 1$

putting h = 1 in the line x + 2y + 5 = 0

$1 + 2k + 5 = 0 \qquad \Rightarrow \quad k = -3$

$-h - \lambda k - 8 = 0$

$-1 + 3\lambda - 8 = 0 \quad \Rightarrow \quad \lambda = 3$

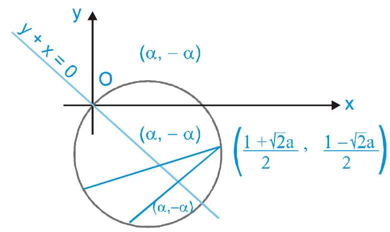
$\therefore$ Equation of the required circle is -

$x^2 + y^2 - 2x - 6y - 8 = 0$

19. $2x^2 + 2y^2 - (1 + \sqrt{2}a)x - (1 - \sqrt{2}a)y = 0$

$$\Rightarrow \quad x^2 + y^2 - \left(\frac{1+\sqrt{2}a}{2}\right)x - \left(\frac{1-\sqrt{2}a}{2}\right)y = 0$$

Since, y + x = 0 bisects two chords of this circle, mid-points of the chords must be of the form (α, –α)



Equation of the chord having (α, –α) as mid-points is

$T = S_1$

$$\Rightarrow \quad x\alpha + y(-\alpha) - \left(\frac{1+\sqrt{2}a}{4}\right)(x+\alpha) - \left(\frac{1-\sqrt{2}a}{4}\right)(y-\alpha)$$

$$= \alpha^2 + (-\alpha)^2 - \left(\frac{1+\sqrt{2}a}{2}\right)\alpha - \left(\frac{1-\sqrt{2}a}{2}\right)(-\alpha)$$

$$\Rightarrow \quad 4x\alpha - 4y\alpha - (1+\sqrt{2}a)x - (1+\sqrt{2}a)\alpha$$

$$-(1-\sqrt{2}a)y + (1-\sqrt{2}a)\alpha$$

$$= 4\alpha^2 + 4\alpha^2 - (1+\sqrt{2}a).2\alpha + (1-\sqrt{2}a).2\alpha$$

$$\Rightarrow \quad 4\alpha x - 4\alpha y - (1+\sqrt{2}a)x - (1-\sqrt{2}a)y$$

$$= 8\alpha^2 - (1+\sqrt{2}a)\alpha + (1-\sqrt{2}a)\alpha$$

But this chord will pass through the point

$$\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$$

$$\therefore \quad 4\alpha\left(\frac{1+\sqrt{2}a}{2}\right) - 4\alpha\left(\frac{1-\sqrt{2}a}{2}\right)$$

$$-\frac{(1+\sqrt{2}a)(1+\sqrt{2}a)}{2} - \frac{(1-\sqrt{2}a)(1-\sqrt{2}a)}{2}$$

$$= 8\alpha^2 - 2\sqrt{2}a\alpha$$

$$\Rightarrow \quad 2\alpha[(1+\sqrt{2}a - 1 + \sqrt{2}a)] = 8\alpha^2 - 2\sqrt{2}a\alpha$$

$$\Rightarrow \quad 4\sqrt{2}a\alpha - \frac{1}{2}[2 + 2(\sqrt{2}a)^2] = 8\alpha^2 - 2\sqrt{2}a\alpha$$

$[\because\ (a+b)^2 + (a-b)^2 = 2a^2 + 2b^2]$

$$\Rightarrow \quad 8\alpha^2 - 6\sqrt{2}a\alpha + 1 + 2a^2 = 0$$

But this quadratic equation will have two distinct roots

if $(6\sqrt{2}a)^2 - 4(8)(1 + 2a^2) > 0$

$\Rightarrow \quad 72a^2 - 32(1 + 2a^2) > 0$

$\Rightarrow \quad 72a^2 - 32 - 64a^2 > 0 \Rightarrow 8a^2 - 32 > 0$

$\Rightarrow \quad a^2 > 4$

$\Rightarrow \quad a < -2 \cup a > 2$

Therefore, $a \in (-\infty, -2) \cup (2, \infty)$.

20. The given circles are

$S_1 = x^2 + y^2 + 4x - 6y + 9 = 0$

$S_2 = x^2 + y^2 - 5x + 4y + 2 = 0$

& variable circle is

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

Now, S & S_1 are orthogonal

$\therefore$ $4g - 3f = c + 9$ ......(i)

S & S_2 are also orthogonal

$\therefore$ $-5g + 4f = c + 2$ ......(ii)

(i) – (ii)

$9g - 10f = 7$

$\therefore$ locus of $(-g, -f)$ is

$-9x + 10y = 7$

$9x - 10y = -7$

$9x - 10y + 7 = 0$

which is the radial axis of the two given circles.

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Length of tangent

$= \sqrt{3^2 + (-4)^2 - 4(3) - 6(-4) + 3} = \sqrt{40}$

$\therefore$ Square of length of tangent = 40

3. When two circles intersect each other, then

Difference between their radii < Distance between centers

$\Rightarrow$ $r - 3 < 5$

$\Rightarrow$ $r < 8$ ...... (i)

Sum of their radii > Distance between centres ..... (ii)

$\Rightarrow$ $r + 3 > 5$ $\Rightarrow$ $r > 2$

Hence by (i) and (ii) $2 < r < 8$

4. Centre of circle = Point of intersection of diameters

$= (1, -1)$

Now area = 154

$\Rightarrow$ $\pi r^2 = 154$ $\Rightarrow$ $r = 7$

Hence the equation of required circle is

$(x - 1)^2 + (y + 1)^2 = 7^2$

$\Rightarrow$ $x^2 + y^2 - 2x + 2y = 47$

5. Let the variable circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ..... (i)

Circle (i) cuts circle $x^2 + y^2 - 4 = 0$ orthogonally

$\Rightarrow$ $2g.0 + 2f.0 = c - 4 \Rightarrow c = 4$

Since circle (i) passes through (a, b)

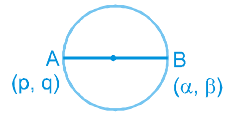
$\therefore$ $a^2 + b^2 + 2ga + 2fb + 4 = 0$

$\therefore$ Locus of centre $(-g, -f)$ is

$2ax + 2by - (a^2 + b^2 + 4) = 0$

6. Equation of circle having AB as diameter is

$(x - p)(x - \alpha) + (y - q)(y - \beta) = 0$



or $x^2 + y^2 - (p + \alpha)x - (q + \beta)y + p\alpha + q\beta = 0$ ..... (i)

as it touches x-axis putting y = 0,

we get $x^2 - (p + \alpha)x + p\alpha + q\beta = 0$ ..... (ii)

