Motion in a Plane

1. **Scalar Quantity**

   It is the physical quantity which has only magnitude but no direction. It is specified completely by a single number, alongwith the proper unit.
   e.g. Temperature, mass, length, time, work, etc.

2. **Vector Quantity**

   It is the physical quantities which have both magnitude and direction and obey the triangle/ parallelogram laws of addition and subtraction.
   It is specified by giving its magnitude by a number and its direction. e.g. Displacement, acceleration, velocity, momentum, force, etc.
   A vector is represented by a **bold face** type and also by **an arrow** placed over a letter as shown : i.e. $\mathbf{F}, \mathbf{a}, \mathbf{b}$ or F, a, b
   The length of the line gives the magnitude and the arrowhead gives the direction.
   Vectors are classified into two types such as polar and axial vectors.

   (*i*) **Polar Vectors** Vectors which have a starting point or a point of application are called polar vectors. e.g. Force, displacement, etc.

   (*ii*) **Axial Vectors** Vectors which represent the rotational effect and acts along the axis of rotation are called axial vectors.
   e.g. Angular velocity, angular momentum, torque, etc.

3. **Modulus of a Vector**

   The magnitude of a vector is called modulus of vector. For a vector **A**, it is represented by $|\mathbf{A}|$ or A.

4. **Unit Vector**

   A vector having magnitude equal to unity but having a specific direction is called a unit vector. A unit vector of **A** is written as $\hat{\mathbf{A}}$ and read as *A* cap.
   It is expressed as $\hat{\mathbf{A}} = \dfrac{\mathbf{A}}{|\mathbf{A}|} = \dfrac{\mathbf{A}}{A}$

   $$= \frac{\text{vector}}{\text{magnitude of the vector}}$$

   $$\mathbf{A} = A\hat{\mathbf{A}}$$

   In Cartesian coordinates, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are the unit vectors along *x*-axis, *y*-axis and *z*-axis.
   The magnitude of a unit vector is unity and has no unit or dimensions.

5. **Equal Vectors**

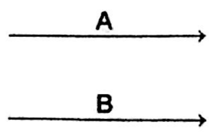
Two vectors are said to be equal, if they have equal magnitude and same direction.

6. **Resultant Vector**

The resultant of two or more vectors is defined as the single vector which produces the same effect as two or more vectors (given vectors) combinedly produce.

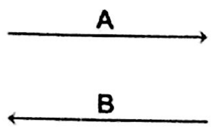
There are two cases

Case I When two vectors are acting in the same direction.



Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B}$

Case II When two vectors are acting in mutually opposite directions.



Resultant vector, $\mathbf{R} = \mathbf{A} - \mathbf{B}$

(*i*) If $\mathbf{B} > \mathbf{A}$, then direction of $\mathbf{R}$ is along $\mathbf{B}$.

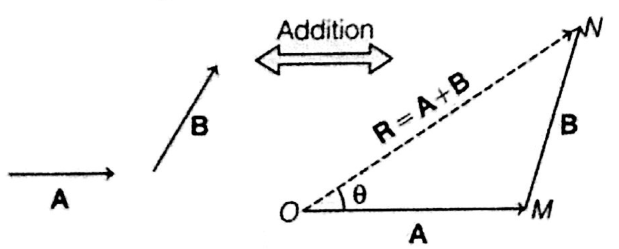
(*ii*) If $\mathbf{A} > \mathbf{B}$, then direction of $\mathbf{R}$ is along $\mathbf{A}$.

7. **Addition of Vectors** (Graphical Method)

Two vectors can be added if both of them are of same nature, e.g. A displacement vector cannot be added to a force but can be added to a displacement vector only. Graphical method of addition of vectors helps us in visualising the vectors and the resultant vector.

This method contains following laws.

(*i*) **Triangle Law of Addition** This law states that if two vectors can be represented both in magnitude and direction by two sides of a triangle taken in the same order, then their resultant is represented completely, both in magnitude and direction, by the third side of the triangle taken in the opposite order.



