CIRCLE

1. (A) DEFINITION :

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point (in the same given plane) remains constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

Equation of a circle :

The curve traced by the moving point is called its circumference i.e. the equation of any circle is satisfied by co-ordinates of all points on its circumference.

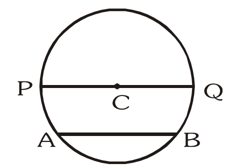
or

The equation of the circle means the equation of its circumference.

or

It is the set of all points lying on the circumference of the circle.

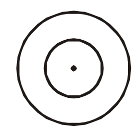
Chord and diameter - the line joining any two points on the circumference is called a chord. If any chord passing through its centre is called its diameter.



AB = chord, PQ = diameter

C = centre

(B) BASIC THEOREMS & RESULTS OF CIRCLES :



(a) **Concentric circles** : Circles having same centre.

(b) **Congruent circles** : Iff their radii are equal.

(c) **Congruent arcs** : Iff they have same degree measure at the centre.

Theorem 1 :

(i) If two arcs of a circle (or of congruent circles) are congruent, the corresponding chords are equal.
Converse : If two chords of a circle are equal then their corresponding arcs are congruent.

(ii) Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
Converse : If the angle subtended by two chords of a circle (or of congruent circles) at the centre are equal, the chords are equal.

Theorem 2 :

(i) The perpendicular from the centre of a circle to a chord bisects the chord.
Converse : The line joining the mid point of a chord to the centre of a circle is perpendicular to the chord.

(ii) Perpendicular bisectors of two chords of a circle intersect at its centre.

Theorem 3 :

(i) There is one and only one circle passing through three non collinear points.

(ii) If two circles intersects in two points, then the line joining the centres is perpendicular bisector of common chords.

Theorem 4 :

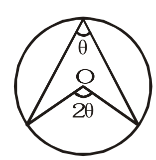
(i) Equal chords of a circle (or of congruent circles) are equidistant from the centre.
Converse : Chords of a circle (or of congruent circles) which are equidistant from the centre are equal in length.

(ii) If two equal chords are drawn from a point on the circle, then the centre of circle will lie on angle bisector of these two chords.

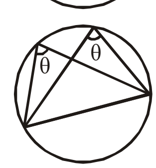
(iii) Of any two chords of a circle larger will be near to centre.

Theorem 5 :

(i) The degree measure of an arc or angle subtended by an arc at the centre is double the angle subtended by it at any point of alternate segment.

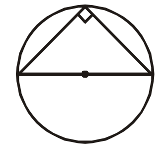


(ii) Angle in the same segment of a circle are equal.



(iii) The angle in a semi circle is right angle.

Converse : The arc of a circle subtending a right angle in alternate segment is semi circle.



Theorem 6 :

Any angle subtended by a minor arc in the alternate segment is acute and any angle subtended by a major arc in the alternate segment is obtuse.

Theorem 7 :

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment, the four points are concyclic, i.e. lie on the same circle.

(d) Cyclic Quadrilaterals :

A quadrilateral is called a cyclic quadrilateral if its all vertices lie on a circle.

Theorem 1 :

The sum of either pair of opposite angles of a cyclic quadrilateral is 180

OR

The opposite angles of a cyclic quadrilateral are supplementary.

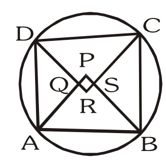
Converse : If the sum of any pair of opposite angle of a quadrilateral is 180 , then the quadrilateral is cyclic.

Theorem 2 :

If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle.

Theorem 3 :

The internal angle bisectors of a cyclic quadrilateral form a quadrilateral which is also cyclic.



Theorem 4 :

If two sides of a cyclic quadrilateral are parallel then the remaining two sides are equal and the diagonals are also equal.

OR

A cyclic trapezium is isosceles and its diagonals are equal.

Converse : If two non-parallel sides of a trapezium are equal, then it is cyclic.

OR

An isosceles trapezium is always cyclic.

Theorem 5 :

When the opposite sides of cyclic quadrilateral (provided that they are not parallel) are produced, then the exterior angle bisectors intersect at right angle.

(C) TANGENTS TO A CIRCLE :

Theorem 1 :

A tangent to a circle is perpendicular to the radius through the point of contact.

Converse : A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle.

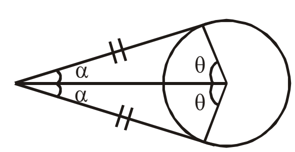
Theorem 2 :

If two tangents are drawn to a circle from an external point, then :

(i) they are equal.

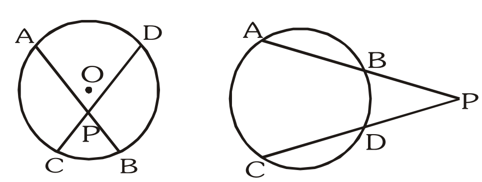
(ii) they subtend equal angles at the centre,

(iii) they are equally inclined to the segment, joining the centre to that point.



Theorem 3 :

If two chords of a circle intersect inside or outside the circle when produced, the rectangle formed by the two segments of one chord is equal in area to the rectangle formed by the two segments of the other chord.



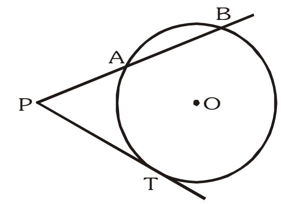
PA PB = PC PD

Theorem 4 :

If PAB is a secant to a circle intersecting the circle at A and B and PT is tangent segment, then $PA \times PB = PT^2$

OR

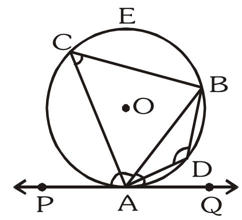
Area of the rectangle formed by the two segments of a chord is equal to the area of the square of side equal to the length of the tangent from the point on the circle.



Theorem 5 :

If a chord is drawn through the point of contact of a tangent to a circle, then the angles which this chord makes with the given tangent are equal respectively to the angles formed in the corresponding alternate segments.

$\angle BAQ = \angle ACB$ and $\angle BAP = \angle ADB$



Converse :

If a line is drawn through an end point of a chord of a circle so that the angle formed with the chord is equal to the angle subtended by the chord in the alternate segment, then the line is a tangent to the circle.

2. STANDARD EQUATIONS OF THE CIRCLE :

(a) Central Form :

If (h, k) is the centre and r is the radius of the circle then its equation is

$(x-h)^2 + (y-k)^2 = r^2$

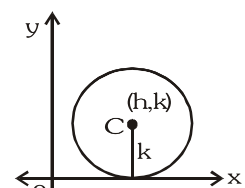
Special Cases :

(i) If centre is origin (0,0) and radius is 'r' then equation of circle is $x^2 + y^2 = r^2$ and this is called the standard form.

(ii) If radius of circle is zero then equation of circle is $(x - h)^2 + (y - k)^2 = 0$. Such circle is called zero circle or **point circle.**

(iii) When circle touches x-axis then equation of the circle is

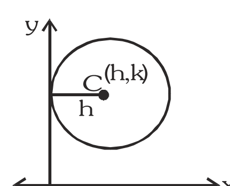
$(x-h)^2 + (y-k)^2 = k^2$.



Touching x-axis

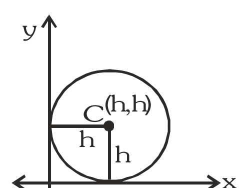
(iv) When circle touches y-axis then equation of circle is

$(x-h)^2 + (y-k)^2 = h^2$



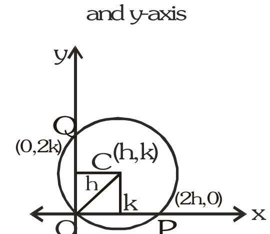
Touching y-axis

(v) When circle touches both the axes (x-axis and y-axis) then equation of circle $(x-h)^2 + (y-h)^2 = h^2$.



Touching x-axis and y-axis

(vi) When circle passes through the origin and centre of the circle is (h,k) then radius $\sqrt{h^2+k^2} = r$ and intercept cut on x-axis OP =2h, and intercept cut on y-axis is OQ = 2k and equation of circle is $(x-h)^2 + (y-k)^2 = h^2 + k^2$ or $x^2 + y^2 - 2hx - 2ky = 0$



Note : Centre of the circle may exist in any quadrant hence for general cases use ± sign before h & k.

(b) General equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$. where g,f,c are constants and centre is $(-g,-f)$

i.e. $\left(-\frac{\text{coefficient of } x}{2}, -\frac{\text{coefficient of } y}{2}\right)$ and radius $r = \sqrt{g^2 + f^2 - c}$

Note :

(i) If $(g^2 + f^2 - c) > 0$, then r is real and positive and the circle is a real circle.

(ii) If $(g^2 + f^2 - c) = 0$, then radius $r = 0$ and circle is a point circle.

(iii) If $(g^2 + f^2 - c) < 0$, then r is imaginary then circle is also an imaginary circle with real centre.

(iv) $x^2 + y^2 + 2gx + 2fy + c = 0$, has three constants and to get the equation of the circle at least three conditions should be known $\Rightarrow$ A unique circle passes through three non collinear points.

(v) **The general second degree in x and y, $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a circle if :**

- coefficient of x^2 = coefficient of y^2 or $a = b \neq 0$
- coefficient of $xy = 0$ or $h = 0$
- $(g^2 + f^2 - c) \geq 0$ (for a real circle)

(c) Intercepts cut by the circle on axes :

The intercepts cut by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on :

(i) x-axis $= 2\sqrt{g^2 - c}$ (ii) y-axis $= 2\sqrt{f^2 - c}$

Note :

(i) If the circle cuts the x-axis at two distinct point, then $g^2 - c > 0$

(ii) If the cirlce cuts the y-axis at two distinct point, then $f^2 - c > 0$

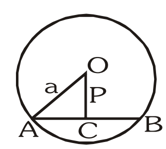
(iii) If circle touches x-axis then $g^2 = c$.

(iv) If circle touches y-axis then $f^2 = c$.

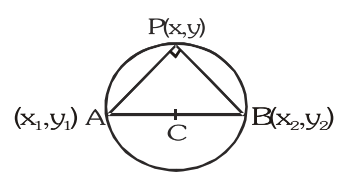
(v) Circle lies completely above or below the x-axis then $g^2 < c$.

(vi) Circle lies completely to the right or left to the y-axis, then $f^2 < c$.

(vii) Intercept cut by a line on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ or length of chord of the circle $= 2\sqrt{a^2 - P^2}$ where a is the radius and P is the length of perpendicular from the centre to the chord.



(d) Equation of circle in diameter form :



If $A(x_1,y_1)$ and $B(x_2,y_2)$ are the end points of the diameter of the circle and $P(x,y)$ is the point other then A and B on the circle then from geometry we know that $\angle APB = 90$.

$\Rightarrow$ (Slope of PA) (Slope of PB) = –1

$\Rightarrow \therefore \left(\frac{y - y_1}{x - x_1}\right)\left(\frac{y - y_2}{x - x_2}\right) = -1$

$\Rightarrow$ $(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0$

Note : This will be the circle of least radius passing through (x_1, y_1) and (x_2, y_2)

(e) Equation of circle in parametric forms :

(i) The parametric equation of the circle $x^2+y^2 = r^2$ are $\mathbf{x = r\cos\theta, y = r\sin\theta}$; $\theta \in [0, 2\pi)$ and $(r\cos\theta, r\sin\theta)$ are called the parametric co-ordinates.

(ii) The parametric equation of the circle $(x - h)^2 + (y - k)^2 = r^2$ is $\mathbf{x = h + r\cos\theta, y = k + r\sin\theta}$ where θ is parameter.

(iii) The parametric equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are $x = -g + \sqrt{g^2 + f^2 - c}\cos\theta$, $y = -f + \sqrt{g^2 + f^2 - c}\sin\theta$ where θ is parameter.

Note : Equation of a straight line joining two point α & β on the circle $x^2 + y^2 = a^2$ is

$x \cos\frac{\alpha+\beta}{2} + y \sin\frac{\alpha+\beta}{2} = a \cos\frac{\alpha-\beta}{2}$.

Illustration 1 : Find the centre and the radius of the circles

(a) $3x^2 + 3y^2 - 8x - 10y + 3 = 0$

(b) $x^2 + y^2 + 2x \sin\theta + 2y \cos\theta - 8 = 0$

(c) $2x^2 + \lambda xy + 2y^2 + (\lambda - 4)x + 6y - 5 = 0$, for some λ.

Solution : (a) We rewrite the given equation as

$$x^2 + y^2 - \frac{8}{3}x - \frac{10}{3}y + 1 = 0 \Rightarrow g = -\frac{4}{3},\ f = -\frac{5}{3},\ c = 1$$

Hence the centre is $\left(\frac{4}{3}, \frac{5}{3}\right)$ and the radius is $\sqrt{\frac{16}{9} + \frac{25}{9} - 1} = \sqrt{\frac{32}{9}} = \frac{4\sqrt{2}}{3}$ units

(b) $x^2 + y^2 + 2x \sin\theta + 2y\cos\theta - 8 = 0$.

Centre of this circle is $(-\sin\theta, -\cos\theta)$

Radius $= \sqrt{\sin^2\theta + \cos^2\theta + 8} = \sqrt{1+8} = 3$ units

(c) $2x^2 + \lambda xy + 2y^2 + (\lambda - 4)x + 6y - 5 = 0$

We rewrite the equation as

$$x^2 + \frac{\lambda}{2}xy + y^2 + \left(\frac{\lambda-4}{2}\right)x + 3y - \frac{5}{2} = 0 \quad \text{........ (i)}$$

Since, there is no term of xy in the equation of circle $\Rightarrow \frac{\lambda}{2} = 0 \Rightarrow \lambda = 0$

So, equation (i) reduces to $x^2 + y^2 - 2x + 3y - \frac{5}{2} = 0$

$\therefore$ centre is $\left(1, -\frac{3}{2}\right)$ Radius $= \sqrt{1 + \frac{9}{4} + \frac{5}{2}} = \frac{\sqrt{23}}{2}$ units.

Illustration 2 : If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then the radius of the circle is -

(A) 3/2 (B) 3/4 (C) 1/10 (D) 1/20

Solution : The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x - 4y + 4 = 0$ and $3x - 4y - \frac{7}{2} = 0$ and so it is equal to $\frac{4+7/2}{\sqrt{9+16}} = \frac{3}{2}$.