Since, circle (i) touches x-axis

Discriminant of equation (ii) = 0

$\Rightarrow$ $(p + \alpha)^2 = 4(p\alpha + q\beta)$

$\Rightarrow \quad (p-\alpha)^2 = 4q\beta$

$\therefore$ Locus of $B(\alpha, \beta)$ is $(p-x)^2 = 4qy$

or $\quad (x-p)^2 = 4qy$

7. According to question two diameters of the circle are

$2x + 3y + 1 = 0$ and $3x - y + 4 = 0$

Solving, we get $x = 1, y = -1$

$\therefore$ Centre of the circle is $(1, -1)$

Given $2\pi r = 10\pi \quad \Rightarrow \quad r = 5$

$\therefore$ Required circle is $(x-1)^2 + (y+1)^2 = 5^2$

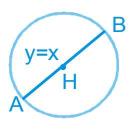
or $\quad x^2 + y^2 - 2x + 2y - 23 = 0$

8. Given, circle is $x^2 + y^2 - 2x = 0$ ...... **(i)**

and line is $y = x$ ...... **(ii)**

Putting $y = x$ in **(i)**,

We get $2x^2 - 2x = 0 \quad \Rightarrow \quad x = 0, 1$



From **(i)**, $y = 0, 1$

Let $A = (0, 0)$, $B = (1, 1)$

Equation of required circle is

$(x-0)(x-1) + (y-0)(y-1) = 0$

or $\quad x^2 + y^2 - x - y = 0$

9. Equation of line PQ (i.e. common chord) is

$5ax + (c-d)y + a + 1 = 0$ ...... **(i)**

Also given equation of line PQ is

$5x + by - a = 0$ ...... **(ii)**

Therefore $\dfrac{5a}{5} = \dfrac{c-d}{b} = \dfrac{a+1}{-a}$; As $\dfrac{a+1}{-a} = a$

$\Rightarrow \quad a^2 + a + 1 = 0$

Therefore no real value of a exists, (as $D < 0$)

10. Let centre $\equiv (h, k)$; As $C_1C_2 = r_1 + r_2$, (Given)

$\Rightarrow \quad \sqrt{(h-0)^2 + (k-3)^2} = |k+2|$

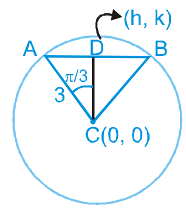
$\Rightarrow \quad h^2 = 5(2k-1)$

Hence locus, $x^2 = 5(2y-1)$, which is parabola

14. Let AB be the chord subtending angle $2\pi/3$ at the centre C of circle

Now, $\angle ACD = \pi/3$

Let the coordinates of midpoint D be (h, k)



In ΔACD, $\cos\dfrac{\pi}{3} = \dfrac{CD}{CA}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{\sqrt{h^2 + k^2}}{3}$

$\Rightarrow \quad x^2 + y^2 = \dfrac{9}{4}$, which is the required locus.

15. Equation of circle $(x-h)^2 + (y-k)^2 = k^2$

It is passing through $(-1, 1)$ then

$(-1-h)^2 + (1-k)^2 = k^2 \; h^2 + 2h - 2k + 2 = 0$

$D \geq 0 \qquad 2k - 1 \geq 0 \quad \Rightarrow \quad k \geq 1/2$

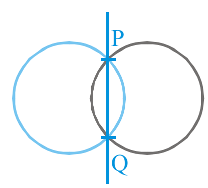
17. Let A, B, C are represented by the point (x, y)

$$\frac{\sqrt{(x-1)^2 + y^2}}{\sqrt{(x+1)^2 + y^2}} = \frac{1}{2}$$

$8x^2 + 8y^2 - 20x + 8 = 0$

Which is the circle which passes through the points A, B, C then circumcentre will be the centre of the circle $\left(\dfrac{5}{4}, 0\right)$.

18. $Eq^{n.}$ of line PQ



$x + 5y + 2p - 5 + p^2 = 0$

P, Q and (1, 1)

will not lie on a circle of (1, 1)

Lies on the line

$x + 5y + p^2 + 2p - 5 = 0$

$\Rightarrow \quad 1 + 5 + p^2 + 2p - 5 = 0$

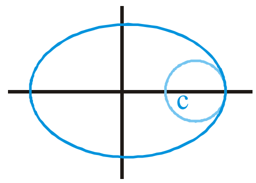
$p^2 + 2p + 1 = 0$

$\Rightarrow \quad p = -1$

Therefore their is a circle passing through P, Q and (1, 1) for all values of p.

Except $p = -1$.

21.



$$\left|\frac{a}{2}\right| = c - \left|\frac{a}{2}\right|$$

$|a| = C$

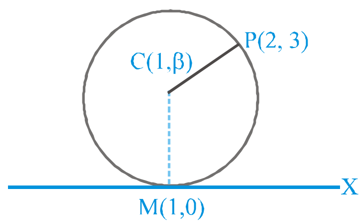
22. (1, 0) and (0, 1) will be ends of diameter

So equation of circle

$(x - 1)(x - 0) + (y - 0)(y - 1)$

$x^2 + y^2 - x - y = 0$

23.



Let center of the circle be $C(1, \beta)$

$\beta^2 = (2 - 1)^2 + (3 - \beta)^2$

$\Rightarrow \beta^2 = -6\beta + 10 + \beta^2$

$\Rightarrow \beta = \frac{5}{3}$

$\therefore r = \frac{5}{3}$

$\text{diameter} = \frac{10}{3}$

24. Let equation of circle be $(x - 3)^2 + (y + r)^2 = r^2$

$\because$ it passes through $(1, -2)$

$\Rightarrow r = 2$

$\Rightarrow$ circle is $(x - 3)^2 + (y + 2)^2 = 4$

$\Rightarrow (5, -2)$

Aliter

$(x - 3)^2 + y^2 + \lambda y = 0$ ....(1)

Putting $(1, -2)$ in (1)

$\Rightarrow \lambda = 4$

Required circle is

$x^2 + y^2 - 6x + 4y + 9 = 0$

point $(5, -2)$ satisfies the equation the equation

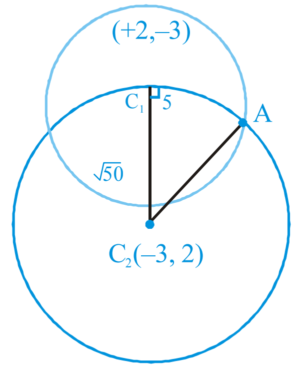
29. Eq. $x^2 + y^2 - 4x + 6y - 12 = 0$

C_1 ; $(2, -3)$, $r_1 = \sqrt{4+9+12} = 5$

$C_2 = (-3, 2)$

$C_1C_2 = \sqrt{5^2 + 5^2} = \sqrt{50}$

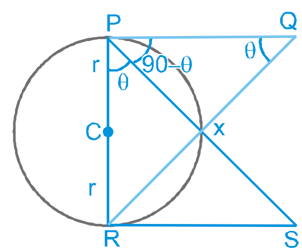
then, $C_2A = \sqrt{5^2 + (\sqrt{50})^2} = \sqrt{75} = 5\sqrt{3}$



Part # II : IIT-JEE ADVANCED

1. Let $\angle RPS = \theta$

$\angle XPQ = 90 - \theta$



$\therefore \angle PQX = \theta$ $(\because \angle PXQ = 90°)$

$\therefore \Delta PRS \sim \Delta QPR$ (AAA similarity)

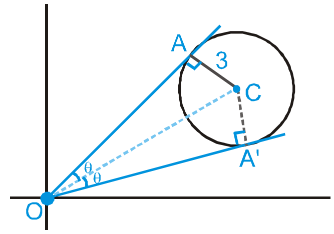
$\therefore \frac{PR}{QP} = \frac{RS}{PR}$

$\Rightarrow PR^2 = PQ.RS$

$\Rightarrow PR = \sqrt{PQ.RS}$

2. The equation $2x^2 - 3xy + y^2 = 0$ represents pair of tangents OA and OA'.

Let angle between these to tangents be 2θ.