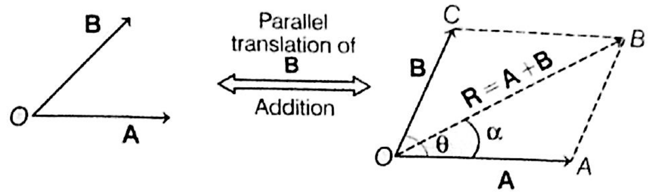
Addition of vectors

The resultant of **A** and **B** is given by

$$\mathbf{R} = \mathbf{OM} + \mathbf{MN} = \mathbf{A} + \mathbf{B}$$

Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B}$

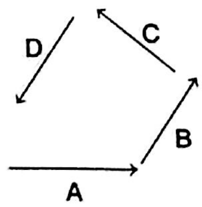
(*ii*) **Parallelogram Law of Addition of Two Vectors** This law states that, if two vectors acting on a particle at the same time be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.



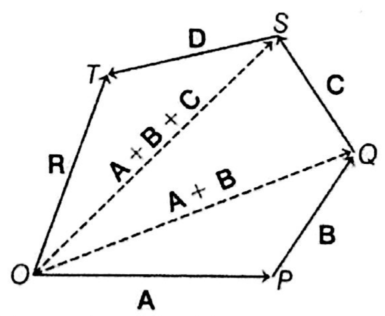
Parallelogram law of addition of two vectors

The resultant vector formed in this method is also same as that formed in triangle law of addition. Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B}$

(*iii*) **Polygon Law of Addition of Vectors** This law states that when the number of vectors are represented in both magnitude and direction by the sides of an open polygon taken in an order, then their resultant is represented in both magnitude and direction by the closing side of the polygon taken in opposite order. Consider the number of vectors **A**, **B**, **C** and **D** be acting in different direction as shown.



Vectors acting in different



Polygon law of addition of vectors

Then, their resultant vector **R** is given by

$$\mathbf{R} = \mathbf{OP} + \mathbf{PQ} + \mathbf{QS} + \mathbf{ST}$$

Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$

Properties of Addition of Vectors

(*i*) It follows commutative law, i.e. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

(*ii*) It follows associative law, $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$

(*iii*) It obeys distributive law, $\lambda(\mathbf{A} + \mathbf{B}) = \lambda\mathbf{A} + \lambda\mathbf{B}$

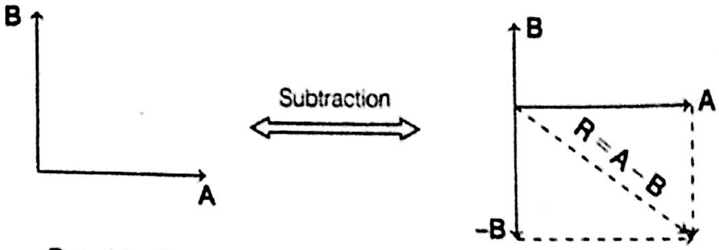
(*iv*) $\mathbf{A} + 0 = \mathbf{A}$

8. Subtraction of Two Vectors (Graphical Method)

If a vector **B** is to be subtracted from vector **A**, then we have to invert the vector **B** and then add it with vector **A** as according to laws of addition of two vectors.

Hence, the subtraction of vector **B** from a vector **A** is expressed as

$$\mathbf{R} = \mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B}$$



Graphical method of subtraction of two vectors

Properties of Subtraction of Vectors

(i) Subtraction of vectors does not follow commutative law

$$\mathbf{A} - \mathbf{B} \neq \mathbf{B} - \mathbf{A}$$

(ii) It does not follow associative law

$$\mathbf{A} - (\mathbf{B} - \mathbf{C}) \neq (\mathbf{A} - \mathbf{B}) - \mathbf{C}$$

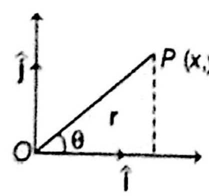
(iii) It follows distributive law

$$\lambda(\mathbf{A} - \mathbf{B}) = \lambda\mathbf{A} - \lambda\mathbf{B}$$

9. Resolution of Vectors in Plane (In Two Dimensions)

The process of splitting a single vector into two or more vectors in different directions which collectively produce the same effect as produced by the single vector alone is known as resolution of a vector.

The various vectors into which the single vector is splitted are known as **components vectors**. Resolution of a vector into two component vectors along the directions of two given vectors is unique.



