Motion in a Straight Line

1. **Mechanics**
It is a branch of physics in which we study the motion of objects.

2. **Rest**
If the position of an object does not change w.r.t. its surrounding with the passage of time, it is said to be at rest. e.g. Book lying on the table, a person sitting on a chair, etc.

3. **Motion**
If the position of an object is continuously changing w.r.t. its surrounding, then it is said to be in the state of motion. Motion is a change in position of an object with time. e.g. The crawling insects, water flowing down a dam, etc.

4. **Types of Motion**
On the basis of the nature of path followed, *motion is classified as*

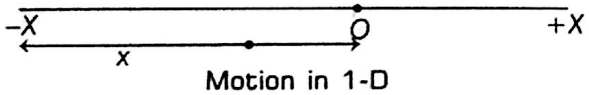
(*i*) **Rectilinear Motion** The motion in which a particle moves along a straight line is called rectilinear motion. e.g. Motion of a sliding body on an inclined plane.

(*ii*) **Circular Motion** The motion in which a particle moves in a circular path is called circular motion. e.g. A string whirled in a circular loop.

(*iii*) **Oscillatory Motion** The motion in which a particle moves to and fro about a given point is known as oscillatory motion. e.g. Simple pendulum.

On the basis of the number of coordinates *required to define the motion, motion is classified as*

(*i*) **One-dimensional Motion (1-D)** The motion of an object is considered as 1-D if only one coordinate is needed to specify the position of the object.

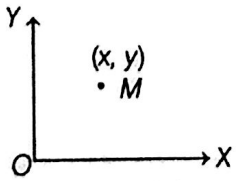


Motion in 1-D

In 1-D motion, the object moves along a straight line. e.g. A boy running on a straight road.

(*ii*) **Two-dimensional Motion (2-D)**

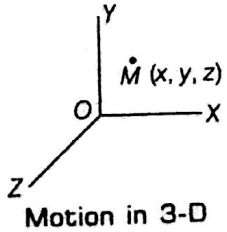
The motion of an object is considered as 2-D, if two coordinates are needed to specify the position of the object. In 2-D motion, the object moves in a plane. e.g. A satellite revolving around the earth.



Motion in 2-D

(*iii*) **Three-dimensional Motion (3-D)**

The motion of an object is considered as 3-D, if all the three coordinates are needed to specify the position of the object. This type of motion takes place in three-dimensional space.



Motion in 3-D

e.g. Butterfly flying in garden, the motion of water molecules, motion of kite in the sky.

5. **Point Object**
An object is considered as point object if the size of the object is much smaller than the distance, it moves in a reasonable duration of time.
e.g. Earth can be considered as a point object in its orbit.

6. **Position**
It is defined as the point where an object is situated. Position can be determined by the coordinate axis, that is marked in units of length and that has positive and negative directions.

7. **Frame of Reference**
It is a coordinate system with a clock w.r.t. which, an observer can describe the position, displacement, acceleration of an object.

8. **Path Length or Distance**
The length of the path covered by the object in a given time-interval, is known as its path length or distance travelled. Path length is a scalar quantity which has only magnitude but no direction.

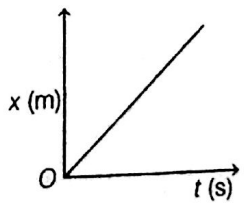
9. **Displacement**
The change in position of an object in a particular direction is termed as displacement, i.e. the difference between the final and initial positions of the object in a given time. It is denoted by Δx.
Mathematically, it is represented by displacement, $\Delta x = x_2 - x_1$ where, x_1 and x_2 are the initial and final positions of the object, respectively.

Cases

(*i*) If $x_2 > x_1$, then Δx is positive.
(*ii*) If $x_1 > x_2$, then Δx is negative.
(*iii*) If $x_1 = x_2$, then Δx is zero.

i.e. the displacement of an object in motion can be positive, negative or zero.
Displacement is a vector quantity as it possesses both, the magnitude and direction.

10. **Uniform Motion in a Straight Line**
A body is said to be in a uniform motion, if it travels equal distances in equal intervals of time along a straight line. A distance (x)-time (t) graph for uniform motion is a straight line.



Uniform motion in a straight line

11. **Non-uniform Motion**
A body is said to be in non-uniform motion, if it travels equal displacements in unequal intervals of time.

12. Speed

The path length or the distance covered by an object divided by the time taken by the object to cover that distance is called the speed of that object.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Speed is a **scalar quantity**. The speed of the object for a given interval of time is always positive.

Unit of speed The unit of speed in MKS (SI) is ms^{-1} and in CGS is cms^{-1}.

Dimensional formula $[M^0LT^{-1}]$.

Types of Speed

(*i*) **Uniform Speed** If an object is moving with a uniform speed, it means that it covers equal distances in equal intervals of time.