Hence radius is $\frac{3}{4}$. **Ans. (B)**

Illustration 3 : If $y = 2x + m$ is a diameter to the circle $x^2 + y^2 + 3x + 4y - 1 = 0$, then find m

Solution : Centre of circle = $(-3/2, -2)$. This lies on diameter $y = 2x + m$

$\Rightarrow -2 = (-3/2)\ 2 + m \Rightarrow m = 1$

Illustration 4 : The equation of a circle which passes through the point $(1, -2)$ and $(4, -3)$ and whose centre lies on the line $3x + 4y = 7$ is

(A) $15(x^2 + y^2) - 94x + 18y - 55 = 0$ (B) $15(x^2 + y^2) - 94x + 18y + 55 = 0$

(C) $15(x^2 + y^2) + 94x - 18y + 55 = 0$ (D) none of these

Solution : Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ ..... (i)

Hence, substituting the points, $(1, -2)$ and $(4, -3)$ in equation (i)

$5 + 2g - 4f + c = 0$ ..... (ii)

$25 + 8g - 6f + c = 0$ ..... (iii)

centre $(-g, -f)$ lies on line $3x + 4y = 7$

Hence $-3g - 4f = 7$

solving for g, f,c, we get

Here $g = \dfrac{-47}{15}$, $f = \dfrac{9}{15}$, $c = \dfrac{55}{15}$

Hence the equation is $15(x^2 + y^2) - 94x + 18y + 55 = 0$ **Ans. (B)**

Illustration 5 : A circle has radius equal to 3 units and its centre lies on the line $y = x - 1$. Find the equation of the circle if it passes through (7, 3).

Solution : Let the centre of the circle be (α, β). It lies on the line $y = x - 1$

$\Rightarrow$ $\beta = \alpha - 1$. Hence the centre is $(\alpha, \alpha - 1)$.

$\Rightarrow$ The equation of the circle is $(x - \alpha)^2 + (y - \alpha + 1)^2 = 9$

It passes through (7, 3) $\Rightarrow$ $(7 - \alpha)^2 + (4 - \alpha)^2 = 9$

$\Rightarrow$ $2\alpha^2 - 22\alpha + 56 = 0$ $\Rightarrow$ $\alpha^2 - 11\alpha + 28 = 0$

$\Rightarrow$ $(\alpha - 4)(\alpha - 7) = 0$ $\Rightarrow$ $\alpha = 4, 7$

Hence the required equations are

$x^2 + y^2 - 8x - 6y + 16 = 0$ and $x^2 + y^2 - 14x - 12y + 76 = 0$. **Ans.**

Do yourself - 1 :

(i) Find the centre and radius of the circle $2x^2 + 2y^2 = 3x - 5y + 7$

(ii) Find the equation of the circle whose centre is the point of intersection of the lines $2x - 3y + 4 = 0$ & $3x + 4y - 5 = 0$ and passes through the origin.

(iii) Find the parametric form of the equation of the circle $x^2 + y^2 + px + py = 0$

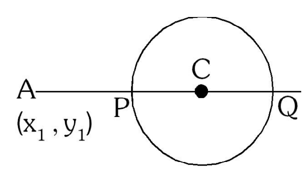
(iv) Find the equation of the circle the end points of whose diameter are the centres of the circles $x^2 + y^2 + 16x - 14y = 1$ & $x^2 + y^2 - 4x + 10y = 2$

3. POSITION OF A POINT W.R.T CIRCLE :

(a) Let the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ and the point is (x_1,y_1) then -

Point (x_1,y_1) lies out side the circle or on the circle or inside the circle according as

$\Rightarrow x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c >, =, < 0$ or $S_1 >, =, < 0$



(b) The greatest & the least distance of a point A from a circle with centre C & radius r is $AC + r$ & $|AC - r|$ respectively.

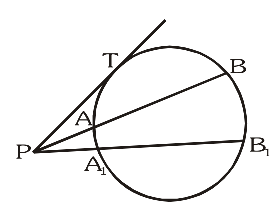
4. POWER OF A POINT W.R.T. CIRCLE :

Theorem : The power of point $P(x_1, y_1)$ w.r.t. the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is S_1

where $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

Note : If P outside, inside or on the circle then power of point is positive, negative or zero respectively.

If from a point $P(x_1, y_1)$, inside or outside the circle, a secant be drawn intersecting the circle in two points A & B, then PA . PB = constant. The product PA . PB is called power of point $P(x_1, y_1)$ w.r.t. the circle



$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, i.e. for number of secants $PA.PB = PA_1 . PB_1 = PA_2 . PB_2 = ... = PT^2 = S_1$

Illustration 6 : If P(2, 8) is an interior point of a circle $x^2 + y^2 - 2x + 4y - p = 0$ which neither touches nor intersects the axes, then set for p is -

(A) $p < -1$ (B) $p < -4$ (C) $p > 96$ (D) ϕ

Solution : For internal point p(2, 8), $4 + 64 - 4 + 32 - p < 0 \Rightarrow p > 96$

and x intercept = $2\sqrt{1+p}$ therefore $1 + p < 0$

$\Rightarrow$ $p < -1$ and y intercept = $2\sqrt{4+p}$ $\Rightarrow$ $p < -4$ **Ans. (D)**

Do yourself - 2 :

(i) Find the position of the points (1, 2) & (6, 0) w.r.t. the circle $x^2 + y^2 - 4x + 2y - 11 = 0$

(ii) Find the greatest and least distance of a point P(7, 3) from circle $x^2 + y^2 - 8x - 6y + 16 = 0$. Also find the power of point P w.r.t. circle.

5. TANGENT LINE OF CIRCLE :

When a straight line meet a circle on two coincident points then it is called the tangent of the circle.

(a) Condition of Tangency :

The line L = 0 touches the circle S = 0 if P the length of the perpendicular from the centre to that line and radius of the circle r are equal i.e. P = r.

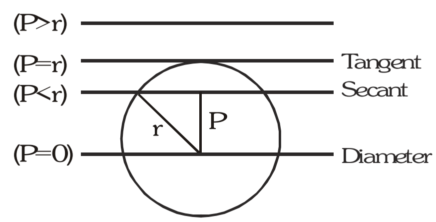


Illustration 7 : Find the range of parameter 'a' for which the variable line y = 2x + a lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 16x - 2y + 61 = 0$ without intersecting or touching either circle.

Solution : The given circles are $C_1 : (x - 1)^2 + (y - 1)^2 = 1$ and $C_2 : (x - 8)^2 + (y - 1)^2 = 4$

The line y − 2x − a = 0 will lie between these circle if centre of the circles lie on opposite sides of the line, i.e. $(1 - 2 - a)(1 - 16 - a) < 0 \Rightarrow a \in (-15, -1)$

Line wouldn't touch or intersect the circles if, $\frac{|1-2-a|}{\sqrt{5}} > 1,\ \frac{|1-16-a|}{\sqrt{5}} > 2$

$\Rightarrow$ $|1 + a| > \sqrt{5}$, $|15 + a| > 2\sqrt{5}$

$\Rightarrow$ $a > \sqrt{5} - 1$ or $a < -\sqrt{5} - 1$, $a > 2\sqrt{5} - 15$ or $a < -2\sqrt{5} - 15$

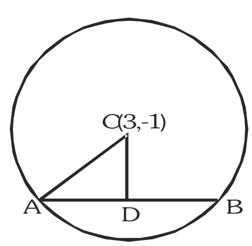
Hence common values of 'a' are $(2\sqrt{5} - 15, -\sqrt{5} - 1)$.

Illustration 8 : The equation of a circle whose centre is (3, −1) and which cuts off a chord of length 6 on the line 2x− 5y+ 18 = 0

(A) $(x - 3)^2 + (y + 1)^2 = 38$ (B) $(x + 3)^2 + (y - 1)^2 = 38$

(C) $(x - 3)^2 + (y + 1)^2 = \sqrt{38}$ (D) none of these

Solution : Let AB(= 6) be the chord intercepted by the line 2x − 5y + 18 = 0 from the circle and let CD be the perpendicular drawn from centre (3, −1) to the chord AB.



i.e., AD = 3, CD = $\frac{2.3-5(-1)+18}{\sqrt{2^2+5^2}} = \sqrt{29}$

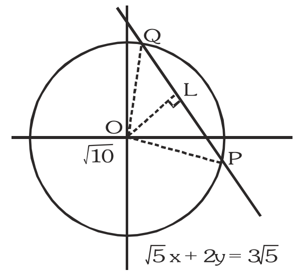
Therefore, $CA^2 = 3^2 + (\sqrt{29})^2 = 38$

Hence required equation is $(x - 3)^2 + (y + 1)^2 = 38$ **Ans. (A)**

Illustration 9 : The area of the triangle formed by line joining the origin to the points of intersection(s) of the line $x\sqrt{5}+2y=3\sqrt{5}$ and circle $x^2 + y^2 = 10$ is

(A) 3 (B) 4 (C) 5 (D) 6

Solution : Length of perpendicular from origin to the line $x\sqrt{5}+2y=3\sqrt{5}$ is



$$OL = \frac{3\sqrt{5}}{\sqrt{(\sqrt{5})^2+2^2}} = \frac{3\sqrt{5}}{\sqrt{9}} = \sqrt{5}$$

Radius of the given circle = $\sqrt{10}$ = OQ = OP

$PQ = 2QL = 2\sqrt{OQ^2 - OL^2} = 2\sqrt{10-5} = 2\sqrt{5}$

Thus area of $\Delta OPQ = \frac{1}{2}\times PQ\times OL = \frac{1}{2}\times 2\sqrt{5}\times\sqrt{5} = 5$ **Ans. (C)**

(b) **Equation of the tangent (T = 0) :**

(i) Tangent at the point (x_1,y_1) on the circle $x^2+ y^2 = a^2$ is $\mathbf{xx_1 + yy_1 = a^2}$.

(ii) (1) The tangent at the point (acos t, asin t) on the circle $x^2 + y^2 = a^2$ is **xcos t + ysin t = a**

(2) The point of intersection of the tangents at the points P(α) and Q(β) is $\left(\frac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \frac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$.

(iii) The equation of tangent at the point (x_1,y_1) on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is
$\mathbf{xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0}$

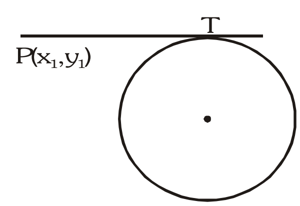
(iv) If line y = mx + c is a straight line touching the circle $x^2 + y^2 = a^2$, then $c = \pm a\sqrt{1+m^2}$ and contact **points are** $\left(\mp\frac{am}{\sqrt{1+m^2}}, \pm\frac{a}{\sqrt{1+m^2}}\right)$ or $\left(\mp\frac{a^2m}{c}, \pm\frac{a^2}{c}\right)$ and equation of tangent is
$\mathbf{y = mx \pm a\sqrt{1+m^2}}$

(v) The equation of tangent with slope m of the circle $(x - h)^2 + (y - k)^2 = a^2$ is
$\mathbf{(y - k) = m(x - h) \pm a\sqrt{1+m^2}}$

Note : To get the equation of tangent at the point $(x_1\ y_1)$ on any second degree curve we replace xx_1 in place of x^2, yy_1 in place of y^2, $\frac{x+x_1}{2}$ in place of x, $\frac{y+y_1}{2}$ in place of y, $\frac{xy_1+yx_1}{2}$ in place of xy and c in place of c.

(c) **Length of tangent ($\sqrt{S_1}$) :**

The length of tangent drawn from point (x_1,y_1) out side the circle
$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is,

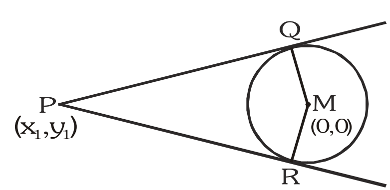


$$PT=\sqrt{S_1} = \sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$$

Note : When we use this formula the coefficient of x^2 and y^2 must be 1.

(d) **Equation of Pair of tangents ($SS_1 = T^2$) :**

Let the equation of circle $S \equiv x^2 + y^2 = a^2$ and $P(x_1,y_1)$ is any point outside the circle. From the point we can draw two real and distinct tangent PQ & PR and combine equation of pair of tangents is -

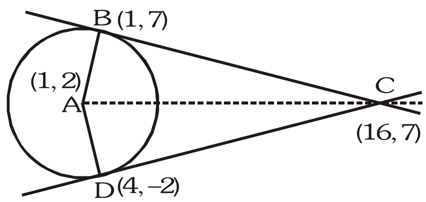


$\mathbf{(x^2 + y^2 - a^2)(x_1^2 + y_1^2 - a^2) = (xx_1 + yy_1 - a^2)^2}$ or
$\mathbf{SS_1 = T^2}$

Illustration 10 : Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ and B(1, 7) and D(4, –2) are points on the circle then, if tangents be drawn at B and D, which meet at C, then area of quadrilateral ABCD is -

(A) 150 (B) 75 (C) 75/2 (D) none of these

Solution :



Here centre A(1, 2) and Tangent at (1, 7) is

$x.1 + y.7 - 1(x + 1) - 2(y + 7) - 20 = 0$ or $y = 7$ .... (i)

Tangent at D(4, –2) is $3x - 4y - 20 = 0$ .... (ii)

Solving (i) and (ii), C is (16, 7)

Area ABCD = AB BC = 5 15 = 75 units. **Ans. (B)**

Do yourself - 3 :

(i) Find the equation of tangent to the circle $x^2 + y^2 - 2ax = 0$ at the point $(a(1 + \cos\alpha), a\sin\alpha)$.

(ii) Find the equations of tangents to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ which are parallel to the line $4x - 3y + 6 = 0$

(iii) Find the equation of the tangents to the circle $x^2 + y^2 = 4$ which are perpendicular to the line $12x - 5y + 9 = 0$. Also find the points of contact.

(iv) Find the value of 'c' if the line y = c is a tangent to the circle $x^2 + y^2 - 2x + 2y - 2 = 0$ at the point (1, 1)

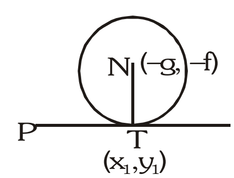
6. NORMAL OF CIRCLE :

Normal at a point is the straight line which is perpendicular to the tangent at the point of contact.

Note : Normal at point of the circle passes through the centre of the circle.

(a) Equation of normal at point (x_1,y_1) of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$y - y_1 = \left(\frac{y_1 + f}{x_1 + g}\right)(x - x_1)$$



(b) The equation of normal on any point (x_1,y_1) of circle $x^2 + y^2 = a^2$ is $\frac{y}{x} = \frac{y_1}{x_1}$.

(c) If $x^2 + y^2 = a^2$ is the equation of the circle then at any point 't' of this circle (a cos t, a sint), the equation of normal is **xsint – ycost = 0.**

Illustration 11 : Find the equation of the normal to the circle $x^2 + y^2 - 5x + 2y - 48 = 0$ at the point (5, 6).

Solution : Since normal to the circle always passes through the centre so equation of the normal will be the line passing through (5, 6) & $\left(\frac{5}{2}, -1\right)$

i.e. $y + 1 = \frac{7}{5/2}\left(x - \frac{5}{2}\right) \Rightarrow 5y + 5 = 14x - 35$

$\Rightarrow 14x - 5y - 40 = 0$ **Ans.**

Illustration 12 : If the straight line ax + by = 2; a, b ≠ 0 touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$, then the values of a and b are respectively

(A) 1, –1 (B) 1, 2 (C) $-\frac{4}{3}, 1$ (D) 2, 1

Solution : Given $x^2 + y^2 - 2x = 3$

∴ centre is (1, 0) and radius is 2

Given $x^2 + y^2 - 4y = 6$

∴ centre is (0, 2) and radius is $\sqrt{10}$. Since line ax + by = 2 touches the first circle

∴ $\frac{|a(1)+b(0)-2|}{\sqrt{a^2+b^2}} = 2$ or $|(a-2)| = [2\sqrt{a^2+b^2}]$ .... (i)

Also the given line is normal to the second circle. Hence it will pass through the centre of the second circle.