Any vector **r** can be expressed as a linear combination of two unit vectors $\hat{i}$ and $\hat{j}$ at right angle, i.e. $\mathbf{r} = x\hat{i} + y\hat{j}$

∴ Resultant vector, $\mathbf{r} = \sqrt{x^2 + y^2}$

If θ is the inclination of **r** with *x*-axis, then

Angle, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

10. Resolution of a Space Vector (In Three Dimensions)

Similarly, we can resolve a general vector **A** into three components along *x*, *y* and *z*-axes in three dimensions (i.e. space). While resolving we have,

$$A_x = A\cos\alpha,\quad A_y = A\cos\beta,\ A_z = A\cos\gamma$$

∴ Resultant vector, $\mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

Magnitude of vector **A** is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

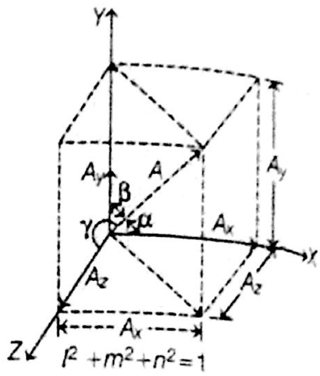
Position vector *r* is given by $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Remember that

$$\cos\alpha = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = l$$

$$\cos\beta = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$$

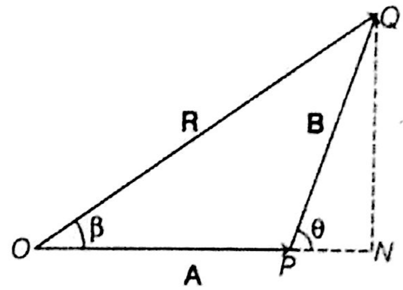
$$\cos\gamma = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$$



Here, *l*, *m* and *n* are known as **direction cosines** of **A**.

11. Addition of Vectors (Analytical Method)

According to triangle law of vector addition, the resultant (**R**) is given by *OQ* but in opposite order.



Resultant, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

This represents the magnitude of resultant vector **R**.

or Direction of resultant **R**, $\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$

Regarding the Magnitude of *R*

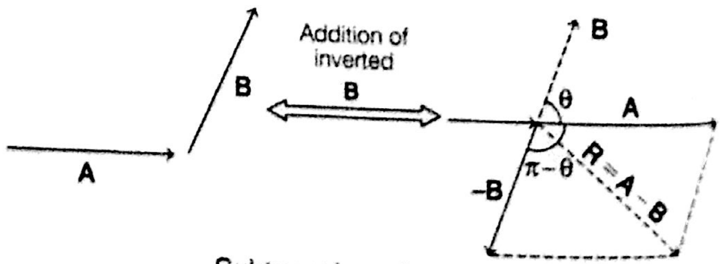
(i) When θ = 0°, then $R = A + B$ (maximum)

(ii) When θ = 90°, then $R = \sqrt{A^2 + B^2}$

(iii) When θ = 180°, then $R = A - B$ (minimum)

12. Subtraction of Vectors (Analytical Method)

There are two vectors **A** and **B** at an angle θ. If we have to subtract **B** from **A**, then first invert the vector **B** and then add with **A** as shown in figure.



Subtraction of vectors

The resultant vector is $\mathbf{R} = \mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B}$

The magnitude of resultant in this case is

$$R = |\mathbf{R}| = \sqrt{A^2 + B^2 + 2AB\cos(\pi - \theta)}$$

Resultant, $R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

Here, θ = angle between **A** and **B**.

Regarding the magnitude of R

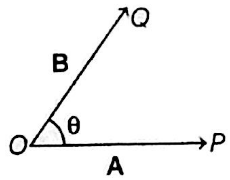
(*i*) When $\theta = 0°$, then $R = A - B$ (minimum)

(*ii*) When $\theta = 90°$, then $R = \sqrt{A^2 + B^2}$

(*iii*) When $\theta = 180°$, then $R = A + B$ (maximum)

13. Dot Product or Scalar Product

It is defined as the product of the magnitudes of vectors **A** and **B** and the cosine angle between them. It is represented by



$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

Case I When the two vectors are parallel, then $\theta = 0°$. We have

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 0° = AB$$

Case II When the two vectors are mutually perpendicular, then, $\theta = 90°$. We have

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 90° = 0$$

Case III When the two vectors are antiparallel, then $\theta = 180°$. We have

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 180° = -AB$$

Properties of Dot Products

(*i*) $\mathbf{a} \cdot \mathbf{a} = (\mathbf{a})^2$ (*ii*) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

(*iii*) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ (*iv*) $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

where, $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$,

$\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$

and $\mathbf{c} = c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{j}} + c_3\hat{\mathbf{k}}$

Here, $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$

$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$

14. Vector Product or Cross Product

It is defined as the product of the magnitudes of vectors **A** and **B** and the sine angle between them.

