Conic Sections

Sections of a cone on intersecting a right circular cone by a plane in different positions, different sections so obtained are called conic section or conics.

1. Circle

A circle is the set of all points in a plane, which are at a fixed distance from a fixed point in the plane. The fixed point is called the **centre** of the circle and the distance from centre to any point on the circle is called the **radius** of the circle.

(i) The **standard equation** of a circle with radius r having centre (h, k) is given by $(x-h)^2 + (y-k)^2 = r^2$.

(ii) The **general equation** of the circle is given by $x^2 + y^2 + 2gx + 2fy + c = 0$, where, g, f and c are constants.

(a) The centre of the circle is $(-g, -f)$.

(b) The radius of the circle is $r = \sqrt{g^2 + f^2 - c}$.

(iii) The general equation of the circle passing through origin is $x^2 + y^2 + 2gx + 2fy = 0$.

(iv) The equation of a circle having end points of a diameter (x_1, y_1) and (x_2, y_2), is given by
$(x - x_1)(x - x_2) + (y - y_1)\cdot(y - y_2) = 0$

Note *The general equation of the circle involves three constants which implies that atleast three conditions are required to determine a circle uniquely.*

Position of a Point with Respect to Circle

Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be the equation of the circle and $P(x_1, y_1)$ be any point in the plane of circle, then point P lies outside, on or inside of the circle, if $S_1 >, =$ or < 0, where $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$

2. Parabola

A parabola is the locus of a point which moves in a plane, so that its distance from a fixed point called focus is always equal to its distance from a fixed straight line called directrix in the same plane.

3. Different Terms Related to the Parabola

Forms of parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Axis of parabola	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directrix of parabola	$x = -a$	$x = a$	$y = -a$	$y = a$
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	(a, 0)	(– a, 0)	(0, a)	(0, – a)
Length of latusrectum	4a	4a	4a	4a
Focal length	$\lvert x + a \rvert$	$\lvert x - a \rvert$	$\lvert y + a \rvert$	$\lvert y - a \rvert$

4. Ellipse

An ellipse is the locus of a point in the plane which moves in the plane in such a way that the ratio of its distance from a fixed point in the plane to its distance from a fixed straight line is always constant. This ratio is called eccentricity (e) which is less than one. The fixed point is called focus and the fixed line is called a directrix.

We have two standard forms of ellipse i.e.

(i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [horizontal ellipse]

(ii) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ [vertical ellipse]

In both the cases,

$a > b$ and $b^2 = a^2(1 - e^2), e < 1$

5. Different Terms Related to the Ellipse

Forms of the ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, a > b$
Equation of major axis	$y = 0$	$x = 0$
Length of major axis	2a	2a
Equation of minor axis	$x = 0$	$y = 0$
Length of minor axis	2b	2b
Value of c	$c = \sqrt{a^2 - b^2}$	$c = \sqrt{a^2 - b^2}$
Eccentricity	$e = \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$	$e = \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$
Equation of directrices	$x = \pm \frac{a^2}{c} = \pm \frac{a}{e}$	$y = \pm \frac{a^2}{c} = \pm \frac{a}{e}$
Vertex	(± a, 0)	(0, ± a)
Foci	(±, c, 0) or (± ae, 0)	(0, ± c) or (0, ± ae)
Length of latusrectum	$2b^2 / a$	$2b^2 / a$

6. Hyperbola

A hyperbola is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point in the same plane to its distance from a fixed line is always constant which is always greater than unity. The fixed point is called the focus, the fixed line is called the directrix and the constant ratio, generally denoted by e, is known as eccentricity of the hyperbola.

We have two standard forms of hyperbola i.e.

(i) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(ii) $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ [conjugate hyperbola]

7. Different Terms Related to the Hyperbola

Forms of the hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Coordinates of centre	(0, 0)	(0, 0)
Coordinates of vertices	(± a, 0)	(0 ± a)
Value of c	$c = \sqrt{a^2 + b^2}$	$c = \sqrt{a^2 + b^2}$
Eccentricity	$e = \frac{c}{a}$	$e = \frac{c}{a}$
Foci	(± ae, 0)	(0, ± ae)
Length of transverse axis	2a	2a
Length of conjugate axis	2b	2b
Equation of directrices	$x = \pm \frac{a^2}{c}$ or $\pm a/e$	$y = \pm \frac{a^2}{c}$ or $\pm a/e$
Eccentricity	$e = \sqrt{\frac{a^2 + b^2}{a^2}}$	$e = \sqrt{\frac{a^2 + b^2}{a^2}}$
Length of latusrectum	$2b^2 / a$	$2b^2 / a$
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = 0$	$y = 0$