∴ a(0) + b(2) = 2 or 2b = 2 or b = 1

Putting this value in equation (i) we get $|a-2| = 2\sqrt{a^2+1^2}$ or $(a-2)^2 = 4(a^2+1)$

or $a^2 + 4 - 4a = 4a^2 + 4$ or $3a^2 + 4a = 0$ or $a(3a+4) = 0$ or $a = 0, -\frac{4}{3}$ $(a \neq 0)$

∴ values of a and b are $\left(-\frac{4}{3}, 1\right)$. **Ans. (C)**

Illustration 13 : Find the equation of a circle having the lines $x^2 + 2xy + 3x + 6y = 0$ as its normal and having size just sufficient to contain the circle $x(x-4) + y(y-3) = 0$.

Solution : Pair of normals are $(x + 2y)(x + 3) = 0$

∴ Normals are $x + 2y = 0$, $x + 3 = 0$.

Point of intersection of normals is the centre of required circle i.e. $C_1(-3, 3/2)$ and centre of given circle is $C_2(2, 3/2)$ and radius $r_2 = \sqrt{4+\frac{9}{4}} = \frac{5}{2}$

Let r_1 be the radius of required circle

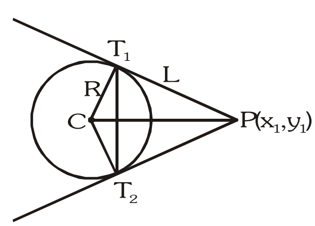
$\Rightarrow$ $r_1 = C_1C_2 + r_2 = \sqrt{(-3-2)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} + \frac{5}{2} = \frac{15}{2}$

Hence equation of required circle is $x^2 + y^2 + 6x - 3y - 45 = 0$

Do yourself - 4 :

(i) Find the equation of the normal to the circle $x^2 + y^2 = 2x$, which is parallel to the line x + 2y = 3.

7. CHORD OF CONTACT (T = 0) :



A line joining the two points of contacts of two tangents drawn from a point out side the circle, is called chord of contact of that point.

If two tangents PT_1 & PT_2 are drawn from the point $P(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is :

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ (i.e. T = 0 same as equation of tangent).

Remember :

(a) Length of chord of contact $T_1T_2 = \frac{2LR}{\sqrt{R^2+L^2}}$.

(b) Area of the triangle formed by the pair of the tangents & its chord of contact $= \frac{RL^3}{R^2+L^2}$, where R is the radius of the circle & L is the length of the tangent from (x_1, y_1) on S = 0.

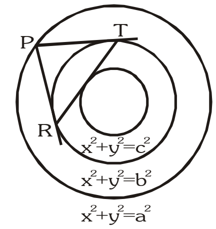
(c) Angle between the pair of tangents from $P(x_1, y_1) = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)$

(d) Equation of the circle circumscribing the triangle PT_1T_2 or quadrilateral CT_1PT_2 is :
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0$.

(e) The joint equation of a pair of tangents drawn from the point $A(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is : $SS_1 = T^2$.
Where $S \equiv x^2 + y^2 + 2gx + 2fy + c$; $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$
$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$.

Illustration 14 : The chord of contact of tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Show that a, b, c are in GP.

Solution : Let $P(a\cos\theta, a\sin\theta)$ be a point on the circle $x^2 + y^2 = a^2$.



Then equation of chord of contact of tangents drawn from $P(a\cos\theta, a\sin\theta)$ to the circle $x^2 + y^2 = b^2$ is $ax\cos\theta + ay\sin\theta = b^2$ ..... (i)

This touches the circle $x^2 + y^2 = c^2$ ..... (ii)

$\therefore$ Length of perpendicular from (0, 0) to (i) = radius of (ii)

$\therefore \quad \frac{|0 + 0 - b^2|}{\sqrt{(a^2\cos^2\theta + a^2\sin^2\theta)}} = c$

or $b^2 = ac \Rightarrow$ a, b, c are in GP.

Do yourself - 5 :

(i) Find the equation of the chord of contact of the point (1, 2) with respect to the circle $x^2 + y^2 + 2x + 3y + 1 = 0$

(ii) Tangents are drawn from the point P(4, 6) to the circle $x^2 + y^2 = 25$. Find the area of the triangle formed by them and their chord of contact.

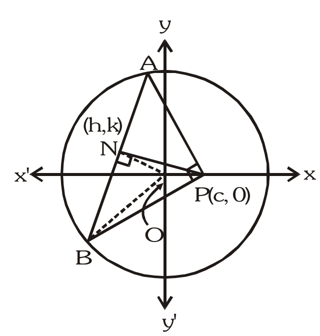
8. EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT ($T = S_1$) :

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $y - y_1 = -\frac{x_1 + g}{y_1 + f}(x - x_1)$. This on simplification can be put in the form $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.

Note that : The shortest chord of a circle passing through a point 'M' inside the circle, is one chord whose middle point is M.

Illustration 15 : Find the locus of middle points of chords of the circle $x^2 + y^2 = a^2$, which subtend right angle at the point (c, 0).

Solution : Let N(h, k) be the middle point of any chord AB, which subtend a right angle at P(c, 0).



Since $\angle APB = 90$

$\therefore$ NA = NB = NP

(since distance of the vertices from middle point of the hypotenuse are equal)

or $(NA)^2 = (NB)^2 = (h - c)^2 + (k - 0)^2$..... (i)

But also $\angle BNO = 90$

$\therefore$ $(OB)^2 = (ON)^2 + (NB)^2$

$\Rightarrow$ $-(NB)^2 = (ON)^2 - (OB)^2 \Rightarrow -[(h - c)^2 + (k - 0)^2] = (h^2 + k^2) - a^2$

or $2(h^2 + k^2) - 2ch + c^2 - a^2 = 0$

$\therefore$ Locus of N(h, k) is $2(x^2 + y^2) - 2cx + c^2 - a^2 = 0$ **Ans.**

Illustration 16 : Let a circle be given by $2x(x - a) + y(2y - b) = 0$ $(a \neq 0, b \neq 0)$

Find the condition on a and b if two chords, each bisected by the x-axis, can be drawn to the circle from (a, b/2).

Solution : The given circle is $2x(x - a) + y(2y - b) = 0$

or $x^2 + y^2 - ax - by/2 = 0$

Let AB be the chord which is bisected by x-axis at a point M. Let its co-ordinates be M(h, 0).

and $S \equiv x^2 + y^2 - ax - by/2 = 0$

$\therefore$ Equation of chord AB is $T = S_1$

$$hx + 0 - \frac{a}{2}(x+h) - \frac{b}{4}(y+0) = h^2 + 0 - ah - 0$$

Since its passes through (a, b/2) we have $ah - \frac{a}{2}(a+h) - \frac{b^2}{8} = h^2 - ah \Rightarrow h^2 - \frac{3ah}{2} + \frac{a^2}{2} + \frac{b^2}{8} = 0$

Now there are two chords bisected by the x-axis, so there must be two distinct real roots of h.

$\therefore$ $B^2 - 4AC > 0$

$\Rightarrow$ $\left(\frac{-3a}{2}\right)^2 - 4.1.\left(\frac{a^2}{2} + \frac{b^2}{8}\right) > 0 \Rightarrow a^2 > 2b^2.$ **Ans.**

Do yourself - 6 :

(i) Find the equation of the chord of $x^2 + y^2 - 6x + 10 - a = 0$ which is bisected at (–2, 4).

(ii) Find the locus of mid point of chord of $x^2 + y^2 + 2gx + 2fy + c = 0$ that pass through the origin.

9. DIRECTOR CIRCLE :

The locus of point of intersection of two perpendicular tangents to a circle is called director circle. Let P(h,k) is the point of intersection of two tangents drawn on the circle $x^2 + y^2 = a^2$. Then the equation of the pair of tangents is $SS_1 = T^2$

i.e. $(x^2 + y^2 - a^2)(h^2 + k^2 - a^2) = (hx + ky - a^2)^2$

As lines are perpendicular to each other then, coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow$ $[(h^2 + k^2 - a^2) - h^2] + [(h^2 + k^2 - a^2) - k^2] = 0$

$\Rightarrow$ $\mathbf{h^2 + k^2 = 2a^2}$

$\therefore$ locus of (h,k) is $x^2 + y^2 = 2a^2$ which is the equation of the director circle.

$\therefore$ director circle is a concentric circle whose radius is $\sqrt{2}$ times the radius of the circle.

Note : The director circle of $x^2 + y^2 + 2gx + 2fy + c = 0$ is $x^2 + y^2 + 2gx + 2fy + 2c - g^2 - f^2 = 0$

Illustration 17 : Let P be any moving point on the circle $x^2 + y^2 - 2x = 1$, from this point chord of contact is drawn w.r.t. the circle $x^2 + y^2 - 2x = 0$. Find the locus of the circumcentre of the triangle CAB, C being centre of the circle and A, B are the points of contact.

Solution : The two circles are

$(x - 1)^2 + y^2 = 1$ .... (i)

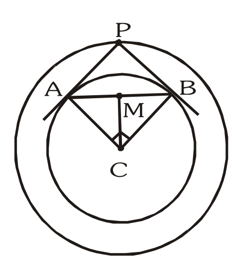
$(x - 1)^2 + y^2 = 2$ .... (ii)

So the second circle is the director circle of the first. So $\angle APB = \pi/2$

Also $\angle ACB = \pi/2$

Now circumcentre of the right angled triangle CAB would lie on the mid point of AB

So let the point be $M \equiv (h, k)$



Now, $CM = CB\sin 45 = \frac{1}{\sqrt{2}}$

So, $(h - 1)^2 + k^2 = \left(\frac{1}{\sqrt{2}}\right)^2$

So, locus of M is $(x - 1)^2 + y^2 = \frac{1}{2}$.

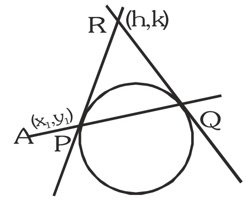
Do yourself - 7 :

(i) Find the equation of the director circle of the circle $(x - h)^2 + (y - k)^2 = a^2$.

(ii) If the angle between the tangents drawn to $x^2 + y^2 + 4x + 8y + c = 0$ from (0, 0) is $\frac{\pi}{2}$, then find value of 'c'

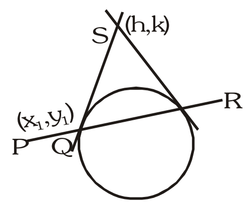
(iii) If two tangents are drawn from a point on the circle $x^2 + y^2 = 50$ to the circle $x^2 + y^2 = 25$, then find the angle between the tangents.

10. POLE AND POLAR :



Let any straight line through the given point $A(x_1,y_1)$ intersect the given circle S =0 in two points P and Q and if the tangent of the circle at P and Q meet at the point R then locus of point R is called polar of the point A and point A is called the pole, with respect to the given circle.

(a) **The equation of the polar of point (x_1,y_1) w.r.t. circle $x^2 + y^2 = a^2$ (T = 0).**



Let PQR is a chord which passes through the point $P(x_1,y_1)$ which intersects the circle at points Q and R and the tangents are drawn at points Q and R meet at point S(h,k) then equation of QR the chord of contact is $x_1h + y_1k = a^2$

∴ locus of point S(h,k) is $\mathbf{xx_1 + yy_1 = a^2}$ which is the equation of the polar.

Note :

(i) The equation of the polar is the T=0, so the polar of point (x_1,y_1) w.r.t circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

(ii) If point is outside the circle then equation of polar and chord of contact is same. So the chord of contact is polar.

(iii) If point is inside the circle then chord of contact does not exist but polar exists.

(iv) If point lies on the circle then polar , chord of contact and tangent on that point are same.

(v) If the polar of P w.r.t. a circle passes through the point Q, then the polar of point Q will pass through P and hence P & Q are conjugate points of each other w.r.t. the given circle.

(vi) If pole of a line w.r.t. a circle lies on second line. Then pole of second line lies on first line and hence both lines are conjugate lines of each other w.r.t. the given circle.

(vii) If O be the centre of a circle and P be any point, then OP is perpendicular to the polar of P.

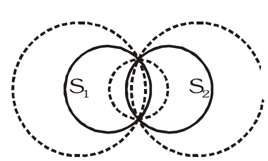
(viii) If O be the centre of a circle and P any point, then if OP (produce, if necessary) meet the polar of P in Q, then OP. OQ = $(\text{radius})^2$

(b) **Pole of a given line with respect to a circle**

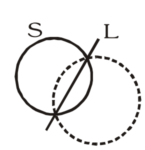
To find the pole of a line we assume the coordinates of the pole then from these coordinates we find the polar. This polar and given line represent the same line. Then by comparing the coefficients of similar terms we can get the coordinates of the pole. The pole of $\ell x + my + n = 0$

w.r.t. circle $x^2 + y^2 = a^2$ will be $\left(\frac{-\ell a^2}{n}, \frac{-ma^2}{n}\right)$

11. FAMILY OF CIRCLES :

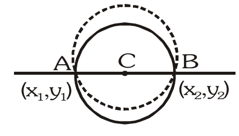


(a) The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ & $S_2 = 0$ is : $S_1 + K S_2 = 0$ $(K \neq -1)$.



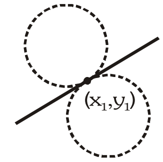
(b) The equation of the family of circles passing through the point of intersection of a circle S = 0 & a line L = 0 is given by S + KL = 0.

(c) The equation of a family of circles passing through two given points (x_1, y_1) & (x_2, y_2) can be written in the form :

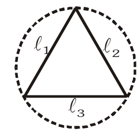


$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + K \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$ where K is a parameter.

(d) The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K[y - y_1 - m(x - x_1)] = 0$, where K is a parameter.



(e) Family of circles circumscribing a triangle whose sides are given by $L_1 = 0$; $L_2 = 0$ & $L_3 = 0$ is given by ; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided coefficient of xy = 0 & coefficient of x^2 = coefficient of y^2.



(f) Equation of circle circumscribing a quadrilateral whose sides in order are represented by the lines $L_1 = 0, L_2 = 0, L_3 = 0$ & $L_4 = 0$ is $L_1L_3 + \lambda L_2L_4 = 0$ provided coefficient of x^2 = coefficient of y^2 and coefficient of xy = 0.