(*ii*) **Variable or Non-Uniform Speed** If an object is moving with a non-uniform speed, it means that it covers unequal distances in equal intervals of time.

(*iii*) **Average Speed** Average speed of an object is defined as the total distance travelled divided by the total time taken.

$$\text{Average speed, } v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

(*iv*) **Instantaneous Speed** Speed at an instant is defined as the limit of the average speed as the time interval (Δt) becomes infinitesimally small or approaches to zero.

Mathematically, instantaneous speed (S_i) at any instant of time (t) is expressed as

$$S_i = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \text{ or } S_i = \frac{ds}{dt}$$

[where, ds is the distance covered in time dt]

13. Velocity

The rate of change in position or displacement of an object with time is called the velocity of that object.

i.e. $$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Velocity is a vector quantity.

Note The velocity of an object can be positive, zero and negative according to its displacement, i.e. positive, zero and negative.

Unit of velocity The unit of velocity in CGS is $cm\text{-}s^{-1}$ and in MKS or SI is ms^{-1}.

Dimensional formula $[M^0LT^{-1}]$.

Types of Velocity

(*i*) **Uniform Velocity** An object could have uniform velocity if it covers equal displacement in equal interval of time.

(*ii*) **Non-uniform Velocity** An object could have non-uniform velocity, if it covers unequal displacements in equal interval of time.

(*iii*) **Average Velocity** Average velocity of a body is defined as the change in position or displacement (Δx) divided by the time interval (Δt) in which that displacement occurs.

$\therefore$ Average velocity,

$$v_{av} = \frac{\text{Total displacement covered } (\Delta x)}{\text{Total time taken } (\Delta t)}$$

(*iv*) **Instantaneous Velocity** Velocity at an instant is defined as the limit of average velocity as the time interval (Δt) becomes infinitesimally small or approaches to zero.

Mathematically, instantaneous velocity (v_i) at an instant of time (t) is given by $v_i = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

or $$v_i = \frac{dx}{dt}$$

where, dx is displacement for time dt.

14. Acceleration

Acceleration of a body can be expressed as the rate of change of velocity with time.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Acceleration is a vector quantity. The SI unit of acceleration is ms^{-2} and in CGS is cms^{-2}. The dimensional formula is $[M^0LT^{-2}]$.

Types of Acceleration

(*i*) **Uniform Acceleration** If an object is moving with uniform acceleration, it means that the change in velocity is equal for equal intervals of time.

(*ii*) **Non-uniform Acceleration** If an object has variable or non-uniform acceleration, it means that the change in velocity is unequal in equal intervals of time.

(*iii*) **Average Acceleration** The average acceleration over a time interval is defined as the change in velocity divided by the time interval.

Average acceleration,

$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1}$$

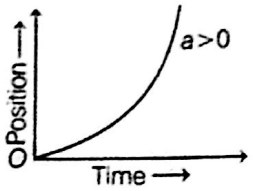
(*iv*) **Instantaneous Acceleration** It is defined as the acceleration of a body at a certain instant or the limiting value of average acceleration when time interval becomes very small or tends to zero. So, instantaneous acceleration,

$$\mathbf{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

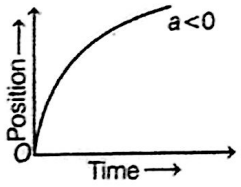
where, $\frac{d\mathbf{v}}{dt}$ is the differential coefficient of $\mathbf{v}$ w.r.t. t.

15. Position-Time Graphs for Positive, Negative and Zero Acceleration

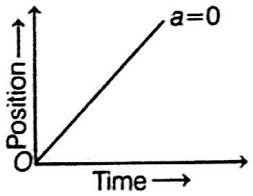
(*i*) **Positive Acceleration** When position of a moving object goes on increasing with time, the object is said to have positive acceleration.



(*ii*) **Negative Acceleration** When position of a moving object goes on decreasing with time, the object is said to have negative acceleration.

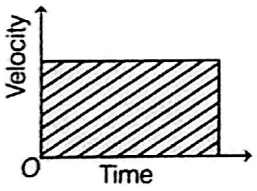


(*iii*) **Zero Acceleration** When the moving object possesses equal change of position in equal interval of time, the object is said to have zero acceleration.



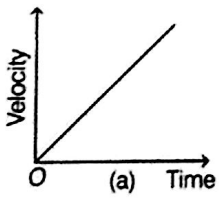
16. Velocity-Time Graphs for an Accelerated Motion

(*i*) **Zero Acceleration** In case of zero acceleration, the velocity of the object does not change with time.

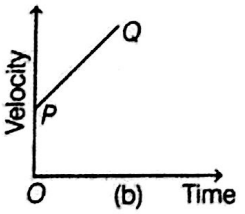


(*ii*) **Positive Acceleration** In case of positive acceleration, the velocity of the object goes on increasing with time.