Then $\tan 2\theta = \dfrac{2\sqrt{\left(\dfrac{-3}{2}\right)^2 - 2\times 1}}{2+1}$

$\left[\text{Using } \tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}\right]$

$\dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{1}{3}$

$\Rightarrow$ $\tan^2\theta + 6\tan\theta - 1 = 0$

$\tan\theta = \dfrac{-6 \pm \sqrt{36+4}}{2} = -3 \pm \sqrt{10}$

As θ is acute $\therefore$ $\tan\theta = \sqrt{10} - 3$

Now we know that line joining the point through which tangents are drawn to the centre bisects the angle between the tangents,

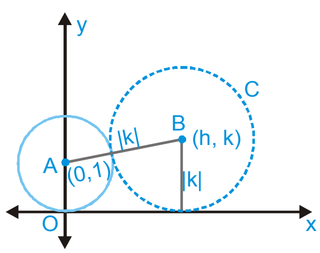
$\therefore$ $\angle AOC = \angle A'OC = \theta$

In ΔOAC $\tan\theta = \dfrac{3}{OA}$

$\Rightarrow$ $OA = \dfrac{3}{\sqrt{10}-3} \times \dfrac{\sqrt{10}+3}{\sqrt{10}+3}$

$\therefore$ $OA = 3(3 + \sqrt{10})$

9. Let the centre of circle C be (h, k). Then as this circle touches axis of x its radius = |k|



Also it touches the given circle $x^2 + (y-1)^2 = 1$, centre (0, 1) radius 1, externally

Therefore

The distance between centres = sum of radii

$\Rightarrow$ $\sqrt{(h-0)^2 + (k-1)^2} = 1 + |k|$

$\Rightarrow$ $h^2 + k^2 - 2k + 1 = (1 + |k|)^2$

$\Rightarrow$ $h^2 + k^2 - 2k + 1 = 1 + 2|k| + k^2$

$\Rightarrow$ $h^2 = 2k + 2|k|$

$\therefore$ Locus of (h, k) is, $x^2 = 2y + 2|y|$

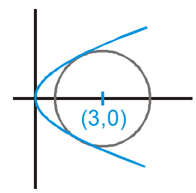
Now if $y > 0$, it becomes $x^2 = 4y$

and if $y \le 0$, it becomes $x = 0$

$\therefore$ Combining the two, the required locus is

$\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

12 $C_1 : y^2 = 4x$ $\qquad C_2 : x^2 + y^2 - 6x + 1 = 0$



$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0 \quad \Rightarrow \quad x = 1$

$y = \pm 2$

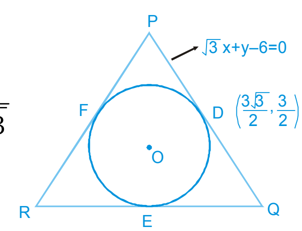
so the curves touches each other at two points (1, 2) & (1, –2)

13. Eq. of circle is $(x+3)^2 + (y-5)^2 = 4$

Distance between the given lines $= \dfrac{6}{\sqrt{13}} <$ radius

So S(**II**) is false & S(**I**) is true

14. (i) $m_{PQ} = -\sqrt{3}$



so slope of $OD = \dfrac{1}{\sqrt{3}}$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\therefore$ $\dfrac{x - \dfrac{3\sqrt{3}}{2}}{\dfrac{\sqrt{3}}{2}} = \dfrac{y - \dfrac{3}{2}}{\dfrac{1}{2}} = \pm 1$

$(2\sqrt{3}, 2)$ (not possible) & $(\sqrt{3}, 1)$

Hence circle is $(x - \sqrt{3})^2 + (y-1)^2 = 1$

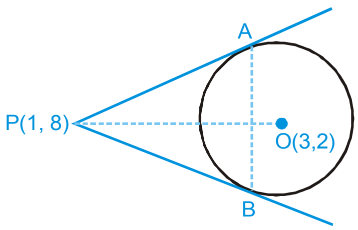
(ii) For point E $\dfrac{x-\sqrt{3}}{-\dfrac{\sqrt{3}}{2}} = \dfrac{y-1}{\dfrac{1}{2}} = 1$ $\quad \left[\therefore\ E\left(\dfrac{\sqrt{3}}{2}, \dfrac{3}{2}\right)\right]$

For point F $\dfrac{x-\sqrt{3}}{0} = \dfrac{y-1}{-1} = 1$ $\quad \left[\therefore\ F\left(\sqrt{3}, 0\right)\right]$

(iii) Equation of line RP $y = 0$

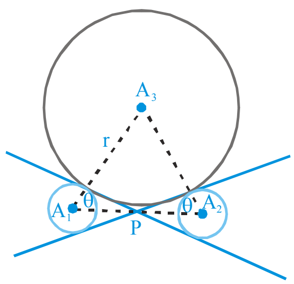
Equation of line QR $y - \frac{3}{2} = \sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right)$

$y = \sqrt{3}\,x$

15. 

The required circle is a circle described on OP as diameter.

16. (8)



In triangle $A_1A_2A_3$

$A_1A_3 = A_3A_2$

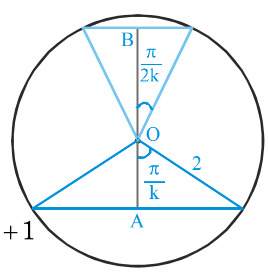
Let angle $A_3A_1A_2 = \theta$, $\cos\theta = \frac{1}{3}$, $\sin\theta = \frac{2\sqrt{2}}{3}$

Apply sine rule in triangle $A_1A_2A_3$

$$\frac{6}{\sin(\pi - 2\theta)} = \frac{r+1}{\sin\theta}$$

$\Rightarrow$ $r = 8$

17. $OA = 2\cos\frac{\pi}{k}$



$OB = 2\cos\frac{\pi}{2k}$

$$2\cos\frac{\pi}{k} + 2\cos\frac{\pi}{2k} = \sqrt{3} + 1$$

$$2\cos^2\frac{\pi}{2k} - 1 + \cos\frac{\pi}{2k} = \frac{\sqrt{3}+1}{2}$$

Let $\cos\frac{\pi}{2k} = t$

$$2t^2 + t - 1 - \frac{\sqrt{3}+1}{2} = 0$$

$\Rightarrow$ $4t^2 + 2t - (3 + \sqrt{3}) = 0$ $\Rightarrow$ $t = \frac{\sqrt{3}}{2}, -\frac{1+\sqrt{3}}{2}$

$t = -\frac{1+\sqrt{3}}{2}$ (not possible)

$t = \frac{\sqrt{3}}{2} = \cos 30° = \cos\frac{\pi}{6}$ $\Rightarrow$ $\cos\frac{\pi}{2k} = \cos\frac{\pi}{6}$

$k = 3$

18. Family of circle which touches y-axis at (0,2) is

$x^2 + (y-2)^2 + \lambda x = 0$

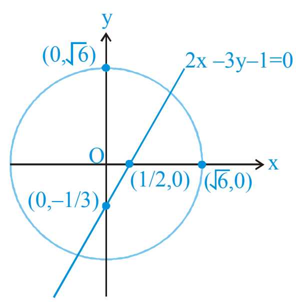
Passing through (–1,0)

$\Rightarrow$ $1 + 4 - \lambda = 0$ $\Rightarrow$ $\lambda = 5$

$\therefore$ $x^2 + y^2 + 5x - 4y + 4 = 0$

which satisfy the point (–4,0).