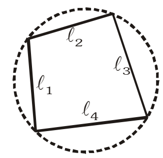


Illustration 18 : The equation of the circle through the points of intersection of $x^2 + y^2 - 1 = 0$, $x^2 + y^2 - 2x - 4y + 1 = 0$ and touching the line $x + 2y = 0$, is -

(A) $x^2 + y^2 + x + 2y = 0$ (B) $x^2 + y^2 - x + 20 = 0$

(C) $x^2 + y^2 - x - 2y = 0$ (D) $2(x^2 + y^2) - x - 2y = 0$

Solution : Family of circles is $x^2 + y^2 - 2x - 4y + 1 + \lambda(x^2 + y^2 - 1) = 0$

$(1 + \lambda)x^2 + (1 + \lambda)y^2 - 2x - 4y + (1 - \lambda) = 0$

$$x^2 + y^2 - \frac{2}{1+\lambda}x - \frac{4}{1+\lambda}y + \frac{1-\lambda}{1+\lambda} = 0$$

Centre is $\left(\frac{1}{1+\lambda}, \frac{2}{1+\lambda}\right)$ and radius = $\sqrt{\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2 - \frac{1-\lambda}{1+\lambda}} = \frac{\sqrt{4+\lambda^2}}{|1+\lambda|}$.

Since it touches the line x + 2y = 0, hence

Radius = Perpendicular distance from centre to the line.

i.e., $$\left|\frac{\frac{1}{1+\lambda} + 2\frac{2}{1+\lambda}}{\sqrt{1^2+2^2}}\right| = \frac{\sqrt{4+\lambda^2}}{|1+\lambda|} \Rightarrow \sqrt{5} = \sqrt{4+\lambda^2} \Rightarrow \lambda = \pm 1$$

$\lambda = -1$ cannot be possible in case of circle. So $\lambda = 1$.

Thus, we get the equation of circle. **Ans. (C)**

Do yourself - 8 :

(i) Prove that the polar of a given point with respect to any one of circles $x^2 + y^2 - 2kx + c^2 = 0$, where k is a variable, always passes through a fixed point, whatever be the value of k.

(ii) Find the equation of the circle passing through the points of intersection of the circle $x^2 + y^2 - 6x + 2y + 4 = 0$ & $x^2 + y^2 + 2x - 4y - 6 = 0$ and with its centre on the line y = x.

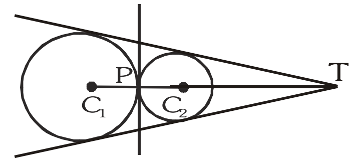
(iii) Find the equation of the circle through the points of intersection of the circles $x^2 + y^2 + 2x + 3y - 7 = 0$ and $x^2 + y^2 + 3x - 2y - 1 = 0$ and passing through the point (1, 2).

12. DIRECT AND TRANSVERSE COMMON TANGENTS :

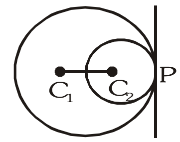
Let two circles having centre C_1 and C_2 and radii, r_1 and r_2 and C_1C_2 is the distance between their centres then :

(a) **Both circles will touch :**

(i) **Externally** if $C_1C_2 = r_1 + r_2$ i.e. the distance between their centres is equal to sum of their radii and point P & T divides C_1C_2 in the ratio $r_1 : r_2$ (internally & externally respectively). In this case there are **three common tangents.**



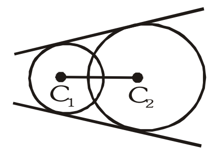
(ii) **Internally** if $C_1C_2 = |r_1 - r_2|$ i.e. the distance between their centres is equal to difference between their radii and point P divides C_1C_2 in the ratio $r_1 : r_2$ **externally** and in this case there will be only **one common tangent.**



(b) **The circles will intersect :**

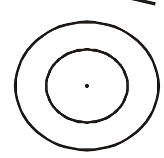
when $|r_1 - r_2| < C_1C_2 < r_1 + r_2$ in this case there are

two common tangents.

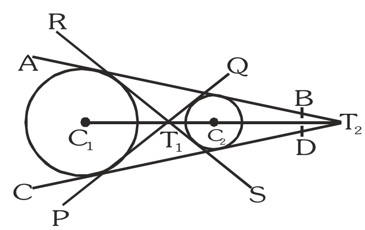


(c) **The circles will not intersect :**

(i) One circle will lie inside the other circle if $C_1C_2 < |r_1 - r_2|$ In this case there will be no common tangent.



(ii) When circle are apart from each other then $C_1C_2 > r_1 + r_2$ and in this case there will be **four common tangents.** Lines PQ and RS are called **transverse** or **indirect** or **internal** common tangents and these lines meet line C_1C_2 on T_1 and T_1 divides the line C_1C_2 in the ratio $r_1 : r_2$ internally and lines AB & CD are called **direct** or **external** common tangents. These lines meet C_1C_2 produced on T_2. Thus T_2 divides C_1C_2 externally in the ratio $r_1 : r_2$.



Note : Length of direct common tangent $= \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2}$

Length of transverse common tangent $= \sqrt{(C_1C_2)^2 - (r_1 + r_2)^2}$

Illustration 19 : Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other, if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

Solution : Given circles are $x^2 + y^2 + 2ax + c^2 = 0$ .... (i)

and $x^2 + y^2 + 2by + c^2 = 0$ .... (ii)

Let C_1 and C_2 be the centres of circles (i) and (ii), respectively and r_1 and r_2 be their radii, then

$C_1 = (-a, 0)$, $C_2 = (0, -b)$, $r_1 = \sqrt{a^2 - c^2}$, $r_2 = \sqrt{b^2 - c^2}$

Here we find the two circles touch each other internally or externally.

For touch, $|C_1C_2| = |r_1 \pm r_2|$

or $\sqrt{(a^2 + b^2)} = \left|\sqrt{(a^2 - c^2)} \pm \sqrt{(b^2 - c^2)}\right|$

On squaring $a^2 + b^2 = a^2 - c^2 + b^2 - c^2 \pm 2\sqrt{(a^2 - c^2)}\sqrt{(b^2 - c^2)}$

or $c^2 = \pm \sqrt{a^2b^2 - c^2(a^2 + b^2) + c^4}$

Again squaring, $c^4 = a^2b^2 - c^2(a^2 + b^2) + c^4$

or $c^2(a^2 + b^2) = a^2b^2$

or $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

Do yourself - 9 :

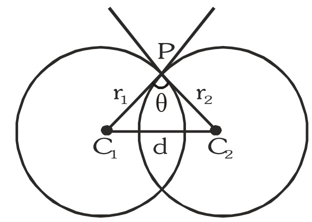
(i) Two circles with radius 5 touches at the point (1, 2). If the equation of common tangent is $4x + 3y = 10$ and one of the circle is $x^2 + y^2 + 6x + 2y - 15 = 0$. Find the equation of other circle.

(ii) Find the number of common tangents to the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x - 6y + 6 = 0$.

13. THE ANGLE OF INTERSECTION OF TWO CIRCLES :

Definition : The angle between the tangents of two circles at the point of intersection of the two circles is called angle of intersection of two circles. If two circles are $S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ and θ is the acute angle between them



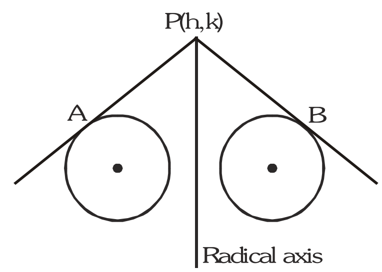
then $\cos\theta = \left|\dfrac{2g_1g_2 + 2f_1f_2 - c_1 - c_2}{2\sqrt{g_1^2 + f_1^2 - c_1}\ \sqrt{g_2^2 + f_2^2 - c_2}}\right|$ or $\cos\theta = \left|\left(\dfrac{r_1^2 + r_2^2 - d^2}{2r_1r_2}\right)\right|$

Here r_1 and r_2 are the radii of the circles and d is the distance between their centres.

If the angle of intersection of the two circles is a right angle then such circles are called **"Orthogonal circles"** and conditions for the circles to be orthogonal is -

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

14. RADICAL AXIS OF THE TWO CIRCLES ($S_1 - S_2 = 0$) :



(a) **Definition :** The locus of a point, which moves in such a way that the length of tangents drawn from it to the circles are equal and is called the radical axis. If two circles are -

$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

Let P(h,k) is a point and PA,PB are length of two tangents on the circles from point P, Then from definition -

$\sqrt{h^2 + k^2 + 2g_1h + 2f_1k + c_1} = \sqrt{h^2 + k^2 + 2g_2h + 2f_2k + c_2}$ or $2(g_1 - g_2)h + 2(f_1 - f_2)k + c_1 - c_2 = 0$

$\therefore$ locus of (h,k)

$2x(g_1 - g_2) + 2y(f_1 - f_2)k + c_1 - c_2 = 0$

$S_1 - S_2 = 0$

which is the equation of radical axis.

Note :

(i) To get the equation of the radical axis first of all make the coefficient of x^2 and $y^2 = 1$

(ii) If circles touch each other then radical axis is the common tangent to both the circles.

(iii) When the two circles intersect on real points then common chord is the radical axis of the two circles.

(iv) The radical axis of the two circles is perpendicular to the line joining the centre of two circles but not always pass through mid point of it.

(v) Radical axis (if exist) bisects common tangent to two circles.

(vi) The radical axes of three circles (taking two at a time) meet at a point.

(vii) If circles are concentric then the radical axis does not always exist but if circles are not concentric then radical axis always exists.

(viii) If two circles are orthogonal to the third circle then radical axis of both circle passes through the centre of the third circle.

(ix) A system of circle, every pair of which have the same radical axis, is called a **coaxial system of circles.**

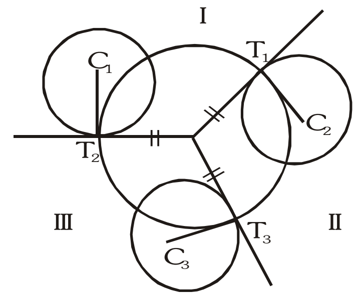
(b) **Radical centre :**

The radical centre of three circles is the point from which length of tangents on three circles are equal i.e. the point of intersection of radical axis of the circles is the radical centre of the circles.

To get the radical axis of three circles $S_1=0$, $S_2=0$, $S_3=0$ we have to solve any two

$S_1-S_2=0,\ S_2-S_3=0,\ S_3-S_1=0$

Note :



(i) The circle with centre as radical centre and radius equal to the length of tangent from radical centre to any of the circle, will cut the three circles orthogonally.

(ii) If three circles are drawn on three sides of a triangle taking them as diameter then its orthocenter will be its radical centre.

(iii) Locus of the centre of a variable circle orthogonal to two fixed circles is the radical axis between the two fixed circles.

(iv) If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P. Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0$, $S_2 = 0$ & $S_3 = 0$ are concurrent is a circle which is orthogonal to all the three circles.

Illustration 20 : A and B are two fixed points and P moves such that PA = nPB where $n \neq 1$. Show that locus of P is a circle and for different values of n all the circles have a common radical axis.

Solution : Let $A \equiv (a, 0)$, $B \equiv (-a, 0)$ and P(h, k)

so PA = nPB

$\Rightarrow \quad (h - a)^2 + k^2 = n^2[(h + a)^2 + k^2]$

$\Rightarrow \quad (1 - n^2)h^2 + (1 - n^2)k^2 - 2ah(1 + n^2) + (1 - n^2)a^2 = 0$

$\Rightarrow \quad h^2 + k^2 - 2ah\left(\frac{1+n^2}{1-n^2}\right) + a^2 = 0$

Hence locus of P is

$x^2 + y^2 - 2ax\left(\frac{1+n^2}{1-n^2}\right) + a^2 = 0$, which is a circle of different values of n.

Let n_1 and n_2 are two different values of n so their radical axis is x = 0 i.e. y-axis. Hence for different values of n the circles have a common radical axis.

Illustration 21 : Find the equation of the circle through the points of intersection of the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$ and cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

Solution : The equation of the circle through the intersection of the given circles is

$x^2 + y^2 - 4x - 6y - 12 + \lambda(-10x - 10y) = 0$ .... (i)

where $(-10x - 10y = 0)$ is the equation of radical axis for the circle

$x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$.

Equation (i) can be re-arranged as

$x^2 + y^2 - x(10\lambda + 4) - y(10\lambda + 6) - 12 = 0$

It cuts the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

Hence $2gg_1 + 2ff_1 = c + c_1$

$\Rightarrow \quad 2(5\lambda + 2)(1) + 2(5\lambda + 3)(0) = -12 - 4 \Rightarrow \lambda = -2$

Hence the required circle is

$x^2 + y^2 - 4x - 6y - 12 - 2(-10x - 10y) = 0$

i.e., $x^2 + y^2 + 16x + 14y - 12 = 0$

Illustration 22 : Find the radical centre of circles $x^2 + y^2 + 3x + 2y + 1 = 0$, $x^2 + y^2 - x + 6y + 5 = 0$ and $x^2 + y^2 + 5x - 8y + 15 = 0$. Also find the equation of the circle cutting them orthogonally.

Solution : Given circles are $S_1 \equiv x^2 + y^2 + 3x + 2y + 1 = 0$

$S_2 \equiv x^2 + y^2 - x + 6y + 5 = 0$

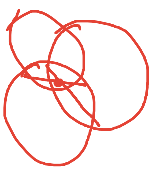
$S_3 \equiv x^2 + y^2 + 5x - 8y + 15 = 0$

Equations of two radical axes are $S_1 - S_2 \equiv 4x - 4y - 4 = 0$ or $x - y - 1 = 0$

and $S_2 - S_3 \equiv -6x + 14y - 10 = 0$ or $3x - 7y + 5 = 0$

Solving them the radical centre is (3, 2). Also, if r is the length of the tangent drawn from the radical centre (3, 2) to any one of the given circles, say S_1, we have

$$r = \sqrt{S_1} = \sqrt{3^2 + 2^2 + 3.3 + 2.2 + 1} = \sqrt{27}$$



Hence (3, 2) is the centre and $\sqrt{27}$ is the radius of the circle intersecting them orthogonally.

$\therefore$ Its equation is $(x - 3)^2 + (y - 2)^2 = r^2 = 27 \Rightarrow x^2 + y^2 - 6x - 4y - 14 = 0$

Alternative Method :

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be the equation of the circle cutting the given circles orthogonally.

$\therefore$ $2g\left(\frac{3}{2}\right) + 2f(1) = c + 1$ or $3g + 2f = c + 1$ .... (i)

$2g\left(-\frac{1}{2}\right) + 2f(3) = c + 5$ or $-g + 6f = c + 5$ .... (ii)

and $2g\left(\frac{5}{2}\right) + 2f(-4) = c + 15$ or $5g - 8f = c + 15$ .... (iii)

Solving (i), (ii) and (iii) we get $g = -3$, $f = -2$ and $c = -14$

$\therefore$ equation of required circle is $x^2 + y^2 - 6x - 4y - 14 = 0$ **Ans.**

Do yourself - 10 :

(i) Find the angle of intersection of two circles
$S : x^2 + y^2 - 4x + 6y + 11 = 0$ & $S' : x^2 + y^2 - 2x + 8y + 13 = 0$

(ii) Find the equation of the radical axis of the circle $x^2 + y^2 - 3x - 4y + 5 = 0$ and $3x^2 + 3y^2 - 7x - 8y + 11 = 0$

(iii) Find the radical centre of three circles described on the three sides $4x - 7y + 10 = 0$, $x + y - 5 = 0$ and $7x + 4y - 15 = 0$ of a triangle as diameters.