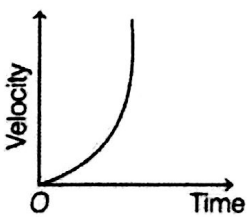
(a) If the object is moving with positive acceleration having zero initial velocity, then the velocity-time graph is a straight line starting from origin.



(b) If the object is moving with positive acceleration having some initial velocity, then the velocity-time graph is a straight line starting from *P*.



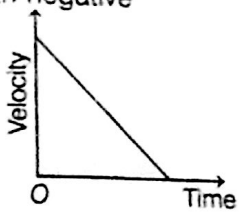
(*iii*) **Increasing Acceleration** In case of increasing acceleration, the velocity of the object goes on increasing exponentially (non-linearly).



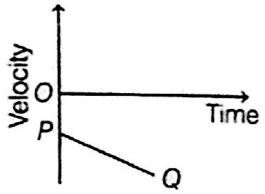
If the object is moving with increasing acceleration having zero initial velocity the slope of *v*-*t* graph gives the instantaneous acceleration.

(*iv*) **Negative Acceleration** In case of negative acceleration, the velocity of the object decreases linearly with time.

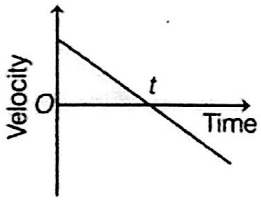
(a) If the object is moving with negative acceleration, having some positive initial velocity, then the velocity- time graph is a straight line having negative slope.



(b) If the object is moving with negative acceleration having some negative initial velocity, then the velocity-time graph is a straight line starting from *P* as point of negative velocity.



(c) If the object is moving with negative acceleration having some positive initial velocity, the direction of its motion can change at time (*t*). The slope of velocity-time graph will remain constant for uniform acceleration.



17. Kinematic Equations for Uniformly Accelerated Motion

If the change in velocity of an object in each unit of time is constant, then object is said to be moving with constant acceleration and such a motion is called uniformly accelerated motion. If an object moves along a straight line with a constant acceleration *a*. Let *u* be the initial velocity at $t = 0$ and *v* be the final velocity of the object after time (*t*).

Velocity-Time Relation $v = u + at$

Position-Time Relation $x = ut + \frac{1}{2}at^2$

Position-Velocity Relation $v^2 = u^2 + 2ax$

where, *x* is the position of the object at time *t*.

18. Displacement of the Particle in *n*th Second

Displacement in the *n*th second, $s(n\text{th}) = u + \frac{a}{2}(2n - 1)$

19. Non-uniformly Accelerated Motion

When acceleration of particle is not constant or acceleration is a function of time, then following relations hold for one-dimensional motion.

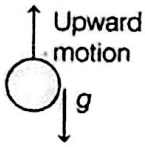
(*i*) $v = \frac{ds}{dt}$ (*ii*) $ds = v\,dt$

(*iii*) $a = \frac{dv}{dt} = v\frac{dv}{dx}$ and (*iv*) $dv = a\,dt$ or $v\,dv = a\,dx$

20. Equations of Motion for the Motion of an Object under Gravity

When an object is thrown upwards or fall towards the earth, then its motion is called motion under gravity.

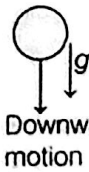
In case of motion under gravity, the equations of motion are given below.



$$v = u + (\mp g)t \quad \text{...(i)}$$

$$h = ut + \frac{1}{2}(\mp g)t^2 \quad \text{...(ii)}$$

$$v^2 = u^2 + 2(\mp g)h \quad \text{...(iii)}$$



In case of upward motion, acceleration due to gravity, g is taken as **negative** and for downward motion, g is taken as **positive**.

21. Stopping Distance for a Vehicle

When brakes are applied to a moving vehicle, the distance it travels before coming to halt is called stopping distance. It is also an important factor for road safety. Its value depends upon the speed at which the vehicle is running and the efficiency of the braking system.

$$\text{Stopping distance, } d_s = \frac{u^2}{2a}$$

where, u = initial velocity of the vehicle and
a = retardation.

22. Reaction Time

Reaction time is defined as the time a person takes to observe, think and act.

e.g. If a person is driving a car and suddenly a boy appears on the road, then the time elapsed before he applies the brakes to the car is the reaction time.

Reaction time depends on

(*i*) an individual's presence of mind

(*ii*) the complexity of the situation.

23. Relative Velocity

It is defined as the time rate of change of relative position of one object w.r.t. to another.

If an object A is moving with velocity vector $\mathbf{v}_A$ and an object B is moving with velocity vector $\mathbf{v}_B$, then the velocity of object A relative to object B is given as

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

The relative velocity of object B relative to object A is

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$