Gravitation

1. Kepler's Laws of Planetary Motion

Kepler's laws of planetary motion are three scientific laws describing motion of planets around the sun.

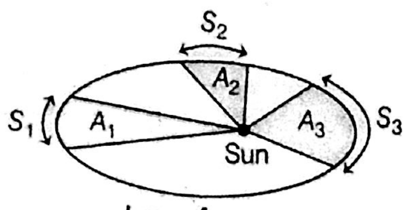
(i) Kepler's First Law of Orbits

According to this law, all planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.

This law identifies that the distance between the sun and earth is constantly changing as the earth goes around its orbit.

(ii) Kepler's Second Law of Areas

According to this law, the speed of planet varies in such a way that the radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time. i.e. the areal velocity of the planet around the sun is constant.



Law of areas

$\therefore$ Areal velocity, $\frac{dA}{dt}$ = constant.

The elliptical orbit of a planet around the sun is shown in figure. According to Kepler's second law, $A_1 = A_2 = A_3$

(iii) Kepler's Third Law of Period

According to this law, the square of the time period of revolution of a planet around the sun is proportional to the cube of the semi-major axis of its elliptical orbit.

i.e. $T^2 \propto a^3$

where, a = semi-major axis

2. Universal Law of Gravitation

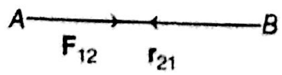
It states that, every body in this universe attracts other body with a force whose magnitude is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Gravitational force, $F = G\frac{m_1 m_2}{r^2}$

where, G is a constant of proportionality known as gravitational constant. It is also known as universal gravitational constant. In CGS system, the value of G is 6.67×10^{-8} dyne cm^2-g^{-2} and its SI value is 6.67×10^{-11} N-m^2-kg^{-2}.

Dimensional formula for G is $[M^{-1}L^3T^{-2}]$.

3. Vector Form of Newton's Law of Gravitation



In vector notation, Newton's law of gravitation is written as follows $\mathbf{F}_{12} = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$ ...(i)

where, F_{12} = gravitational force exerted on A by B and $\hat{r}_{21}$ is a unit vector pointing towards A.
Negative sign shows that the gravitational force is attractive in nature.

Similarly, $$\mathbf{F}_{21} = -G\frac{m_1m_2}{r_{12}^2}\hat{r}_{12} \quad \ldots(ii)$$

where, $\hat{r}_{12}$ is a unit vector pointing towards B.

Equating Eqs. (i) and (ii), we have

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

As $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are directed towards the centres of the two particles, so, gravitational force is a central force.

4. Central Forces

Central force is that force which acts along the line joining towards the centres of two interacting bodies. As the angular momentum of the particle is conserved, the torque vanishes as **F** is zero or along **r**.

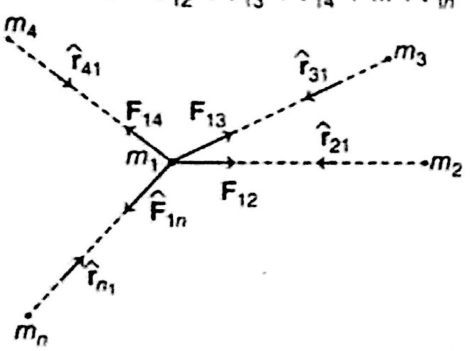
$$\therefore \quad \frac{d\mathbf{L}}{dt} = \mathbf{r}\times\mathbf{F} = 0 \text{ as } \tau = \mathbf{r}\times\mathbf{F}$$

Thus, central force satisfies this condition. A central force is always directed towards the centre from a fixed point.