15. SOME IMPORTANT RESULTS TO REMEMBER :

(a) If the circle $S_1 = 0$, bisects the circumference of the circle $S_2 = 0$, then their common chord will be the diameter of the circle $S_2 = 0$.

(b) The radius of the director circle of a given circle is $\sqrt{2}$ times the radius of the given circle.

(c) The locus of the middle point of a chord of a circle subtend a right angle at a given point will be a circle.

(d) The length of side of an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$ is $\sqrt{3}a$

(e) If the lengths of tangents from the points A and B to a circle are ℓ_1 and ℓ_2 respectively, then if the points A and B are conjugate to each other, then $(AB)^2 = \ell_1^2 + \ell_2^2$.

(f) Length of transverse common tangent is less than the length of direct common tangent.

Do yourself - 11 :

(i) When the circles $x^2 + y^2 + 4x + 6y + 3 = 0$ and $2(x^2 + y^2) + 6x + 4y + c = 0$ intersect orthogonally, then find the value of c is

(ii) Write the condition so that circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch externally.

Miscellaneous Illustrations :

Illustration 23 : Find the equation of a circle which passes through the point (2, 0) and whose centre is the limit of the point of intersection of the lines $3x + 5y = 1$ and $(2 + c)x + 5c^2y = 1$ as $c \to 1$.

Solution : Solving the equations $(2 + c)x + 5c^2y = 1$ and $3x + 5y = 1$

then $(2 + c)x + 5c^2\left(\frac{1-3x}{5}\right) = 1$ or $(2 + c)x + c^2(1 - 3x) = 1$

$\therefore \quad x = \frac{1-c^2}{2+c-3c^2}$ or $x = \frac{(1+c)(1-c)}{(3c+2)(1-c)} = \frac{1+c}{3c+2}$

$\therefore \quad x = \lim_{c\to 1}\frac{1+c}{3c+2}$ or $x = \frac{2}{5}$

$\therefore \quad y = \frac{1-3x}{5} = \frac{1-\frac{6}{5}}{5} = -\frac{1}{25}$

Therefore the centre of the required circle is $\left(\frac{2}{5}, \frac{-1}{25}\right)$ but circle passes through (2, 0)

$\therefore$ Radius of the required circle $= \sqrt{\left(\frac{2}{5}-2\right)^2 + \left(-\frac{1}{25}-0\right)^2} = \sqrt{\frac{64}{25}+\frac{1}{625}} = \sqrt{\frac{1601}{625}}$

Hence the required equation of the circle is $\left(x-\frac{2}{5}\right)^2 + \left(y+\frac{1}{25}\right)^2 = \frac{1601}{625}$

or $25x^2 + 25y^2 - 20x + 2y - 60 = 0$ **Ans.**

Illustration 24 : Two straight lines rotate about two fixed points. If they start from their position of coincidence such that one rotates at the rate double that of the other. Prove that the locus of their point of intersection is a circle.

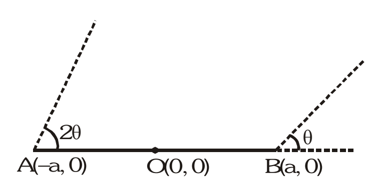
Solution : Let $A \equiv (-a, 0)$ and $B \equiv (a, 0)$ be two fixed points.

Let one line which rotates about B an angle θ with the x-axis at any time t and at that time the second line which rotates about A make an angle 2θ with x-axis.

Now equation of line through B and A are respectively

$y - 0 = \tan\theta(x - a)$ ...... (i)

and $y - 0 = \tan 2\theta(x + a)$ ...... (ii)



From (ii), $y = \frac{2\tan\theta}{1-\tan^2\theta}(x+a)$

$= \left\{\frac{\frac{2y}{(x-a)}}{1-\frac{y^2}{(x-a)^2}}\right\}(x+a)$ (from (i))

$\Rightarrow \quad y = \frac{2y(x-a)(x+a)}{(x-a)^2-y^2} \quad \Rightarrow \quad (x - a)^2 - y^2 = 2(x^2 - a^2)$

or $x^2 + y^2 + 2ax - 3a^2 = 0$ which is the required locus.

Illustration 25 : If the circle $x^2 + y^2 + 6x - 2y + k = 0$ bisects the circumference of the circle $x^2 + y^2 + 2x - 6y - 15 = 0$, then k =

(A) 21 (B) −21 (C) 23 (D) −23

Solution : $2g_2(g_1 - g_2) + 2f_2(f_1 - f_2) = c_1 - c_2$

$2(1)(3 - 1) + 2(-3)(-1 + 3) = k + 15$

$4 - 12 = k + 15$ or $-8 = k + 15 \Rightarrow k = -23$ **Ans. (D)**

Illustration 26 : Find the equation of the circle of minimum radius which contains the three circles.

$S_1 \equiv x^2 + y^2 - 4y - 5 = 0$

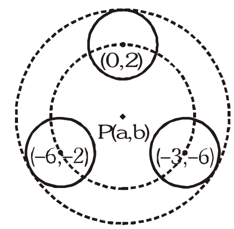
$S_2 \equiv x^2 + y^2 + 12x + 4y + 31 = 0$

$S_3 \equiv x^2 + y^2 + 6x + 12y + 36 = 0$

Solution : For S_1, centre = (0, 2) and radius = 3

For S_2, centre = (−6, −2) and radius = 3

For S_3, centre = (−3, −6) and radius = 3



let P(a, b) be the centre of the circle passing through the centres of the three given circles, then

$(a - 0)^2 + (b - 2)^2 = (a + 6)^2 + (b + 2)^2$

$\Rightarrow (a + 6)^2 - a^2 = (b - 2)^2 - (b + 2)^2$

$(2a + 6)6 = 2b(-4)$

$$b = \frac{2 \times 6(a+3)}{-8} = -\frac{3}{2}(a+3)$$

again $(a - 0)^2 + (b - 2)^2 = (a + 3)^2 + (b + 6)^2$

$\Rightarrow (a + 3)^2 - a^2 = (b - 2)^2 - (b + 6)^2$

$(2a + 3)3 = (2b + 4)(-8)$

$$(2a + 3)3 = -16\left[-\frac{3}{2}(a+3)+2\right]$$

$6a + 9 = -8(-3a - 5)$

$6a + 9 = 24a + 40$

$18a = -31$

$$a = -\frac{31}{18},\ b = -\frac{23}{12}$$

$$\text{radius of the required circle} = 3+\sqrt{\left(-\frac{31}{18}\right)^2+\left(-\frac{23}{12}-2\right)^2} = 3+\frac{5}{36}\sqrt{949}$$

$\therefore$ equation of the required circle is $\left(x+\frac{31}{18}\right)^2+\left(y+\frac{23}{12}\right)^2=\left(3+\frac{5}{36}\sqrt{949}\right)^2$

Illustration 27 : Find the equation of the image of the circle $x^2 + y^2 + 16x - 24y + 183 = 0$ by the line mirror $4x + 7y + 13 = 0$.

Solution : Centre of given circle = (−8, 12), radius = 5

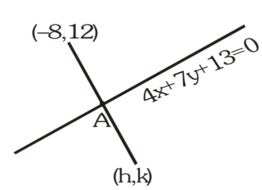
the given line is $4x + 7y + 13 = 0$

let the centre of required circle is (h, k)

since radius will not change. so radius of required circle is 5.

Now (h, k) is the reflection of centre (−8, 12) in the line $4x + 7y + 13 = 0$

$$\text{Co-ordinates of A} = \left(\frac{-8+h}{2}, \frac{12+k}{2}\right)$$



$$\Rightarrow \frac{4(-8+h)}{2}+\frac{7(12+k)}{2}+13=0$$

$-32 + 4h + 84 + 7k + 26 = 0$

$4h + 7k + 78 = 0$ ....(i)

Also $\frac{k-12}{h+8} = \frac{7}{4}$

$4k - 48 = 7h + 56$

$4k = 7h + 104$ ....(ii)

solving (i) & (ii)

$h = -16,\ k = -2$

$\therefore$ required circle is $(x + 16)^2 + (y + 2)^2 = 5^2$

Illustration 28 : The circle $x^2 + y^2 - 6x - 10y + k = 0$ does not touch or intersect the coordinate axes and the point (1, 4) is inside the circle. Find the range of the value of k.

Solution : Since (1, 4) lies inside the circle

$\Rightarrow$ $S_1 < 0$

$\Rightarrow$ $(1)^2 + (4)^2 - 6(1) - 10(4) + k < 0$

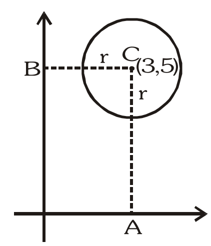
$\Rightarrow$ $k < 29$

Also centre of given circle is (3, 5) and circle does not touch or intersect the coordinate axes

$\Rightarrow$ $r < CA$ & $r < CB$

$CA = 5$

$CB = 3$



$\Rightarrow$ $r < 5$ & $r < 3$

$\Rightarrow$ $r < 3$ or $r^2 < 9$

$r^2 = 9 + 25 - k$

$r^2 = 34 - k$ $\Rightarrow$ $34 - k < 9$

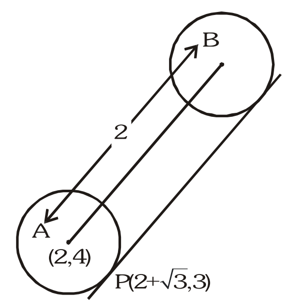
$k > 25$

$\Rightarrow$ $k \in (25, 29)$

Illustration 29 : The circle $x^2 + y^2 - 4x - 8y + 16 = 0$ rolls up the tangent to it at $(2 + \sqrt{3}, 3)$ by 2 units, find the equation of the circle in the new position.

Solution : Given circle is $x^2 + y^2 - 4x - 8y + 16 = 0$

let $P \equiv (2 + \sqrt{3}, 3)$



Equation of tangent to the circle at $P(2 + \sqrt{3}, 3)$ will be

$(2 + \sqrt{3})x + 3y - 2(x + 2 + \sqrt{3}) - 4(y + 3) + 16 = 0$

or $\sqrt{3}x - y - 2\sqrt{3} = 0$

slope $= \sqrt{3}$ $\Rightarrow$ $\tan\theta = \sqrt{3}$

$\theta = 60$

line AB is parallel to the tangent at P

$\Rightarrow$ coordinates of point B = (2 + 2cos60 , 4 + 2sin60)

thus $B = (3,\ 4 + \sqrt{3})$

radius of circle $= \sqrt{2^2 + 4^2 - 16} = 2$

$\therefore$ equation of required circle is $(x - 3)^2 + (y - 4 - \sqrt{3})^2 = 2^2$

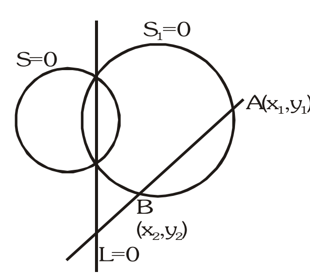
Illustration 30 : A fixed circle is cut by a family of circles all of which, pass through two given points $A(x_1, y_1)$ an $B(x_2, y_2)$. Prove that the chord of intersection of the fixed circle with any circle of the family passe through a fixed point.

Solution : Let S = 0 be the equation of fixed circle

let $S_1 = 0$ be the equation of any circle through A and B which intersect S = 0 in two points.

$L \equiv S - S_1 = 0$ is the equation of the chord of intersection of S = 0 and $S_1 = 0$

let $L_1 = 0$ be the equation of line AB



let S_2 be the equation of the circle whose diametrical ends are $A(x_1, y_1)$ & $B(x_2, y_2)$

then $S_1 \equiv S_2 - \lambda L_1 = 0$

$\Rightarrow \quad L \equiv S - (S_2 - \lambda L_1) = 0 \quad$ or $\quad L \equiv (S - S_2) + \lambda L_1 = 0$

or $\quad L \equiv L' + \lambda L_1 = 0 \qquad$ ....(i)

(i) implies each chord of intersection passes through the fixed point, which is the point of intersection of lines $L' = 0$ & $L_1 = 0$. Hence proved.

Illustration 31 : Let L_1 be a straight line through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 & L_2 are equal, then which of the following equations can represent L_1?

(A) $x + y = 0$ (B) $x - y = 0$ (C) $x + 7y = 0$ (D) $x - 7y = 0$

Solution : Let L_1 be $y = mx$

lines L_1 & L_2 will be at equal distances from centre of the circle centre of the circle is $\left(\frac{1}{2}, -\frac{3}{2}\right)$

$$\Rightarrow \frac{\left|\frac{1}{2}m + \frac{3}{2}\right|}{\sqrt{1+m^2}} = \frac{\left|\frac{1}{2} - \frac{3}{2} - 1\right|}{\sqrt{2}} \quad \Rightarrow \quad \frac{(m+3)^2}{(1+m^2)} = 8$$

$$\Rightarrow 7m^2 - 6m - 1 = 0 \quad \Rightarrow \quad (m - 1)(7m + 1) = 0$$

$$\Rightarrow m = 1, m = -\frac{1}{7} \quad \Rightarrow \quad y = x, 7y + x = 0$$

Ans. (B, C)

ANSWERS FOR DO YOURSELF

1 : (i) Centre $\left(\frac{3}{4}, -\frac{5}{4}\right)$, Radius $\frac{3\sqrt{10}}{4}$ (ii) $17(x^2 + y^2) + 2x - 44y = 0$

(iii) $x = \frac{p}{2}(-1 + \sqrt{2}\cos\theta)$; $y = \frac{p}{2}(-1 + \sqrt{2}\sin\theta)$ (iv) $x^2 + y^2 + 6x - 2y - 51 = 0$

2 : (i) (1, 2) lie inside the circle and the point (6, 0) lies outside the circle

(ii) min = 0, max = 6, power = 0

3 : (i) $x\cos\alpha + y\sin\alpha = a(1 + \cos\alpha)$ (ii) $4x - 3y + 7 = 0$ & $4x - 3y - 43 = 0$

(iii) $5x + 12y = \pm 26$; $\left(\mp\frac{10}{13}, \mp\frac{24}{13}\right)$ (iv) 1

4 : (i) $x + 2y = 1$

5 : (i) $4x + 7y + 10 = 0$ (ii) $\frac{405\sqrt{3}}{52}$ sq. units

6 : (i) $5x - 4y + 26 = 0$ (ii) $x^2 + y^2 + gx + fy = 0$

7 : (i) $(x - h)^2 + (y - k)^2 = 2a^2$ (ii) 10 (iii) angle between the tangents = 90

8 : (ii) $x^2 + y^2 - \frac{10x}{7} - \frac{10y}{7} - \frac{12}{7} = 0$ (iii) $x^2 + y^2 + 4x - 7y + 5 = 0$

9 : (i) $(x - 5)^2 + (y - 5)^2 = 25$ (ii) 4

10 : (i) 135 (ii) $x + 2y = 2$ (iii) (1, 2)