19.



If the point lies inside the smaller part, then origin and point should give opposite signs w.r.t. line & point should lie inside the circle.

for origin : $2 \times 0 - 3 \times 0 - 1 = -1$ (–ve)

for $(2, \frac{3}{4})$: $2 \times 2 - 3 \times \frac{3}{4} - 1$

$= \frac{3}{4}$ (+ve); point lies inside the circle

for $(\frac{5}{2}, \frac{3}{4})$: $2 \times \frac{5}{2} - 3 \times \frac{3}{4} - 1 = \frac{7}{4}$ (+ve) ; point lies outside the circle

For $\left(\frac{1}{4}, -\frac{1}{4}\right)$: $2 \times \frac{1}{4} - 3\left(-\frac{1}{4}\right) - 1 = \frac{1}{4}$ (+ve) ; point lies inside the circle

For $\left(\frac{1}{8}, \frac{1}{4}\right)$: $2 \times \frac{1}{8} - 3\left(\frac{1}{4}\right) - 1 = \frac{-3}{2}$ (–ve) ; point lies inside the circle.

$\therefore$ 2 points lie inside smaller part.

20. Let mid point be (h, k),

Then chord of contact :

$hx + ky = h^2 + k^2$ ......**(i)**

Let any point on the line $4x - 5y = 20$ be

$$\left(x_1, \frac{4x_1 - 20}{5}\right)$$

$\therefore$ Chord of contact :

$5x_1x + (4x_1 - 20)y = 45$ ......**(ii)**

(i) and **(ii)** are same

$$\therefore \frac{5x_1}{h} = \frac{4x_1 - 20}{k} = \frac{45}{h^2 + k^2}$$

$$\Rightarrow x_1 = \frac{9h}{h^2 + k^2}$$

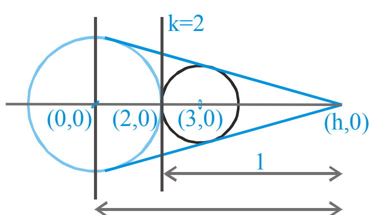
$$\text{and } x_1 = \frac{45k + 20(h^2 + k^2)}{4(h^2 + k^2)}$$

$$\Rightarrow \frac{9h}{h^2 + k^2} = \frac{45k + 20(h^2 + k^2)}{4(h^2 + k^2)}$$

$\Rightarrow 20(h^2 + k^2) - 36h + 45k = 0$

$\therefore$ Locus is $20(x^2 + y^2) - 36x + 45y = 0$

21. $$h = \frac{2 \times 3 - 1 \times 0}{2 - 1} = 6$$



equation of tangents from (6, 0) :

$y - 0 = m(x - 6) \Rightarrow y - mx + 6m = 0$

use p = r

$$\left|\frac{6m}{\sqrt{1 + m^2}}\right| = 2 \Rightarrow 36m^2 = 4 + 4m^2$$

$$32m^2 = 4 \Rightarrow m^2 = 1/8 \Rightarrow m = \pm\frac{1}{2\sqrt{2}}$$

$$\text{at } m = -\frac{1}{2\sqrt{2}}$$

equation of tangent will be $x + 2\sqrt{2}y = 6$

22. Equation of tangent at P will be $\sqrt{3}x + y = 4$

Slope of line L will be $\frac{1}{\sqrt{3}}$

Let equation of L be : $y = \frac{x}{\sqrt{3}} + c$

$\Rightarrow x - \sqrt{3}y + \sqrt{3}c = 0$

Now this L is tangent to 2^{nd} circle

So $\frac{3 + \sqrt{3}c}{2} = \pm 1 \Rightarrow c = -\frac{1}{\sqrt{3}}$

or $c = -\frac{5}{\sqrt{3}}$

using $c = -\frac{1}{\sqrt{3}}$

$$y = \frac{x}{\sqrt{3}} - \frac{1}{\sqrt{3}} \Rightarrow x - \sqrt{3}y = 1.$$

Hence **(A)**

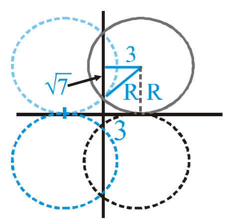
23. As per figure,

$R^2 = 3^2 + \left(\sqrt{7}\right)^2$

$\Rightarrow R = 4$

$\therefore$ centre $\equiv$ (3,4)

radius 4

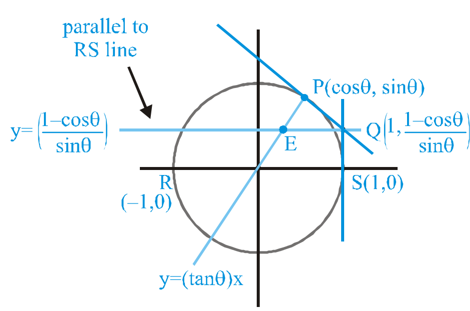


$\therefore$ equation $x^2 + y^2 - 6x - 8y + 9 = 0$

such a circle can lie in all 4 quadrants as shown in figure.

$\therefore$ equation can be $x^2 + y^2 \pm 6x \pm 8y + 9 = 0$

25.



$$E\left(\left(\frac{1 - \cos\theta}{\sin\theta\tan\theta}\right), \left(\frac{1 - \cos\theta}{\sin\theta}\right)\right) \Rightarrow E\left(\frac{\tan\frac{\theta}{2}}{\tan\theta}, \tan\frac{\theta}{2}\right)$$

$$\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{k}{h} \quad \Rightarrow \quad \left(\frac{2k}{1-k^2}\right) = \frac{k}{h}$$

$\therefore \quad 2xy = y(1 - y^2)$

26. $y^2 + 2y - 3 = 0$

$y = 1,\ y = -3$

$p\left(\sqrt{2}, -1\right)$

tangent is $x\sqrt{2} + y = 3$

$C_2(0, \alpha) \perp$ distance $= 2\sqrt{3}$

$$\frac{|\alpha - 3|}{3} = 2\sqrt{3}$$

$\alpha - 3 = \pm 6$

$\alpha = 3, \pm 6$

$\alpha = 9, -3$

$(0, 9), (0, -3)$

$$L_{DCT} = \sqrt{(C_2C_1)^2 - (R+r)^2} = \sqrt{144 - 16\times 3} = 4\sqrt{6}$$

(C) $A = \frac{1}{2}R_3R_2 \times \perp$ form$(0, 0) = 2\sqrt{6} \times \frac{3}{\sqrt{3}} = 6\sqrt{2}$

(D) Area $= \frac{1}{2}\begin{vmatrix} 0 & -3 & 1 \\ 0 & 9 & 1 \\ \sqrt{2} & 1 & 1 \end{vmatrix} = 6\sqrt{2}$

MOCK TEST

1. $x^2 + y^2 - 5x + 2y - 5 = 0$

$$\Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 - 5 - \frac{25}{4} - 1 = 0$$

$$\Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 = \frac{49}{4}$$

$\Rightarrow$ So the axes are shifted to $\left(\frac{5}{2}, -1\right)$

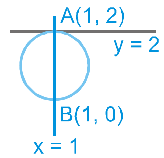
New equation of circle must be $x^2 + y^2 = \frac{49}{4}$

2. **(D)**

$S(x, 2) = 0$ given two identical solutions $x = 1$.