It is represented as

$$\mathbf{A} \times \mathbf{B} = AB\sin\theta\,\hat{\mathbf{n}}$$

where, $\hat{\mathbf{n}}$ is a unit vector in the direction of $\mathbf{A} \times \mathbf{B}$.

Cross Product of Two Vectors in Terms of Their Components

If $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$, then

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2)\hat{\mathbf{i}} - (a_1b_3 - a_3b_1)\hat{\mathbf{j}} + (a_1b_2 - a_2b_1)\hat{\mathbf{k}}$$

where, $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$

and $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}},\ \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}},\ \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}},$

$\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}},\ \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}},\ \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$

Properties of Cross Product

(i) $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

(ii) $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

(iii) $(\mathbf{a} \times \mathbf{b}) + (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \times \mathbf{c}) + (\mathbf{a} \times \mathbf{d}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{b} \times \mathbf{d})$

(iv) $m\mathbf{a} \times \mathbf{b} = \mathbf{a} \times m\mathbf{b}$

(v) $(\mathbf{b} + \mathbf{c})\mathbf{a} = \mathbf{b} \times \mathbf{a} + \mathbf{c} \times \mathbf{a}$

(vi) $\mathbf{a} \times \mathbf{a} = 0$

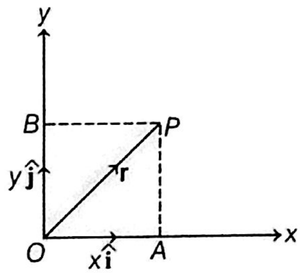
(vii) $\mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{a} \times \mathbf{b} - \mathbf{a} \times \mathbf{c}$

(viii) $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - |\mathbf{a} \cdot \mathbf{b}|^2$

(ix) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$

15. Position Vector

A vector that extends from a reference point to the point at which particle is located is called position vector.



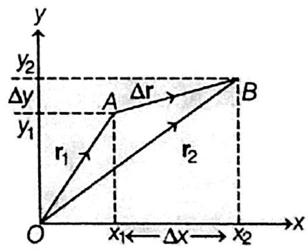
$$\mathbf{OP} = \mathbf{OA} + \mathbf{OB}$$

Position vector, $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

In three dimensions, the position vector is represented as $r = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

16. Displacement

It is the distance representing separation between two defind points.



Displacement, $\mathbf{AB} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}}$

Similarly, in three dimensions the displacement can be represented as

$$\Delta\mathbf{r} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}$$

17. Velocity

Rate of change of displacement of a body is called velocity. It is of two types as given below.

(*i*) **Average Velocity** It is defined as the ratio of the displacement and the corresponding time interval.

Thus, average velocity $= \dfrac{\text{displacement}}{\text{time taken}}$

Average velocity, $\mathbf{v}_{av} = \dfrac{\Delta\mathbf{r}}{\Delta t} = \dfrac{\mathbf{r}_2 - \mathbf{r}_1}{t_2 - t_1} = \dfrac{\Delta x}{\Delta t}\hat{\mathbf{i}} + \dfrac{\Delta y}{\Delta t}\hat{\mathbf{j}}$

Velocity can be expressed in the component form as

$$\mathbf{v} = \mathbf{v}_x\hat{\mathbf{i}} + \mathbf{v}_y\hat{\mathbf{j}}$$

where, v_x and v_y are the components of velocity along x-direction and y-direction, respectively.

The magnitude of $\mathbf{v}$ is given by $|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$

and the direction of $\mathbf{v}$ is given by angle θ,

$\Rightarrow$ Direction of velocity, $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

(ii) **Instantaneous Velocity** The velocity at an instant of time (t) is known as instantaneous velocity.

The average velocity will become instantaneous, if Δt approaches to zero. The instantaneous velocity is expressed as

$$\mathbf{v}_i = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$$

18. Acceleration

The rate of change of velocity of a body is called acceleration. It is of two types as given below.