11 : (i) 18 (ii) $a^{-2} + b^{-2} = c^{-1}$

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. The lines 2x – 3y = 5 and 3x – 4y = 7 are diameters of a circle of area 154 sq. units. The equation of the circle is -
(A) $x^2 + y^2 - 2x - 2y = 47$ (B) $x^2 + y^2 - 2x - 2y = 62$
(C) $x^2 + y^2 - 2x + 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 62$

2. If a be the radius of a circle which touches x-axis at the origin, then its equation is -
(A) $x^2 + y^2 + ax = 0$ (B) $x^2 + y^2 \pm 2ya = 0$ (C) $x^2 + y^2 \pm 2xa = 0$ (D) $x^2 + y^2 + ya = 0$

3. The equation of the circle which touches the axis of y at the origin and passes through (3,4) is -
(A) $4(x^2 + y^2) - 25x = 0$ (B) $3(x^2 + y^2) - 25x = 0$
(C) $2(x^2 + y^2) - 3x = 0$ (D) $4(x^2 + y^2) - 25x + 10 = 0$

4. The equation of the circle passing through (3,6) and whose centre is (2,–1) is -
(A) $x^2 + y^2 - 4x + 2y = 45$ (B) $x^2 + y^2 - 4x - 2y + 45 = 0$
(C) $x^2 + y^2 + 4x - 2y = 45$ (D) $x^2 + y^2 - 4x + 2y + 45 = 0$

5. The equation to the circle whose radius is 4 and which touches the negative x-axis at a distance 3 units from the origin is -
(A) $x^2 + y^2 - 6x + 8y - 9 = 0$ (B) $x^2 + y^2 \pm 6x - 8y + 9 = 0$
(C) $x^2 + y^2 + 6x \pm 8y + 9 = 0$ (D) $x^2 + y^2 \pm 6x - 8y - 9 = 0$

6. The equation of a circle which passes through the three points (3,0) (1,–6),(4,–1) is -
(A) $2x^2 + 2y^2 + 5x - 11y + 3 = 0$ (B) $x^2 + y^2 - 5x + 11y - 3 = 0$
(C) $x^2 + y^2 + 5x - 11y + 3 = 0$ (D) $2x^2 + 2y^2 - 5x + 11y - 3 = 0$

7. $y = \sqrt{3}x + c_1$ & $y = \sqrt{3}x + c_2$ are two parallel tangents of a circle of radius 2 units, then $|c_1 - c_2|$ is equal to -
(A) 8 (B) 4 (C) 2 (D) 1

8. Number of different circles that can be drawn touching 3 lines, no two of which are parallel and they are neither coincident nor concurrent, are -
(A) 1 (B) 2 (C) 3 (D) 4

9. B and C are fixed points having co-ordinates (3, 0) and (–3, 0) respectively. If the vertical angle BAC is 90 , then the locus of the centroid of the ΔABC has the equation -
(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = 2$ (C) $9(x^2 + y^2) = 1$ (D) $9(x^2 + y^2) = 4$

10. If a circle of constant radius 3k passes through the origin 'O' and meets co-ordinate axes at A and B then the locus of the centroid of the triangle OAB is -
(A) $x^2 + y^2 = (2k)^2$ (B) $x^2 + y^2 = (3k)^2$ (C) $x^2 + y^2 = (4k)^2$ (D) $x^2 + y^2 = (6k)^2$

11. The area of an equilateral triangle inscribed in the circle $x^2 + y^2 - 2x = 0$ is :
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{4}$ (C) $\frac{3\sqrt{3}}{8}$ (D) none

12. The length of intercept on y-axis, by a circle whose diameter is the line joining the points (–4,3) and (12, –1) is -
(A) $3\sqrt{2}$ (B) $\sqrt{13}$ (C) $4\sqrt{13}$ (D) none of these

13. The gradient of the tangent line at the point $(a \cos \alpha, a \sin \alpha)$ to the circle $x^2 + y^2 = a^2$, is -
(A) $\tan (\pi - \alpha)$ (B) $\tan \alpha$ (C) $\cot \alpha$ (D) $- \cot \alpha$

14. $\ell x + my + n = 0$ is a tangent line to the circle $x^2 + y^2 = r^2$, if -
(A) $\ell^2 + m^2 = n^2 r^2$ (B) $\ell^2 + m^2 = n^2 + r^2$ (C) $n^2 = r^2(\ell^2 + m^2)$ (D) none of these

15. Line $3x + 4y = 25$ touches the circle $x^2 + y^2 = 25$ at the point -

(A) (4, 3) (B) (3, 4) (C) (–3, –4) (D) none of these

16. The equations of the tangents drawn from the point (0,1) to the circle $x^2 + y^2 - 2x + 4y = 0$ are -

(A) $2x - y + 1 = 0,\ x + 2y - 2 = 0$ (B) $2x - y - 1 = 0,\ x + 2y - 2 = 0$

(C) $2x - y + 1 = 0,\ x + 2y + 2 = 0$ (D) $2x - y - 1 = 0,\ x + 2y + 2 = 0$

17. The greatest distance of the point P(10,7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is -

(A) 5 (B) 15 (C) 10 (D) None of these

18. The equation of the normal to the circle $x^2 + y^2 = 9$ at the point $\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ is -

(A) $x - y = \frac{\sqrt{2}}{3}$ (B) $x + y = 0$ (C) $x - y = 0$ (D) none of these

19. The parametric coordinates of any point on the circle $x^2 + y^2 - 4x - 4y = 0$ are-

(A) $(-2 + 2\cos\alpha, -2 + 2\sin\alpha)$ (B) $(2 + 2\cos\alpha, 2 + 2\sin\alpha)$

(C) $(2 + 2\sqrt{2}\cos\alpha, 2 + 2\sqrt{2}\sin\alpha)$ (D) $(-2 + 2\sqrt{2}\cos\alpha, -2 + 2\sqrt{2}\sin\alpha)$

20. The length of the tangent drawn from the point (2,3) to the circles $2(x^2 + y^2) - 7x + 9y - 11 = 0$ -

(A) 18 (B) 14 (C) $\sqrt{14}$ (D) $\sqrt{28}$

21. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. The equation of the pair of tangents is -

(A) $x^2 + y^2 + 5xy = 0$ (B) $x^2 + y^2 + 10xy = 0$ (C) $2x^2 + 2y^2 + 5xy = 0$ (D) $2x^2 + 2y^2 - 5xy = 0$

22. Tangents are drawn from (4, 4) to the circle $x^2 + y^2 - 2x - 2y - 7 = 0$ to meet the circle at A and B. The length of the chord AB is -

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $2\sqrt{6}$ (D) $6\sqrt{2}$

23. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals -

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) none

24. Pair of tangents are drawn from every point on the line $3x + 4y = 12$ on the circle $x^2 + y^2 = 4$. Their variable chord of contact always passes through a fixed point whose co-ordinates are -

(A) $\left(\frac{4}{3}, \frac{3}{4}\right)$ (B) $\left(\frac{3}{4}, \frac{3}{4}\right)$ (C) (1, 1) (D) $\left(1, \frac{4}{3}\right)$

25. The locus of the mid-points of the chords of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ which subtend 60 at the centre is -

(A) $x^2 + y^2 - 4x - 2y - 7 = 0$ (B) $x^2 + y^2 + 4x + 2y - 7 = 0$

(C) $x^2 + y^2 - 2x - 4y - 7 = 0$ (D) $x^2 + y^2 + 2x + 4y + 7 = 0$

26. The locus of the centres of the circles such that the point (2,3) is the mid point of the chord $5x + 2y = 16$ is -

(A) $2x - 5y + 11 = 0$ (B) $2x + 5y - 11 = 0$ (C) $2x + 5y + 11 = 0$ (D) none

27. The locus of the centre of a circle which touches externally the circle, $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis is given by the equation -

(A) $x^2 - 6x - 10y + 14 = 0$ (B) $x^2 - 10x - 6y + 14 = 0$

(C) $y^2 - 6x - 10y + 14 = 0$ (D) $y^2 - 10x - 6y + 14 = 0$

28. The equation of the circle having the lines $y^2 - 2y + 4x - 2xy = 0$ as its normals & passing through the point (2,1) is -

(A) $x^2 + y^2 - 2x - 4y + 3 = 0$ (B) $x^2 + y^2 - 2x + 4y - 5 = 0$

(C) $x^2 + y^2 + 2x + 4y - 13 = 0$ (D) none

29. A circle is drawn touching the x-axis and centre at the point which is the reflection of (a, b) in the line $y - x = 0$. The equation of the circle is -

(A) $x^2 + y^2 - 2bx - 2ay + a^2 = 0$
(B) $x^2 + y^2 - 2bx - 2ay + b^2 = 0$
(C) $x^2 + y^2 - 2ax - 2by + b^2 = 0$
(D) $x^2 + y^2 - 2ax - 2by + a^2 = 0$

30. The length of the common chord of circles $x^2 + y^2 - 6x - 16 = 0$ and $x^2 + y^2 - 8y - 9 = 0$ is -

(A) $10\sqrt{3}$
(B) $5\sqrt{3}$
(C) $5\sqrt{3}/2$
(D) none of these

31. The number of common tangents of the circles $x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2y - 7 = 0$ -

(A) 1
(B) 3
(C) 2
(D) 4

32. If the circle $x^2 + y^2 = 9$ touches the circle $x^2 + y^2 + 6y + c = 0$, then c is equal to -

(A) –27
(B) 36
(C) –36
(D) 27

33. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touches each other, then -

(A) $f_1g_1 = f_2g_2$
(B) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$
(C) $f_1f_2 = g_1g_2$
(D) none

34. The tangent from the point of intersection of the lines $2x - 3y + 1 = 0$ and $3x - 2y - 1 = 0$ to the circle $x^2 + y^2 + 2x - 4y = 0$ is -

(A) $x + 2y = 0,\ x - 2y + 1 = 0$
(B) $2x - y - 1 = 0$
(C) $y = x,\ y = 3x - 2$
(D) $2x + y + 1 = 0$

35. The locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is -

(A) $9x + 10y - 7 = 0$
(B) $x - y + 2 = 0$
(C) $9x - 10y + 11 = 0$
(D) $9x + 10y + 7 = 0$

<u>SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)</u>

36. Equation $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$, may represents -

(A) Equation of straight line, if θ is constant and r is variable.

(B) Equation of a circle, if r is constant & θ is variable.

(C) A straight line passing through a fixed point & having a known slope.

(D) A circle with a known centre and given radius.

37. If r represent the distance of a point from origin & θ is the angle made by line joining origin to that point from line x-axis, then $r = |\cos\theta|$ represents -

(A) two circles of radii $\frac{1}{2}$ each.
(B) two circles centred at $\left(\frac{1}{2}, 0\right)$ & $\left(-\frac{1}{2}, 0\right)$
(C) two circles touching each other at the origin.
(D) pair of straight line

38. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length 8 has a slope equal to $\frac{3}{4}$, then coordinates of centre of C_2 are -

(A) $\left(\frac{9}{5}, -\frac{12}{5}\right)$
(B) $\left(-\frac{9}{5}, \frac{12}{5}\right)$
(C) $\left(\frac{9}{5}, \frac{12}{5}\right)$
(D) $\left(-\frac{9}{5}, -\frac{12}{5}\right)$

39. For the equation $x^2 + y^2 + 2\lambda x + 4 = 0$ which of the following can be true -

(A) It represents a real circle for all $\lambda \in R$.

(B) It represents a real circle for $|\lambda| > 2$.

(C) The radical axis of any two circles of the family is the y-axis.

(D) The radical axis of any two circles of the family is the x-axis.

40. If y = c is a tangent to the circle $x^2 + y^2 - 2x + 2y - 2 = 0$, then the value of c can be -

(A) 1 (B) 3 (C) –1 (D) –3

41. For the circles $S_1 \equiv x^2 + y^2 - 4x - 6y - 12 = 0$ and $S_2 \equiv x^2 + y^2 + 6x + 4y - 12 = 0$ and the line $L \equiv x + y = 0$

(A) L is common tangent of S_1 and S_2

(B) L is common chord of S_1 and S_2

(C) L is radical axis of S_1 and S_2

(D) L is perpendicular to the line joining the centre of S_1 & S_2

CHECK YOUR GRASP				ANSWER KEY					EXERCISE-1
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B	B	A	C	D	A	D	A	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	D	C	B	A	B	C	C	C
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	B	A	D	C	A	D	A	B	B
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	A	A	B	B	C	A,B,C,D	A,B,C	A,B	B,C	A,D
Que.	41									
Ans.	B,C,D									

EXERCISE - 02 BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. If $\left(a,\frac{1}{a}\right)$, $\left(b,\frac{1}{b}\right)$, $\left(c,\frac{1}{c}\right)$ & $\left(d,\frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, abcd =
(A) 4 (B) 1/4 (C) 1 (D) 16

2. What is the length of shortest path by which one can go from (–2, 0) to (2, 0) without entering the interior of circle, $x^2 + y^2 = 1$?
(A) $2\sqrt{3}$ (B) $\sqrt{3}+\frac{2\pi}{3}$ (C) $2\sqrt{3}+\frac{\pi}{3}$ (D) none of these

3. Three equal circles each of radius r touch one another. The radius of the circle touching all the three given circles internally is -
(A) $(2+\sqrt{3})r$ (B) $\frac{(2+\sqrt{3})}{\sqrt{3}}r$ (C) $\frac{(2-\sqrt{3})}{\sqrt{3}}r$ (D) $(2-\sqrt{3})r$

4. If $a^2 + b^2 = 1$, $m^2 + n^2 = 1$, then which of the following is true for all values of m, n, a, b -
(A) $|am + bn| \le 1$ (B) $|am - bn| \ge 1$ (C) $|am + bn| \ge 1$ (D) $|am - bn| \le 1$

5. Circles are drawn touching the co-ordinate axis and having radius 2, then -
(A) centre of these circles lie on the pair of lines $y^2 - x^2 = 0$
(B) centre of these circles lie only on the line y = x
(C) Area of the quadrilateral whose vertices are centre of these circles is 16 sq.unit
(D) Area of the circle touching these four circles internally is $4\pi(3+2\sqrt{2})$

6. The distance between the chords of contact of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and from the point (g,f) is -
(A) $\sqrt{g^2+f^2}$ (B) $\frac{\sqrt{g^2+f^2-c}}{2}$ (C) $\frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$ (D) $\frac{\sqrt{g^2+f^2+c}}{2\sqrt{g^2+f^2}}$

7. $x^2 + y^2 + 6x = 0$ and $x^2 + y^2 - 2x = 0$ are two circles, then -
(A) They touch each other externally
(B) They touch each other internally
(C) Area of triangle formed by their common tangents is $3\sqrt{3}$ sq. units.
(D) Their common tangents do not form any triangle.

8. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles, $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$, λ being the variable. The locus of the point of intersection of these tangents is -
(A) $2x - y + 10 = 0$ (B) $x + 2y - 10 = 0$ (C) $x - 2y + 10 = 0$ (D) $2x + y - 10 = 0$

9. 3 circle of radii 1, 2 and 3 and centres at A, B and C respectively, touch each other. Another circle whose centre is P touches all these 3 circles externally and has radius r. Also $\angle PAB = \theta$ & $\angle PAC = \alpha$ -
(A) $\cos\theta = \frac{3-r}{3(1+r)}$ (B) $\cos\alpha = \frac{2-r}{2(1+r)}$ (C) $r = \frac{6}{23}$ (D) $r = \frac{6}{\sqrt{23}}$

10. Slope of tangent to the circle $(x - r)^2 + y^2 = r^2$ at the point (x, y) lying on the circle is -
(A) $\frac{x}{y-r}$ (B) $\frac{r-x}{y}$ (C) $\frac{y^2-x^2}{2xy}$ (D) $\frac{y^2+x^2}{2xy}$

11. The circle passing through the distinct points (1,t) , (t,1) & (t,t) for all values of 't', passes through the point -
(A) (–1, –1) (B) (–1, 1) (C) (1, –1) (D) (1,1)

12. AB is a diameter of a circle. CD is a chord parallel to AB and 2CD = AB. The tangent at B meets the line AC produced at E then AE is equal to -
(A) AB (B) $\sqrt{2}AB$ (C) $2\sqrt{2}AB$ (D) 2AB

13. The locus of the mid points of the chords of the circle $x^2 + y^2 - ax - by = 0$ which subtend a right angle at $\left(\frac{a}{2}, \frac{b}{2}\right)$ is -

(A) $ax + by = 0$ (B) $ax + by = a^2 + b^2$

(C) $x^2 + y^2 - ax - by + \frac{a^2 + b^2}{8} = 0$ (D) $x^2 + y^2 - ax - by - \frac{a^2 + b^2}{8} = 0$

14. A variable circle is drawn to touch the x-axis at the origin. The locus of the pole of the straight line $\ell x + my + n = 0$ w.r.t. the variable circle has the equation -

(A) $x(my - n) - \ell y^2 = 0$ (B) $x(my + n) - \ell y^2 = 0$ (C) $x(my - n) + \ell y^2 = 0$ (D) none

15. (6,0) , (0,6) and (7,7) are the vertices of a triangle. The circle inscribed in the triangle has the equation -

(A) $x^2 + y^2 - 9x + 9y + 36 = 0$ (B) $x^2 + y^2 - 9x - 9y + 36 = 0$

(C) $x^2 + y^2 + 9x - 9y + 36 = 0$ (D) $x^2 + y^2 - 9x - 9y - 36 = 0$

16. Number of points (x, y) having integral coordinates satisfying the condition $x^2 + y^2 < 25$ is -

(A) 69 (B) 80 (C) 81 (D) 77

17. The centre(s) of the circle(s) passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is/are -

(A) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (C) $\left(\frac{1}{2}, 2^{1/2}\right)$ (D) $\left(\frac{1}{2}, -2^{1/2}\right)$

18. The equation(s) of the tangent at the point (0, 0) to the circle, making intercepts of length 2a and 2b units on the co-ordinate axes, is (are) -

(A) $ax + by = 0$ (B) $ax - by = 0$ (C) $x = y$ (D) $bx + ay = 0$

19. Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'P_1' and 'P_2'. Possible co-ordinates of 'P' so that area of triangle PP_1P_2 is minimum is/are -

(A) (10, 0) (B) $(10\sqrt{2}, 0)$ (C) (–10, 0) (D) $(-10\sqrt{2}, 0)$

20. The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular if -

(A) $h = r$ (B) $h = -r$ (C) $r^2 + h^2 = 1$ (D) $r^2 + h^2 = 2$

21. The common chord of two intersecting circles C_1 and C_2 can be seen from their centres at the angles of 90 & 60 respectively. If the distance between their centres is equal to $\sqrt{3} + 1$ then the radii of C_1 and C_2 are -

(A) $\sqrt{3}$ and 3 (B) $\sqrt{2}$ and $2\sqrt{2}$ (C) $\sqrt{2}$ and 2 (D) $2\sqrt{2}$ and 4

22. In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to -

(A) $\frac{AB.AD}{\sqrt{AB^2 + AD^2}}$ (B) $\frac{AB.AD}{AB+AD}$ (C) $\sqrt{AB.AD}$ (D) $\frac{AB.AD}{\sqrt{AB^2 - AD^2}}$

23. A circle touches a straight line $\ell x + my + n = 0$ and cuts the circle $x^2 + y^2 = 9$ orthogonally. The locus of centres of such circles is -

(A) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$ (B) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

(C) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 + 9)$ (D) none of these

BRAIN TEASERS — ANSWER KEY — EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	B	A,D	A,C,D	C	A,C	A	A,B,C	B,C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	D	D	C	A	B	A	C,D	A,B	A,C	A,B
Que.	21	22	23							
Ans.	C	D	A							

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**.

1.

	Column-I		Column-II
(A)	If point of intersection and number of common tangents of two circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$ are λ and μ respectively, then	(p)	$\mu - \lambda = 3$
(B)	If point of intersection and number of tangents of two circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 2x = 0$ are λ and μ respectively, then	(q)	$\mu + \lambda = 5$
(C)	If the straight line $y = mx \ \forall \ m \in I$ touches or lies outside the circle $x^2 + y^2 - 20y + 90 = 0$ and the maximum and minimum values of $\|m\|$ are μ & λ respectively then	(r)	$\mu - \lambda = 4$
(D)	If two circle $x^2 + y^2 + px + py - 7 = 0$ and $x^2 + y^2 - 10x + 2py + 1 = 0$ cut orthogonally and the value of p are λ & μ respectively then	(s)	$\mu + \lambda = 4$

ASSERTION & REASON

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. Consider two circles $C_1 \equiv x^2 + y^2 + 2x + 2y - 6 = 0$ & $C_2 \equiv x^2 + y^2 + 2x + 2y - 2 = 0$.

Statement-I : Two tangents are drawn from a point on the circle C_1 to the circle C_2, then tangents always perpendicular.

Because

Statement-II : C_1 is the director circle of C_2.

(A) A (B) B (C) C (D) D

2. **Statement-I** : The line $(x - 3)\cos\theta + (y - 3)\sin\theta = 1$ touches a circle $(x - 3)^2 + (y - 3)^2 = 1$ for all values of θ.

Because

Statement-II : $x\cos\theta + y\sin\theta = a$ is a tangent of circle $x^2 + y^2 = a^2$ for all values of θ.

(A) A (B) B (C) C (D) D

3. Consider the circles $C_1 \equiv x^2 + y^2 - 6x - 4y + 9 = 0$ and $C_2 \equiv x^2 + y^2 - 8x - 6y + 23 = 0$.

Statement-I : Circle C_1 bisects the circumference of the circle C_2.

Because

Statement-II : Centre of C_1 lie on C_2.

(A) A (B) B (C) C (D) D

4. **Statement-I** : Circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x + 7 = 0$ intersect each other at two distinct points

Because

Statement-II : Circles with centres C_1 and C_2 and radii r_1 and r_2 intersect at two distinct points, if $|C_1C_2| < r_1 + r_2$

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1 :

Let A ≡ (–3, 0) and B ≡ (3, 0) be two fixed points and P moves on a plane such that PA = nPB $(n > 0)$.

On the basis of above information, answer the following questions :

1. If $n \neq 1$, then locus of a point P is -
   (A) a straight line (B) a circle (C) a parabola (D) an ellipse
2. If $n = 1$, then the locus of a point P is -
   (A) a straight line (B) a circle (C) a parabola (D) a hyperbola
3. If $0 < n < 1$, then -
   (A) A lies inside the circle and B lies outside the circle
   (B) A lies outside the circle and B lies inside the circle
   (C) both A and B lies on the circle (D) both A and B lies inside the circle
4. If $n > 1$, then -
   (A) A lies inside the circle and B lies outside the circle (B) A lies outside the circle and B lies inside the circle
   (C) both A and B lies on the circle (D) both A and B lies inside the circle
5. If locus of P is a circle, then the circle -
   (A) passes through A and B (B) never passes through A and B
   (C) passes through A but does not pass through B (D) passes through B but does not pass through A

Comprehension # 2 :

P is a variable point of the line L = 0. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram PQSR is completed.

On the basis of above information, answer the following questions :

1. If L ≡ 2x + y – 6 = 0, then the locus of circumcetre of ΔPQR is -
   (A) $2x - y = 4$ (B) $2x + y = 3$ (C) $x - 2y = 4$ (D) $x + 2y = 3$
2. If P ≡ (6, 8), then the area of ΔQRS is -
   (A) $\frac{(6)^{3/2}}{25}$ sq. units (B) $\frac{(24)^{3/2}}{25}$ sq. units (C) $\frac{48\sqrt{6}}{25}$ sq. units (D) $\frac{192\sqrt{6}}{25}$ sq. units
3. If P ≡ (3, 4), then coordinate of S is -
   (A) $\left(-\frac{46}{25},-\frac{63}{25}\right)$ (B) $\left(-\frac{51}{25},-\frac{68}{25}\right)$ (C) $\left(-\frac{46}{25},-\frac{68}{25}\right)$ (D) $\left(-\frac{68}{25},-\frac{51}{25}\right)$

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE-3

- ***Match the Column***

  1. (A) → (r, s) ; (B) → (s); (C) → (p); (D) → (q)
- ***Assertion & Reason***

  1. A 2. A 3. B 4. C
- ***Comprehension Based Questions***

  Comprehension # 1 : 1. B 2. A 3. A 4. B 5. B

  Comprehension # 2 : 1. B 2. D 3. B

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. Find the equations of the circles which have the radius $\sqrt{13}$ & which touch the line $2x - 3y + 1 = 0$ at (1, 1).

2. (x_1, y_1) & (x_2, y_2) are the ends of a diameter of a circle such that x_1 & x_2 are the roots of $ax + bx + c = 0$ & y_1 & y_2 are roots of $py + qy + r = 0$. Find the equation of the circle, its centre & radius.

3. If the lines $a_1 x + b_1 y + c_1 = 0$ & $a_2 x + b_2 y + c_2 = 0$ cut the coordinate axes in concyclic points. Prove that $a_1 a_2 = b_1 b_2$.

4. A (–a, 0) ; B (a, 0) are fixed points. C is a point which divides internally AB in a constant ratio $\tan\alpha$. If AC & CB subtend equal angles at P, prove that the equation of the locus of P is $x + y + 2ax \sec 2\alpha + a = 0$.

5. Let A be the centre of the circle $x + y - 2x - 4y - 20 = 0$. Suppose that the tangents at the points B(1 , 7) & D(4 , -2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.

6. Determine the nature of the quadrilateral formed by four lines $3x + 4y - 5 = 0$; $4x - 3y - 5 = 0$; $3x + 4y + 5 = 0$ and $4x - 3y + 5 = 0$. Find the equation of the circle inscribed and circumscribing this quadrilateral.

7. A variable circle passes through the point A (a, b) & touches the x-axis ; show that the locus of the other end of the diameter through A is $(x - a) = 4by$.

8. A circle is drawn with its centre on the line $x + y = 2$ to touch the line $4x - 3y + 4 = 0$ and pass through the point (0, 1). Find its equation.

9. Obtain the equations of the straight lines passing through the point A(2, 0) & making 45 angle with the tangent at A to the circle $(x + 2) + (y - 3) = 25$. Find the equations of the circles each of radius 3 whose centres are on these straight lines at a distance of $5\sqrt{2}$ from A.

10. Suppose the equation of the circle which touches both the coordinates axes and passes through the point with abscissa – 2 and ordinate 1 has the equation $x^2 + y^2 + Ax + By + C = 0$, find all the possible ordered triplet (A, B, C).

11. The foot of the perpendicular from the origin to a variable tangent of the circle $x^2 + y^2 - 2x = 0$ is N. Find the equation of the locus of N.

12. The line $\ell x + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the point P and Q. The circle on PQ as diameter passes through the origin. Prove that $n^2(a + b) = \ell^2 + m^2$.

13. Find the equation of the circle which passes through the point (1, 1) & which touches the circle $x + y + 4x - 6y - 3 = 0$ at the point (2, 3) on it.

14. A circle S = 0 is drawn with its centre at (–1, 1) so as to touch the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ externally. Find the intercept made by the circle S = 0 on the coordinates axes.

15. Find the equation of the circle which cuts each of the circles $x + y = 4$, $x + y - 6x - 8y + 10 = 0$ & $x + y + 2x - 4y - 2 = 0$ at the extremities of a diameter.

16. If the line $x \sin\alpha - y + a \sec\alpha = 0$ touches the circle with radius 'a' and centre at the origin then find the most general values of 'α' and sum of the values of 'α' lying in $[0, 100\pi]$.

17. Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$. Find the condition on a & b if two chords, each bisected by the x-axis, can be drawn to the circle from the point $\left(a, \frac{b}{2}\right)$.

18. Find the equation of a line with gradient 1 such that the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal length on it.

19. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x + y = 100$ into two arcs whose lengths are in the ratio 2 : 1.

20. Find the locus of the middle points of portions of the tangents to the circle $x^2 + y^2 = a^2$ terminated by the coordinate axes.

21. Show that the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in two points at equal distances 'd' from a point (x_1 , y_1) on its circumference is $xx_1 + yy_1 - a^2 + \frac{d^2}{2} = 0$.

22. A point moving around circle $(x + 4)^2 + (y + 2)^2 = 25$ with centre C broke away from it either at the point A or point B on the circle and move along a tangent to the circle passing through the point D(3, –3).

Find the following :

(a) Equation of the tangents at A and B.

(b) Coordinates of the points A and B.

(c) Angle ADB and the maximum and minimum distances of the point D from the circle.

(d) Area of quadrilateral ADBC and the ΔDAB.

(e) Equation of the circle circumscribing the ΔDAB and also the intercepts made by the this circle on the coordinates axes.

23. Show that the equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ represents, for different values of λ, a system of circles passing through two fixed points A, B on the x – axis, and find the equation of that circle of the system the tangents to which at A & B meet on the line $x + 2y + 5 = 0$.

24. Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$.

25. A triangle has two of its sides along the coordinate axes, its third side touches the circle $x + y - 2ax - 2ay + a = 0$. Prove that the locus of the circumcentre of the triangle is : $a - 2a(x + y) + 2xy = 0$.

26. Find the equations to the four common tangents to the circles $x + y = 25$ and $(x - 12) + y = 9$.

27. Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centre of the circles which cut the circles $x + y + 4x - 6y + 9 = 0$ & $x + y - 5x + 4y + 2 = 0$ orthogonally.