$\Rightarrow$ line $y = 2$ is a tangent to the circle $S(x, y) = 0$ at the point $(1, 2)$ and $S(1, y) = 0$ gives two distinct solutions $y = 0, 2$

$\Rightarrow$ Line $x = 1$ cut the circle $S(x, y) = 0$ at points $(1, 0)$ and $(1, 2)$



A(1, 2) and B(1, 0) are diametrically opposite points.

$\therefore$ equation of the circle is $(x-1)^2 + y(y-2) = 0$

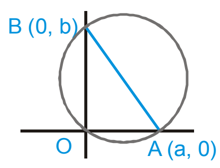
$x^2 + y^2 - 2x - 2y + 1 = 0$

3. Equation of circum circle of triangle OAB

$x^2 + y^2 - ax - by = 0.$

Equation of tangent at origin $ax + by = 0.$

$$d_1 = \frac{|a^2|}{\sqrt{a^2+b^2}} \text{ and } d_2 = \frac{|b^2|}{\sqrt{a^2+b^2}}$$



$d_1 + d_2 = \sqrt{a^2+b^2}$ = diameter

4. **(B)**

Equation of the family of circles passing through A(3 , 7) and B (6, 5) is

$(x-3)(x-6)+(y-7)(y-5)+\lambda(2x+3y-27)=0$.

Equation of given circle is $x^2+y^2-4x-6y-3=0$

$\Rightarrow$ Equation of common chord is :- $S_1-S_2=0$

$\Rightarrow$ $(2\lambda-5)x+(3\lambda-6)y+(56-27\lambda)=0$

$\Rightarrow$ $\lambda(2x+3y-27)-(5x+6y-56)=0$

$\Rightarrow$ This represents family of lines passing through the point of intersection of

$2x+3y-27=0$ & $5x+6y-56=0$

$\Rightarrow$ fixed point $=\left(2,\dfrac{23}{3}\right)$

5. $\because$ $\tan 60^\circ=\dfrac{OA}{1}=\sqrt{3}$

$\therefore$ $A(\sqrt{3},0)$ and $C(-\sqrt{3},0)$

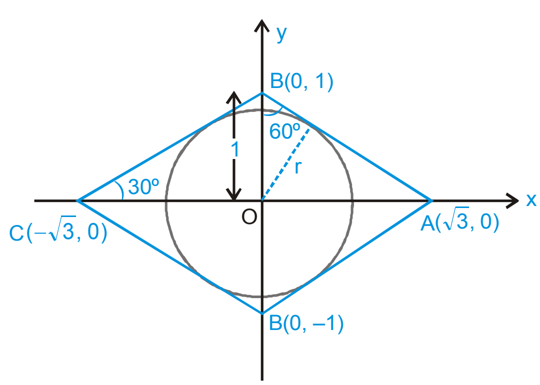
$\because$ $\sin 60^\circ=\dfrac{r}{1}=\dfrac{\sqrt{3}}{2}$

Let coordinates of any point P on the circle be P ≡ (r cosθ , r sinθ)

$\therefore$ $PA^2=(\sqrt{3}-r\cos\theta)^2+(r\sin\theta)^2$

$PB^2=(r\cos\theta)^2+(1-r\sin\theta)^2$

$PC^2=(r\cos\theta+\sqrt{3})^2+(r\sin\theta)^2$



and $PD^2=(r\cos\theta)^2+(r\sin\theta+1)^2$

$\therefore$ $PA^2+PB^2+PC^2+PD^2=4r^2+8=11$

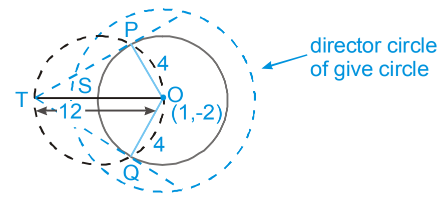
$\because$ $r=\sqrt{3}/2$

6. **(D)**

$(x-1)^2+(y+2)^2=16$

$(x-1)^2+(y+2)^2=32$

$\Rightarrow$ $OS=4\sqrt{2}$



$\therefore$ required distance $TS=OT-SO=12-4\sqrt{2}$

7. $\because$ $\theta=\tan^{-1}\left(\dfrac{2}{3}\right)$ $\Rightarrow$ $\tan\theta=\dfrac{2}{3}$

$\therefore$ $\sin\theta=\dfrac{2}{\sqrt{13}}$ and $\cos\theta=\dfrac{3}{\sqrt{13}}$

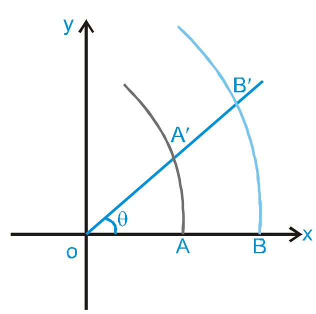
$\therefore$ $A'\equiv(OA\cos\theta, OA\sin\theta)$

$\Rightarrow$ $A'\equiv(3,2)$

Similarly $B'\equiv(OB\cos\theta, OB\sin\theta)\equiv(6,4)$

Now it can be checked that circles C_1 and C_2 touch each other.

Let the point of contact be C.



$\therefore$ $C\equiv\left(5,\dfrac{10}{3}\right)$

$\therefore$ required radical axis is a line perpendicular to A′B′ and passing through point C

$y-\dfrac{10}{3}=-\dfrac{3}{2}(x-5)$

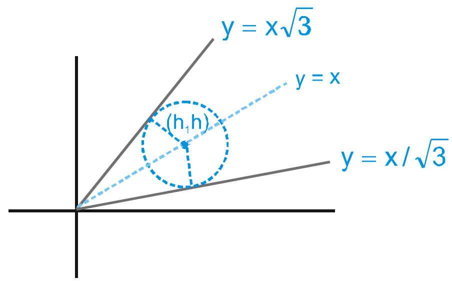
8. (C)

$\because$ centre lies on line $y = x$

$\therefore$ let centre (h, h)