5. Principle of Superposition

The resultant force **F** can be expressed in vector addition of various forces

$$\mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \ldots + \mathbf{F}_{1n}$$



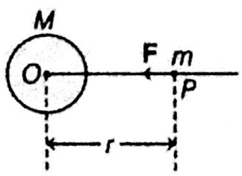
Superposition principle of gravitational forces

Resultant force,

$$\mathbf{F} = -Gm_1\left(\frac{m_2}{r_{21}^2}\hat{r}_{21} + \frac{m_3}{r_{31}^2}\hat{r}_{31} + \ldots + \frac{m_n}{r_{n1}^2}\hat{r}_{n1}\right)$$

6. Intensity of Gravitational Field at a Point

Let, M be the mass of a body around which a gravitational field exists. In order to get the gravitational field intensity at a point P in the gravitational field, a test mass m is placed at the point P.



Gravitational field intensity

If a test mass m at a point P in a gravitational field experience a force **F**, then

intensity of gravitational field, $I = \frac{F}{m} = \frac{GM}{r^2}$

where, r is the distance of P from the centre of the body producing the gravitational field.

7. Acceleration due to Gravity of Earth

The force of gravity acting on a body having unit mass placed on or near the surface of the earth is known as acceleration due to gravity. It is represented by symbol g and its value is $9.8\ \text{ms}^{-2}$ on the surface of the earth.

Acceleration due to gravity, $g = \frac{GM}{R^2}$

Substituting the values of $M = 6\times10^{24}$ kg and $R = 6.4\times10^6$ m, we get $g = 9.8\ \text{ms}^{-2}$.

(i) Acceleration due to Gravity above the Surface of Earth

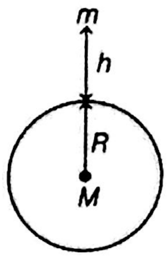
Acceleration due to gravity at height h,

$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

If $h << R$, then h/R is very small.

Acceleration due to gravity at height h, $g_h = g\left(1-\frac{2h}{R}\right)$

The value of acceleration due to gravity decreases with increase in height above the surface of the earth.



Variation of g with altitude

(ii) Acceleration due to Gravity below the Earth

Acceleration due to gravity at depth d,

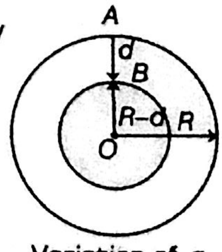
$$g_d = g\left(1-\frac{d}{R}\right)$$

The acceleration due to gravity decreases as we move down into the surface of the earth.

At the centre of the earth, $d = R$, we get

$$g_d = g\left(1-\frac{R}{R}\right) = 0$$

Hence, acceleration due to gravity at the centre of the earth is zero.



Variation of g with depth

8. Gravitational Potential

Gravitational potential at a point in the gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

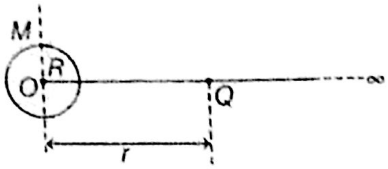
i.e. $$V = -\frac{W}{m} = -\int\frac{\mathbf{F}\cdot d\mathbf{r}}{m} = -\int \mathbf{I}\cdot d\mathbf{r} \quad \left(\text{as, } \frac{F}{m} = I\right)$$

It is a scalar quantity. The unit of gravitational potential in SI system is J-kg^{-1} and in CGS system is erg-g^{-1}.

Dimensional formula for gravitational potential

$$= [M^0L^2T^{-2}]$$

Expression for Gravitational Potential



Gravitational potential

Gravitational potential, $V = \dfrac{-GM}{r}$ ...(i)

where, V is total gravitational potential at point Q, due to a spherical body or earth of mass M.

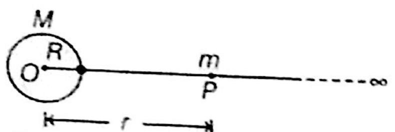
Special Cases

(i) When $r = \infty$ then from Eq. (i), $V = 0$, hence gravitational potential is maximum (zero) at infinity.

(ii) At surface of the earth $r = R$, then $V = \dfrac{-GM}{R}$.