CONCEPTUAL SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-4(A)

1. $x + y - 6x + 4y = 0$ or $x + y + 2x - 8y + 4 = 0$ **2.** $x + y + \left(\frac{b}{a}\right)x + \left(\frac{q}{p}\right)y + \left(\frac{c}{a} + \frac{r}{p}\right) = 0$

5. 75 sq.units **6.** square of side 2; $x^2 + y^2 = 1$; $x^2 + y^2 = 2$

8. $x^2 + y^2 - 2x - 2y + 1 = 0$ or $x^2 + y^2 - 42x + 38y - 39 = 0$

9. $x - 7y = 2$, $7x + y = 14$; $(x - 1)^2 + (y - 7)^2 = 3^2$; $(x - 3)^2 + (y + 7)^2 = 3^2$; $(x - 9)^2 + (y - 1)^2 = 3^2$; $(x + 5)^2 + (y + 1)^2 = 3^2$

10. $x^2 + y^2 + 10x - 10y + 25 = 0$ or $x^2 + y^2 + 2x - 2y + 1 = 0$, (10, –10, 25) (2, –2, 1)

11. $(x + y^2 - x)^2 = x^2 + y^2$ **13.** $x + y + x - 6y + 3 = 0$ **14.** zero, zero **15.** $x + y - 4x - 6y - 4 = 0$

16. $\alpha = n\pi$, 5050π **17.** $a > 2b^2$ **18.** $2x - 2y - 3 = 0$ **19.** $4x - 3y - 25 = 0$ or $3x + 4y - 25 = 0$

20. $a^2(x^2 + y^2) = 4x^2y^2$

22. (a) $3x - 4y = 21$; $4x + 3y = 3$; (b) A(0, 1) and B(–1, –6); (c) 90 , $5(\sqrt{2} \pm 1)$ units;(d) 12.5 sq. units;

(e) $x^2 + y^2 + x + 5y - 6 = 0$, x intercept 5; y intercept 7

23. $x^2 + y^2 - 2x - 6y - 8 = 0$ **26.** $2x - \sqrt{5}\,y - 15 = 0$, $2x + \sqrt{5}\,y - 15 = 0$, $x - \sqrt{35}\,y - 30 = 0$, $x + \sqrt{35}\,y - 30 = 0$

27. $9x - 10y + 7 = 0$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. Find the equation of the circle inscribed in a triangle formed by the lines $3x + 4y = 12$; $5x + 12y = 4$ & $8y = 15x + 10$ without finding the vertices of the triangle.

2. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line is drawn from the point P intersects the curve at points Q & R. If the product PQ PR is independent of the slope of the line, then show that the curve is a circle.

3. Find the equation of a circle which is co-axial with circles $2x + 2y - 2x + 6y - 3 = 0$ & $x + y + 4x + 2y + 1 = 0$. It is given that the centre of the circle to be determined lies on the radical axis of these two circles.

4. If $3\ell^2 + 6\ell + 1 - 6m^2 = 0$, then find the equation of the circle for which $\ell x + my + 1 = 0$ is a tangent.

5. Circle are drawn which are orthogonal to both the circles $S \equiv x^2 + y^2 - 16 = 0$ and $S' \equiv x^2 + y^2 - 8x - 12y + 16 = 0$. If tangents are drawn from the centre of the variable circles to S. Then find the locus of the mid point of the chord of contact of these tangents.

6. Show that the locus of the point the tangents from which to the circle $x + y - a = 0$ include a constant angle α is $(x + y - 2a)$ $\tan \alpha = 4a (x + y - a)$.

7. Find the locus of the mid point of the chord of a circle $x + y = 4$ such that the segment intercepted by the chord on the curve $x - 2x - 2y = 0$ subtends a right angle at the origin.

8. Prove that the length of the common chord of the two circles $x + y = a$ and $(x - c) + y = b$ is $\frac{1}{c}\sqrt{(a+b+c)(a-b+c)(a+b-c)(-a+b+c)}$, where a, b, c > 0.

9. Find the equation of the circles passing through the point (2, 8), touching the lines $4x - 3y - 24 = 0$ & $4x + 3y - 42 = 0$ & having x coordinate of the centre of the circle less than or equal to 8.

10. Lines $5x + 12y - 10 = 0$ & $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 & cuts intercepts of length 8 on these lines.

11. A circle touches the line y = x at a point P such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point (–10,2) in its interior and the length of its chord on the line x + y = 0 is $6\sqrt{2}$. Determine the equation of the circle. **[JEE 1990]**

12. Find the intervals of values of 'a' for which the line y + x = 0 bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2 + 2y^2 - \left(1+\sqrt{2}a\right)x - \left(1-\sqrt{2}a\right)y = 0$. **[JEE 1996]**

13. Find the equations of the circles passing through (–4, 3) and touching the lines x + y = 2 and x – y = 2.

14. P is a variable point on the circle with centre at C . CA & CB are perpendiculars from C on x-axis & y-axis respectively. Show that the locus of the centroid of the triangle PAB is a circle with centre at the centroid of the triangle CAB & radius equal to one third of the radius of the given circle.

BRAIN STORMING SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-4(B)

1. $x^2 + y^2 - 2x - 2y + 1 = 0$
3. $4x^2 + 4y^2 + 6x + 10y - 1 = 0$
4. $x^2 + y^2 - 6x + 3 = 0$
5. $x^2 + y^2 - 4x - 6y = 0$
7. $x + y - 2x - 2y = 0$
9. centre (2,3), r = 5; centre $\left(-\frac{182}{9}, 3\right)$, $r = \frac{205}{9}$
10. $x^2 + y^2 - 10x - 4y + 4 = 0$
11. $(x - 9)^2 + (y - 1)^2 = 50$
12. $a \in (-\infty, -2) \cup (2, \infty)$
13. $x^2 + y^2 + 2(10 \pm \sqrt{54})x + 55 \pm 8\sqrt{54} = 0$

EXERCISE - 05 [A] **JEE-[MAIN] : PREVIOUS YEAR QUESTIONS**

1. The square of the length of tangent from (3, –4) on the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ [AIEEE-2002]
(1) 20 (2) 30 (3) 40 (4) 50

2. Radical axis of the circles $x^2 + y^2 + 6x - 2y - 9 = 0$ and $x^2 + y^2 - 2x + 9y - 11 = 0$ is- [AIEEE-2002]
(1) $8x - 11y + 2 = 0$ (2) $8x + 11y + 2 = 0$ (3) $8x + 11y - 2 = 0$ (4) $8x - 11y - 2 = 0$

3. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then- [AIEEE-2003]
(1) $r > 2$ (2) $2 < r < 8$ (3) $r < 2$ (4) $r = 2$

4. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is- [AIEEE-2003]
(1) $x^2 + y^2 - 2x + 2y = 62$ (2) $x^2 + y^2 + 2x - 2y = 62$
(3) $x^2 + y^2 + 2x - 2y = 47$ (4) $x^2 + y^2 - 2x + 2y = 47$

5. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is- [AIEEE-2004]
(1) $2ax + 2by + (a^2 + b^2 + 4) = 0$ (2) $2ax + 2by - (a^2 + b^2 + 4) = 0$
(3) $2ax - 2by + (a^2 + b^2 + 4) = 0$ (4) $2ax - 2by - (a^2 + b^2 + 4) = 0$

6. A variable circle passes through the fixed point A(p, q) and touches x-axis. The locus of the other end of the diameter through A is- [AIEEE-2004]
(1) $(x - p)^2 = 4qy$ (2) $(x - q)^2 = 4py$ (3) $(y - p)^2 = 4qx$ (4) $(y - q)^2 = 4px$

7. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is- [AIEEE-2004]
(1) $x^2 + y^2 - 2x + 2y - 23 = 0$ (2) $x^2 + y^2 - 2x - 2y - 23 = 0$
(3) $x^2 + y^2 + 2x + 2y - 23 = 0$ (4) $x^2 + y^2 + 2x - 2y - 23 = 0$

8. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is- [AIEEE-2004]
(1) $x^2 + y^2 - x - y = 0$ (2) $x^2 + y^2 - x + y = 0$ (3) $x^2 + y^2 + x + y = 0$ (4) $x^2 + y^2 + x - y = 0$

9. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct point P and Q then the line $5x + by - a = 0$ passes through P and Q for- [AIEEE-2005]
(1) exactly one value of a (2) no value of a
(3) infinitely many values of a (4) exactly two values of a

10. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is- [AIEEE-2005]
(1) an ellipse (2) a circle
(3) a hyperbola (4) a parabola

11. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is- [AIEEE-2005]
(1) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$ (2) $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(3) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$ (4) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

12. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then- [AIEEE-2005]
(1) $3a^2 - 10ab + 3b^2 = 0$ (2) $3a^2 - 2ab + 3b^2 = 0$
(3) $3a^2 + 10ab + 3b^2 = 0$ (4) $3a^2 + 2ab + 3b^2 = 0$

13. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is- [AIEEE-2006]
(1) $x^2 + y^2 + 2x - 2y - 62 = 0$ (2) $x^2 + y^2 - 2x + 2y - 62 = 0$
(3) $x^2 + y^2 - 2x + 2y - 47 = 0$ (4) $x^2 + y^2 + 2x - 2y - 47 = 0$

14. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is - [AIEEE-2006, IIT-1996]

(1) $x^2 + y^2 = 1$ (2) $x^2 + y^2 = \frac{27}{4}$ (3) $x^2 + y^2 = \frac{9}{4}$ (4) $x^2 + y^2 = \frac{3}{2}$

15. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval- [AIEEE-2007]

(1) $0 < k < 1/2$ (2) $k \ge 1/2$ (3) $-1/2 \le k \le 1/2$ (4) $k \le 1/2$

16. The point diametrically opposite to the point (1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is- [AIEEE-2008]

(1) (3, –4) (2) (–3, 4) (3) (–3, –4) (4) (3, 4)

17. Three distinct points A, B and C are given in the 2–dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point :- [AIEEE-2009]

(1) $\left(\frac{5}{2}, 0\right)$ (2) $\left(\frac{5}{3}, 0\right)$ (3) (0, 0) (4) $\left(\frac{5}{4}, 0\right)$

18. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for :- [AIEEE-2009]

(1) All except two values of p (2) Exactly one value of p

(3) All values of p (4) All except one value of p

19. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is :- [AIEEE-2010]

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$ (4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

20. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if :- [AIEEE-2010]

(1) $-85 < m < -35$ (2) $-35 < m < 15$ (3) $15 < m < 65$ (4) $35 < m < 85$

21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (c>0) touch each other if :- [AIEEE-2011]

(1) $a = 2c$ (2) $|a| = 2c$ (3) $2|a| = c$ (4) $|a| = c$

22. The equation of the circle passing through the points (1, 0) and (0, 1) and having the smallest radius is:

(1) $x^2 + y^2 + x + y - 2 = 0$ (2) $x^2 + y^2 - 2x - 2y + 1 = 0$ [AIEEE-2011]

(3) $x^2 + y^2 - x - y = 0$ (4) $x^2 + y^2 + 2x + 2y - 7 = 0$

23. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is : [AIEEE-2012]

(1) 5/3 (2) 10/3 (3) 3/5 (4) 6/5

24. The circle passing through (1, – 2) and touching the axis of x at (3, 0) also passes through the point : [JEE(Main)-2013]

(1) (–5, 2) (2) (2, –5) (3) (5, –2) (4) (–2, 5)

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	3	1	2	4	2	1	1	1	2	4	4	4	3	3	2
Que.	16	17	18	19	20	21	22	23	24						
Ans	3	4	4	3	2	4	3	2	3						

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then 2r equals

(A) $\sqrt{PQ \cdot RS}$ (B) $\frac{PQ+RS}{2}$ (C) $\frac{2PQ \cdot RS}{PQ+RS}$ (D) $\sqrt{\frac{(PQ)^2+(RS)^2}{2}}$

[JEE 2001 (Screening) 1]

2. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA.

[JEE 2001 (Mains), 5]

3. Find the equation of the circle which passes through the points of intersection of circles $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ and intersects the circle $x^2 + y^2 + 4x + 6y + 4 = 0$ orthogonally. [REE 2001 (Mains), 3]

4. Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of centre of the circumcircle of triangle TPQ. [REE 2001 (Mains), 5]

5. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

[JEE 2002 (Scr), 3]

6. If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ (B) $\frac{\sqrt{a^2-4b^2}}{2b}$ (C) $\frac{2b}{a-2b}$ (D) $\frac{b}{a-2b}$

[JEE 2002 (Scr), 3]

7. The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$

(A) 1 (B) 2 (C) 3 (D) $\sqrt{3}$ [JEE 2004 (Scr)]

8. Line $2x + 3y + 1 = 0$ is a tangent to a circle at (1, −1). This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points (0, −1) and (− 2, 3). Find equation of circle. [JEE 2004, 4]

9. A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is [JEE 2005 (Scr)]

(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ (B) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{x, y) : y \le 0\}$

(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ (D) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

10. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is [JEE 2007, 3]

(A) 3 (B) 2 (C) 3/2 (D) 1

11. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

Statement-1 : The tangents are mutually perpendicular.

because

Statement-2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true. [JEE 2007, 3]

12. Consider the two curves $C_1 : y^2 = 4x$; $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other **[JEE 2008, 3]**

13. Consider, $L_1 : 2x + 3y + p - 3 = 0$; $L_2 : 2x + 3y + p + 3 = 0$,

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$.

Statement-1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

and

Statement-2 : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement-1 is True, Statement-2 is True; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True; statement-2 is **NOT** a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True **[JEE 2008, 3]**

14. **Comprehension (3 questions together):**

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

(i) The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ (B) $(x - 2\sqrt{3})^2 + (y + \frac{1}{2})^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ (D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

(ii) Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(iii) Equations of the sides RP, RQ are

(A) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x, y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}x, y = 0$ **[JEE 2008, 4+4+4]**

15. Tangents drawn from the point P(l, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$ **[JEE 2009, 3]**

16. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is **[JEE 2009, 4]**

17. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is **[JEE 10, 3]**

[**Note** : [k] denotes the largest integer less than or equal to k]

18. The circle passing through the point (–1,0) and touching the y-axis at (0, 2) also passes through the point -

(A) $\left(-\frac{3}{2},0\right)$ (B) $\left(-\frac{5}{2},2\right)$ (C) $\left(-\frac{3}{2},\frac{5}{2}\right)$ (D) (–4,0)

[JEE 2011, 3, –1]

19. The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is **[JEE 2011, 4]**

20. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line 4x – 5y = 20 to the circle $x^2 + y^2 = 9$ is- **[JEE 2012, 3, –1]**

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question 21 and 22

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P\left(\sqrt{3},1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

21. A common tangent of the two circles is **[JEE 2012, 3, –1]**

(A) x = 4 (B) y = 2 (C) $x+\sqrt{3}y=4$ (D) $x+2\sqrt{2}y=6$

22. A possible equation of L is **[JEE 2012, 3, –1]**

(A) $x-\sqrt{3}y=1$ (B) $x+\sqrt{3}y=1$ (C) $x-\sqrt{3}y=-1$ (D) $x+\sqrt{3}y=5$

23. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ or y-axis is (are) **[JEE(Advanced) 2013, 3, (–1)]**

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$

(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [B]

1. A	2. OA = $3(3+\sqrt{10})$		3. $x^2 + y^2 + 14x - 6y + 6 = 0$;		4. 2px + 2qy = r
5. C	6. A	7. C	8. $2x^2 + 2y^2 - 10x - 5y + 1 = 0$		9. D
10. B	11. A	12. B	13. C	14. (i) D, (ii) A, (iii) D	15. B
16. 8	17. 3	18. D	19. 2	20. A	21. D	22. A
23. A,C					