$\therefore \dfrac{\left|h - h\sqrt{3}\right|}{2} = 1$

$\Rightarrow h = (\sqrt{3} + 1)$



$\therefore$ equation of required circle is

$x^2 + y^2 - 2x(\sqrt{3} + 1) - 2y(\sqrt{3} + 1) + 7 + 4\sqrt{3} = 0$

9. Let the coordinates of P and Q are $(a, 0)$ and $(0, b)$ respectively

$\therefore$ equation of PQ is $bx + ay - ab = 0$ ....**(i)**

$\because$ $a^2 + b^2 = 4r^2$ .....**(ii)**

$\because$ $OM \perp PQ$

$\therefore$ equation of OM is $ax - by = 0$ ....**(iii)**

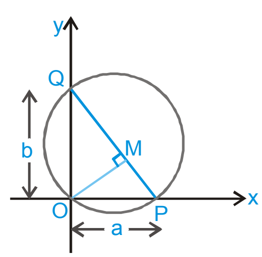
Let $M(h, k)$

$\therefore$ $bh + ak - ab = 0$ ....**(iv)**

and $ah - bk = 0$ ....**(v)**

On solving equations **(iv)** and **(v)**, we get

$a = \dfrac{h^2 + k^2}{h}$ and $b = \dfrac{h^2 + k^2}{k}$



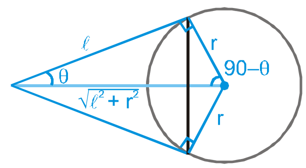
put a and b in **(ii)**, we get

$(h^2 + k^2)^2 (h^{-2} + k^{-2}) = 4r^2$

$\therefore$ locus of $M(h, k)$ is $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$

10. (C)

S_1 **:** Area $= \dfrac{1}{2} 2r\cos\theta \,.\, \ell\cos\theta$



$= r\ell\cos^2\theta = \dfrac{r\ell^3}{r^2 + \ell^2}$

S_2 **:** Product of x-intercepts = product of y-intercepts

$\therefore \left(-\dfrac{c_1}{a_1}\right)\left(-\dfrac{c_2}{a_2}\right) = \left(-\dfrac{c_1}{b_1}\right)\left(-\dfrac{c_2}{b_2}\right)$

i.e. $a_1 a_2 = b_1 b_2$

S_3 **:** $r = \dfrac{\Delta}{s} = \dfrac{a}{2\sqrt{3}}$

$\therefore$ area of square inscribed $= \dfrac{2a^2}{12} = \dfrac{a^2}{6}$

S_4 **:** Length of median $= 3a$

$\therefore$ length of side $= 2\sqrt{3}\,a$

$\therefore R = \dfrac{2\sqrt{3}\,a}{2\sin A} = \dfrac{\sqrt{3}\,a \,.\, 2}{\sqrt{3}} = 2a$

$\therefore$ equation of the circumcircle is $x^2 + y^2 = 4a^2$

11. (A, C, D)

Coordinates of O are $(5, 3)$ and radius $= 2$

Equation of tangent at

$A(7, 3)$ is $7x + 3y - 5(x + 7) - 3(y + 3) + 30 = 0$

i.e. $2x - 14 = 0$ i.e. $x = 7$

Equation of tangent at

$B(5, 1)$ is $5x + y - 5(x + 5) - 3(y + 1) + 30 = 0$

i.e. $-2y + 2 = 0$ i.e. $y = 1$

$\therefore$ coordinate of C are $(7, 1)$

$\therefore$ area of OACB $= 4$

Equation of AB is $x - y = 4$ (radical axis)

Equation of the smallest circle is

$(x - 7)(x - 5) + (y - 3)(y - 1) = 0$

i.e. $x^2 + y^2 - 12x - 4y + 38 = 0$

12. Equation of circle passing through (0, 0) and (1, 0) is

$x^2 + y^2 - x + 2fy = 0$ ....(i)

$\because$ $x^2 + y^2 = 9$ ......(ii)

(i) & (ii) touch each other.

so equation of Radical axis is $x = 2fy + 9$ ......(iii)

line (iii) is also tangent to the circle (ii)

$\therefore$ on solving (ii) & (iii), we get

$(1 + 4f^2)y^2 + 36fy + 72 = 0$ ....(iv)

$\therefore$ $D = 0 \Rightarrow f = \pm\sqrt{2}$.

13. (B,C)

$x^2 + y^2 - 8x - 16y + 60 = 0$ ....(i)

Equation of chord of contact from

(– 2, 0) is $-2x - 4(x-2) - 8y + 60 = 0$

$3x + 4y - 34 = 0$ ......(ii)

From (i) and (ii)

$$x^2 + \left(\frac{34-3x}{4}\right)^2 - 8x - 16\left(\frac{34-3x}{4}\right) + 60 = 0$$

$16x^2 + 1156 - 204x + 9x^2 - 128x - 2176 + 192x + 960 = 0$

$5x^2 - 28x - 12 = 0$

$\Rightarrow$ $(x-6)(5x+2) = 0$

$$x = 6, -\frac{2}{5}$$

$\therefore$ points are (6, 4), $\left(-\frac{2}{5}, \frac{44}{5}\right)$.

14. $\because$ $a\ell^2 - bm^2 + 2\ell d + 1 = 0$ ......(i)

and $a + b = d^2$ ......(ii)

Put $a = d^2 - b$ in equation (1), we get

$(\ell d + 1)^2 = b(\ell^2 + m^2)$

$$\Rightarrow \frac{|\ell d + 1|}{\sqrt{\ell^2 + m^2}} = \sqrt{b} \quad ......(iii)$$

From (3) we can say that the line $\ell x + my + 1 = 0$ touches a fixed circle having centre at (d,0) and radius = $\sqrt{b}$

15. (A,D)

Area of the quadrilateral = $\sqrt{c} \times \sqrt{9 + 25 - c} = 15$

$\therefore$ c = 9, 25

16. Centre (– 2, –6). Substituting in L

$-2(k+7) + 6(k-1) - 4(k-5)$

$= (-2k + 6k - 4k) - 14 - 6 + 20 = 0$

Hence every member of L passing through the centre of the circle

$\Rightarrow$ cuts it at 90°.

Hence S-1 is true and S-2 is false.

17. $$\frac{(AK)}{(OA)} = \cos\theta = \frac{AB}{AK}$$

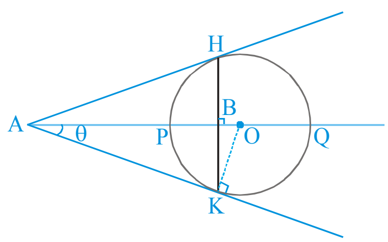
$\Rightarrow$ $(AK)^2 = (AB)(OA) = (AP)(AQ)$

[$AK^2 = AP \cdot AQ$ using power of point A]

Also $$OA = \frac{AP + AQ}{2}$$

[$AQ - AO = r = AO - AP \Rightarrow 2AO = AQ + QP$]

$$\Rightarrow (AP)(AQ) = AB\left(\frac{AP + AQ}{2}\right)$$



$$\Rightarrow AB = \frac{2(AP)(AQ)}{(AP + AQ)}$$

18. (D) Since $S_1 = 0$ and $S_3 = 0$ has no radical axis

$\therefore$ radical centre does not exist

19. Equation of director's circle is $(x-3)^2 + (y+4)^2 = 200$ and point (13, 6) satisfies the given circle $(x-3)^2 + (y+4)^2 = 100$