(i) **Average Acceleration** It is defined as the ratio of change in velocity ($\Delta \mathbf{v}$) and the corresponding time interval (Δt). *It can be expressed as*

$$\mathbf{a}_{av} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1}$$

(ii) **Instantaneous Acceleration** It is defined as the limiting value of the average acceleration as the time interval approaches to zero.

It can be expressed as, $\mathbf{a}_i = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$

Instantaneous acceleration, $\mathbf{a}_i = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$

In terms of x and y, a_x and a_y can be expressed as

$$a_x = \frac{dv_x}{dt} \quad \text{and} \quad a_y = \frac{dv_y}{dt}$$

The magnitude of instantaneous acceleration is given by $a_i = \sqrt{a_x^2 + a_y^2}$

Direction of acceleration, $\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

19. Motion in a Plane with Uniform Velocity

A body is said to be moving with uniform velocity if it suffers equal displacements in equal intervals of time, however small, the intervals may be.

Consider an object moving with uniform velocity $\mathbf{v}$ in xy-plane. Let $\mathbf{r}(0)$ and $\mathbf{r}(t)$ be its position vectors at $t = 0$ and $t = t$, respectively. Then,

$$\mathbf{v} = \frac{\mathbf{r}(t) - \mathbf{r}(0)}{t - 0}$$

$$\Rightarrow \quad \mathbf{r}(t) = \mathbf{r}(0) + \mathbf{v}t$$

In terms of rectangular coordinates, we get

$$\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

20. Motion in a Plane with Constant Acceleration

A body is said to be moving with uniform acceleration, if its velocity vector suffers the same change in the same interval of time however small the intervals may be.

According to the definition of uniform acceleration, we have $\mathbf{a} = \frac{\mathbf{v} - \mathbf{v}_0}{t - 0} \Rightarrow \mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$

In terms of rectangular components, we can express it as $v_x = v_{0x} + a_x t$ and $v_y = v_{0y} + a_y t$

21. Relative Velocity in Two Dimensions

The relative velocity of an object A w.r.t. object B, when both are in motion, is the rate of change of position of object A w.r.t. object B.

Suppose two objects A and B are moving with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ w.r.t. ground or the earth. Then, relative velocity of object A w.r.t. object B,

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

Relative velocity of object B w.r.t. object A,

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

Clearly, $\mathbf{v}_{AB} = -\mathbf{v}_{BA}$

and $|\mathbf{v}_{AB}| = |\mathbf{v}_{BA}|$.

22. Projectile Motion

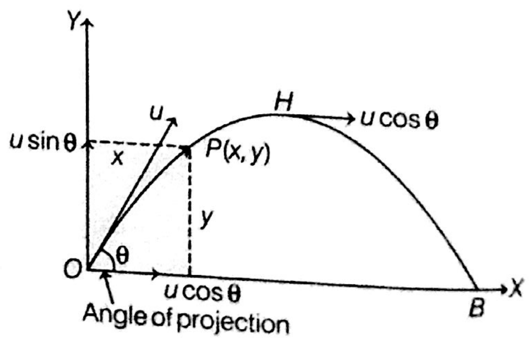
Projectile motion is a form of motion in which an object or a particle is thrown with some initial velocity near the earth's surface and it moves along a curved path under the action of gravity alone. The path followed by a projectile is called its **trajectory**. e.g.,

(i) A tennis ball or a baseball in a flight.

(ii) A bullet fired from a rifle.

23. Equation of Path of a Projectile

Suppose at any time t_1, the object reaches at point $P(x, y)$.



So, x = horizontal distance travelled by object in time t.

y = vertical distance travelled by object in time t.

Motion along Horizontal Direction

The velocity of an object in horizontal direction, i.e. OX is constant, so the acceleration a_x in horizontal direction is zero.

$\therefore$ Position of the object at time t along horizontal direction is given by $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

But $x_0 = 0, u_x = u \cos\theta, a_x = 0$ and $t = t$

$\therefore \quad x = u \cos\theta\, t$

$\Rightarrow$ Time, $\quad t = \dfrac{x}{u \cos\theta} \quad \dots\text{(i)}$

Motion along Vertical Direction

The vertical velocity of the object is decreasing from O to P due to gravity, so acceleration a_y is $-g$.