9. Gravitational Potential Energy

Gravitational potential energy of a body at a point is defined as the amount of work done in bringing the given body from infinity to that point against the gravitational force.



Gravitational potential energy

Gravitational potential energy, $U = \left(-\dfrac{GM}{r}\right) \times m$

Note *Gravitational potential inside a spherical shell is constant. For calculation of gravitational potential or potential energy of outside points, a spherical body or earth can be treated as a point mass placed at its centre.*

10. Escape Speed

Escape speed on the earth (or any other planet) is defined as the minimum speed with which a body should be projected vertically upwards from the surface of the earth, so that it just escapes out from gravitational field of the earth and never returns on its own.

$\therefore$ Escape velocity, $v_e = \sqrt{2gR}$

where, R is the radius of the earth.

Also, escape velocity, $v_e = R\sqrt{\dfrac{8}{3}\pi G\rho}$

where, ρ is the mean density of the earth.

11. Earth's Satellites

A satellite is a body which is constantly revolving in an orbit around a comparatively much larger body. e.g. The moon is a natural satellite while INSAT-1B is an artificial satellite of the earth. Condition for establishment of satellite is that the centre of orbit of satellite must coincide with centre of the earth or satellite must move around greater circle of the earth.

12. Energy of an Orbiting Satellite

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of the earth) and kinetic energy (due to orbital motion). If m is the mass of the satellite and v is its orbital velocity, then

KE of the satellite,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{r} \quad [\because v = \sqrt{GM/r}]$$

Kinetic energy of the satellite,

$$K = \frac{GMm}{2(R+h)} \quad [\because r = R + h] \quad \text{...(i)}$$

PE of the satellite,

$$U = mV = -\frac{GMm}{R+h} \quad \text{...(ii)}$$

Total mechanical energy of satellite,

$$E = K + U$$

$$E = -\frac{GMm}{2(R+h)}$$

Satellites are always at finite distance from the earth and hence their energies cannot be positive or zero.

13. Geostationary Satellite

These satellites are in a circular orbits around the earth in the equatorial plane with $T = 24$ h and is also known as geosynchronous satellites.

(*i*) It should revolve in an orbit concentric and coplanar with the equatorial plane.

(*ii*) These satellites appears stationary due to its ro relative velocity w.r.t. that place on earth.

(*iii*) It should be at a height around 36000 km.

(*iv*) These satellites are used for communication purpose like radio broadcast, TV broadcast, etc.

14. Polar Satellite

They are low-altitude satellites $n = a^3D$ which circle in a North-South orbit passing over the North and South poles. It is also known as sun synchronous satellite.

(*i*) The time period is about 100 min.

(*ii*) These satellites are used for military purpose.

15. Weightlessness

At one particular position, the two gravitational pulls may be equal and opposite and the net pull on the body becomes zero. This is zero gravity region or the null point and the body is said to be weightless. *The state of weightlessness can be observed in the following situations*:

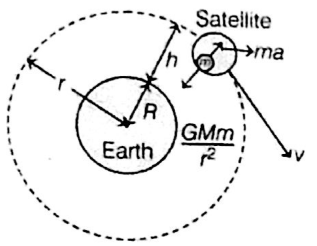
(*i*) When objects fall freely under gravity.

(*ii*) When a satellite revolves in its orbit around the earth.

(*iii*) When bodies are at null points in outer space

Weightlessness in a Satellite

A satellite which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. If a body of mass m placed on a surface inside a satellite moving around the earth.



Weightlessness in a satellite

The gravitational pull of the earth $= \frac{GMm}{r^2}$

The reaction by the surface $= R$

By Newton's law, $\frac{GMm}{r^2} - R = ma$

$$\frac{GMm}{r^2} - R = m\left(\frac{GM}{r^2}\right)$$

$$\left[\because a = \frac{GM}{r^2}, \text{acceleration of satellite}\right]$$

$$R = 0$$

Thus, the surface does not exert any force on the body and hence its apparent weight is zero.