21. (A) $x^2 + k^2x^2 - 20kx + 90 = 0$

$x^2(1 + k^2) - 20kx + 90 = 0$

$D \le 0$

$400k^2 - 4 \times 90(1 + k^2) \le 0$

$10k^2 - 9 - 9k^2 \le 0$

$k^2 - 9 \le 0 \Rightarrow k \in [-3, 3]$

(B) $2\left(\frac{p}{2}\times5+\frac{p}{2}\times p\right)=-6$

$\Rightarrow\ -5p+p^2+6=0$

$\Rightarrow\ p^2-5p+6=0 \quad \Rightarrow\ p=2 \text{ or } 3$

(C) $r_1^{\,2}=\lambda^2-4\geq0$

$\lambda\in(-\infty,-2]\cup[2,\infty)$ ....**(i)**

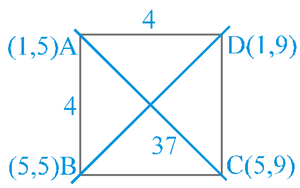
$r_2^{\,2}=4\lambda^2-8\geq0$

$\lambda^2-2\geq0$

$\lambda\in(-\infty,-\sqrt{2}\,]\cup[\sqrt{2},\infty)$ ....**(i)**

$(1)\cap(2)$ is $\lambda\in(-\infty,-2]\cup[2,\infty)$

(D)



22. (A)→(r), (B)→(s), (C)→(q), (D)→(p)

(A) Since (2, 3) lies on $ax+by-5=0$

$\therefore\ 2a+3b-5=0$

Since line is at greatest distance from centre

$\Rightarrow\ \left(\frac{4-3}{3-2}\right)\left(-\frac{a}{b}\right)=-1$ i.e. $a=b$

$\therefore\ a=1, b=1 \quad \therefore\ |a+b|=2$

(B) Let P be the point (α,β), then $\alpha^2+\beta^2+2\alpha+2\beta=0$

mid point of OP is $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$

$\therefore$ locus of $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$ is $4x^2+4y^2+4x+4y=0$

i.e. $x^2+y^2+x+y=0$

$\therefore\ 2g=1, 2f=1$

$\therefore\ g+f=1$

(C) Centres of the circles are $(1,2),(5,-6)$

Equation of C_1C_2 is $y-2=-\frac{8}{4}(x-1)$

i.e. $2x+y-4=0$

Equation of radical axis is $8x-16y-56=0$

i.e. $x-2y-7=0$

Points of intersection is $(3,-2)$

(D) $x^2+y^2-6\sqrt{3}\,x-6y+27=0$

Equation of the pair of tangents is given by

$(-3\sqrt{3}x-3y+27)^2=27\,(x^2+y^2-6\sqrt{3}x-6y+27)$

$27x^2+9y^2+27^2+18\sqrt{3}\,xy-6\times27\sqrt{3}\,x-6\times27y$

$=27x^2+27y^2-6\times27\sqrt{3}\,x-6\times27y+27^2$

$18y^2-18\sqrt{3}\,xy=0$

$y(y-\sqrt{3}\,x)=0$

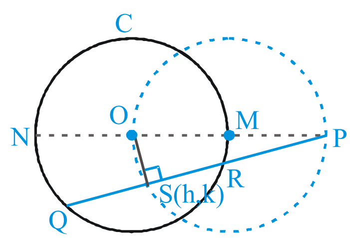
$\therefore$ the tangents are $y=0 \quad y=\sqrt{3}\,x$

$\therefore$ angle between the tangents is $\frac{\pi}{3}$

$\therefore\ 2\sqrt{3}\tan\theta=2\sqrt{3}\times\sqrt{3}=6$

23.

1. Locus of S is a part of circle with OP as diameter passing inside the circle 'C'



2. $(PR)(PQ)=PT^2=(PN)(PM)=(d-r)(d+r)=d^2-r^2$

$=(PS-SR)(PS+SQ)=PS^2-SQ^2$

$(\therefore\ SQ=SR)$

$=PS^2-(SQ)(SR)$

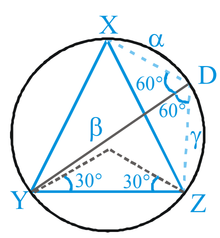
$\therefore\ (PQ)(PR)\neq(PS)^2$

3. Using Ptolemy's theorem,

$(YD)(XZ)=(XY)(ZD)+(YZ)(XD)$

$=XZ\,(ZD+XD) \qquad \{\because (XY=YZ=ZX)\}$

$\Rightarrow\ \beta=\gamma+\alpha \quad \Rightarrow$ **(A)**



24.

1. (B)

From the figure

Since ΔOAB is equilateral triangle

$\therefore \quad \angle OAB = 60°$

2. (C)

Let T be the point of intersection of tangents

Since $\angle AOC = 120°$

$\Rightarrow$ Angle between tangents is 60°.

3. (C)

Locus of point of intersection of tangents at A and C is a circle whose centre is O(0, 0) and radius is

$OT = \sqrt{a^2 + a^2 \cot^2 30} = 2a$

So locus is $x^2 + y^2 = 4a^2$

25.

1. for zeroes to be on either side of origin

$f(0) < 0$

$a^2 + a - 2 < 0 \quad \Rightarrow \quad (a+2)(a-1) < 0$

$\Rightarrow \quad -2 < a < 1 \quad \Rightarrow$ 2 integers i.e. $\{-1, 0\}$

$\Rightarrow$ **(B)**

2. Vertex of C_a is $(2a, a-2)$

hence $h = 2a$ and $k = a - 2$

$h = 2(k+2)$

locus $\quad x = 2y + 4 \quad \Rightarrow \quad x - 2y - 4 = 0$ **Ans.**

3. Let $y = mx + c$ is a common tangent to

$y = \frac{x^2}{4} - 3x + 10$ ......**(i)** (for $a = 3$)

and $y = 2 - \frac{x^2}{4}$ ......**(ii)**

where $m = m_1$ or m_2 and $c = c_1$ or c_2

solving $y = mx + c$ with **(i)**

$$mx + c = \frac{x^2}{4} - 3x + 10$$

or $\frac{x^2}{4} - (m+3)x + 10 - c = 0$

D = 0 gives

$(m+3)^2 = 10 - c \quad \Rightarrow \quad c = 10 - (m+3)^2$ ......**(iii)**

$mx + c = 2 - \frac{x^2}{4} \quad \Rightarrow \quad \frac{x^2}{4} + mx + c - 2 = 0$

D = 0 gives

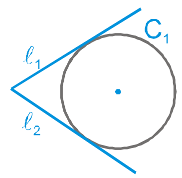
$m^2 = c - 2 \quad \Rightarrow \quad c = 2 + m^2$ ......**(iv)**

from **(iii)** and **(iv)**

$10 - (m+3)^2 = 2 + m^2 \quad \Rightarrow \quad 2m^2 + 6m + 1 = 0$

$\Rightarrow \quad m_1 + m_2 = -\frac{6}{2} = -3$

26. Centre of C_1 lies over angle bisector of ℓ_1 & ℓ_2
Equations of angle bisectors are



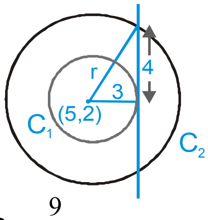
$$\frac{5x + 12y - 10}{13} = \pm \frac{5x - 12y - 40}{13}$$

$\Rightarrow \quad x = 5$ or $y = -\frac{5}{4}$

Since centre lies in first quadrant

so it should be on x = 5.

So let centre be $(5, \alpha)$



$\Rightarrow \quad 3 = \frac{|25 + 12\alpha - 10|}{13} \quad \Rightarrow \quad \alpha = 2, -\frac{9}{2}$

From the figure $r = \sqrt{16+9} = 5$

But $\alpha \neq -\frac{9}{2}$ so $\alpha = 2$.