$\therefore$ Position of the object at any time t along the vertical direction i.e. OY is

Total vertical distance, $y = x\tan\theta - \left(\dfrac{1}{2}\dfrac{g}{u^2\cos^2\theta}\right)x^2$

This equation represents a parabola and is known as **equation of trajectory of a projectile.**

24. Time of Flight

It is defined as the total time for which projectile is in flight, i.e. time during the motion of projectile from O to B. It is denoted by T.

$$\text{Time of flight, } T = \frac{2u\sin\theta}{g}$$

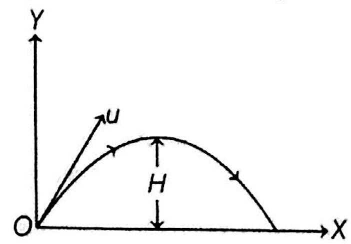
Time of flight consist of two parts such as

(*i*) Time taken by an object to go from point O to H. It is also known as **time of ascent** (t).

(*ii*) Time taken by an object to go from point H to B. It is also known as **time of descent** (t).

25. Maximum Height of a Projectile

It is defined as the maximum vertical height attained by an object above the point of projection during its flight. It is denoted by H.



Maximum height of motion

$$\text{Maximum height, } H = \frac{u^2\sin^2\theta}{2g}$$

26. Horizontal Range of a Projectile

The horizontal range of the projectile is defined as the horizontal distance covered by the projectile during its time of flight. It is denoted by R.

If the object having uniform velocity $u\cos\theta$ (i.e. horizontal component) and the total time of flight T, then the horizontal range covered by the object.

$$\therefore \quad R = u\cos\theta \times T = u\cos\theta \times 2u\frac{\sin\theta}{g}$$

$$\left[\because T = 2u\frac{\sin\theta}{g}\right]$$

$$\text{Horizontal range, } R = \frac{u^2\sin 2\theta}{g}$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

The horizontal range will be maximum, if

$$\sin 2\theta = \text{maximum} = 1$$

$$\sin 2\theta = \sin 90^\circ \quad \text{or} \quad \theta = 45^\circ$$

$\therefore$ Maximum horizontal range,

$$R_m = \frac{u^2}{g}$$

27. Uniform Circular Motion

When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion.

e.g.

(*i*) Motion of the tip of the second hand of a clock.

(*ii*) Motion of a point on the rim of a wheel rotating uniformly.

28. Terms Related to Circular Motion

Angular Displacement It is defined as the angle traced out by the radius vector at the centre of the circular path in the given time. It is denoted by $\Delta\theta$ and expressed in radians. It is a dimensionless quantity.

Angular Velocity It is defined as the time rate of change of its angular displacement. It is denoted by ω and is measured in radians per second. Its dimensional formula is $[M^0L^0T^{-1}]$. It is a vector quantity.

It is expressed as $\omega = \dfrac{\text{Angle track}}{\text{Time taken}}$

$$= \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular Acceleration It is defined as the time rate of change of angular velocity of a particle. It is measured in radian per second square and has dimensions $[M^0L^0T^{-2}]$.

Time Period It is defined as the time taken by a particle to complete one revolution along its circular path. It is denoted by T and is measured in second.

Frequency It is defined as the number of revolutions completed per unit time. It is denoted by f and is measured in Hz.

(*i*) **Relation between Time Period and Frequency**

$$\text{Time period, } T = \frac{1}{f}$$

(ii) **Relation between Angular Velocity, Frequency and Time Period**

Angular velocity, $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi f$

(iii) **Relation between Linear Velocity (v) and Angular velocity (ω)**

Linear velocity, $v = r\frac{\Delta\theta}{\Delta t} = r\omega$

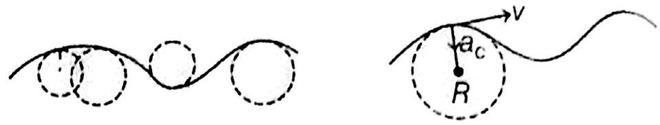
29. Centripetal Acceleration

The acceleration associated with a uniform circular motion is called centripetal acceleration.

Centripetal acceleration, $a = \frac{v^2}{r}$

Radius of Curvature

Any curved path can be assumed to be of infinite circular arc. Radius of curvature at point is defined as the radius of that circular arc which fits at the particular point on the curve as shown in figure.



In the expression, $a_c = v^2/R$, the term R is known as radius of curvature.