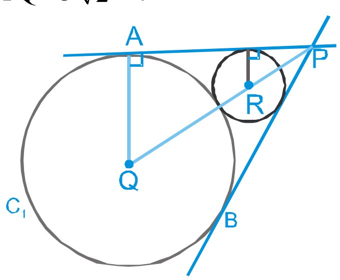
So equation of circle C_2 is

$(x-5)^2 + (y-2)^2 = 5^2$

$x^2 + y^2 - 10x - 4y + 4 = 0$.

27. $AQ = 3 + 2\sqrt{2}$

$PQ = 3\sqrt{2} + 4$



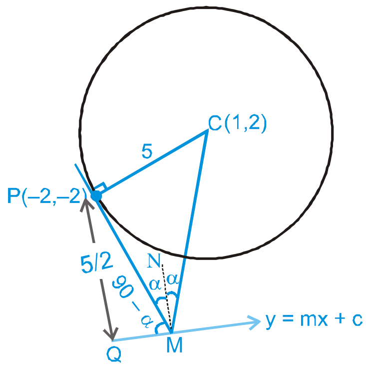
Let 'r' be required radius

$3\sqrt{2} + 4 = 3 + 2\sqrt{2} + r + r\sqrt{2}$

$\sqrt{2} + 1 = r(1 + \sqrt{2}) \quad \Rightarrow \quad r = 1$

28. Let the equation of required straight line be

$y = mx + c.$



$\Rightarrow \quad \frac{5}{2} = \frac{|-2+2m-c|}{\sqrt{1+m^2}}$ .....**(i)**

For ΔPCM $\quad \frac{PC}{PM} = \tan 2\alpha.$

$\Rightarrow \quad PM = 5\cot 2\alpha$ .....**(ii)**

For ΔPQM $\frac{5}{2} = PM \sin(90-\alpha)$

$\Rightarrow \quad \frac{5}{2} = \frac{5\cos 2\alpha}{\sin 2\alpha} \cos\alpha.$

on solving, we get $\alpha = 30°$

Equation of tangent at P(– 2, – 2) is

$3x + 4y + 14 = 0.$

$\tan 60° = \left|\frac{m+3/4}{1-3m/4}\right|$

$\sqrt{3} = \frac{m+3/4}{1-3m/4} \quad \Rightarrow \quad m = \frac{4\sqrt{3}-3}{4+3\sqrt{3}}$

Now on substituting value of 'm' in equation **(i)**, we get

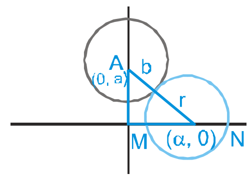
$c = \frac{11+2\sqrt{3}}{4+3\sqrt{3}} \quad \text{or} \quad \frac{-39+2\sqrt{3}}{4+3\sqrt{3}}$

but c should be (–ve)

So equation of line $y = \frac{(4\sqrt{3}-3)}{4+3\sqrt{3}} x + \left(\frac{-39+2\sqrt{3}}{4+3\sqrt{3}}\right)$

29. Let radius $= r$

$\therefore$ from figure $\sqrt{\alpha^2+a^2} = b + r$ ....**(i)**



Consider a point P (0, k) on the y-axis

$M(\alpha - r, 0)$ and $N(\alpha + r, 0)$

Now, slope of MP $= \frac{-k}{\alpha - r}$, slope of NP $= \frac{-k}{\alpha + r}$

If $\angle MPN = \theta$

$\Rightarrow \quad \tan\theta = \left|\frac{\frac{-k}{\alpha-r} - \frac{-k}{\alpha+r}}{1+\frac{k^2}{\alpha^2-r^2}}\right| = \left|\frac{2kr}{\alpha^2-r^2+k^2}\right|$

According to the given condition, θ is a constant for any choice α.

$$\frac{2kr}{\alpha^2-r^2+k^2} = \text{constant}$$

i.e. $\quad \frac{r}{\alpha^2-r^2+k^2} = \text{constant}$

i.e. $\quad \frac{\sqrt{\alpha^2+a^2}-b}{\alpha^2-\left(\sqrt{\alpha^2+a^2}-b\right)^2+k^2} = \text{constant}$

(from equation **(i)**)

i.e. $\quad \frac{\sqrt{\alpha^2+a^2}-b}{2b\sqrt{\alpha^2+a^2}-a^2-b^2+k^2} = \text{constant}$

$$\frac{\sqrt{\alpha^2+a^2}-b}{\sqrt{\alpha^2+a^2}-\lambda} = \text{constant}$$

$\left\{\text{putting } \frac{a^2+b^2-k^2}{2b} = \lambda\right\}$

which is possible only if $\lambda = b$

$$\frac{a^2+b^2-k^2}{2b} = b \Rightarrow k = \pm\sqrt{a^2-b^2}$$

$\therefore \quad P \equiv \left(0, \pm\sqrt{a^2-b^2}\right)$

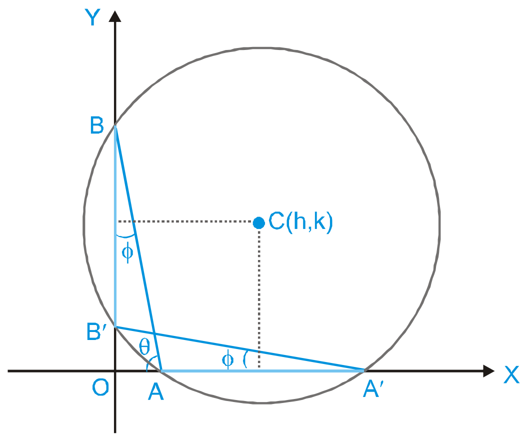
30. Let $\angle OA'B' = \phi$ and $\angle OAB = \theta$

$\Rightarrow \quad \theta + \phi = \dfrac{\pi}{2}$ and $\angle OBA = \phi$

$\because$ length of AB is 'a' and length of A′B′ is 'b'

$\therefore$ from the figure

A′ $(b \cos \phi, 0)$ and A$(a \cos \theta, 0)$



similarly B$(0, a\sin\theta)$ and B′ $(0, b\sin\phi)$

Let $c(h, k)$ be the centre of circle

$\therefore \quad 2h = a\cos\theta + b\cos\phi$

$\because \quad \phi = \dfrac{\pi}{2} - \theta$

$\therefore \quad 2h = a\cos\theta + b\sin\theta \quad$ ....**(i)**

and $2k = a\sin\theta + b\sin\phi \qquad \because \quad \phi = \dfrac{\pi}{2} - \theta$

$\therefore \quad 2k = a\sin\theta + b\cos\theta \quad$ ....**(ii)**

on solving **(i)** and **(ii)**, we get $\cos\theta = \dfrac{2ah - 2bk}{a^2 - b^2}$

and $\quad \sin\theta = \dfrac{2ak - 2bh}{a^2 - b^2}$

$\because \quad \sin^2\theta + \cos^2\theta = 1$

$\therefore$ locus of C(h, k) is

$(2ax - 2by)^2 + (2bx - 2ay)^2 = (a^2 - b^2)^2$