DCAM classes

Dynamic Classes for Academic Mastery

Work, Energy and Power

1. Work

Work is said to be done by a force, when the body is displaced actually through some distance in the direction of the force. *Thus, work is done on a body only if the following two conditions are satisfied*

(*i*) A force acts on the body.

(*ii*) The point of application of the force moves in the direction of the force.

2. Work Done by a Constant Force

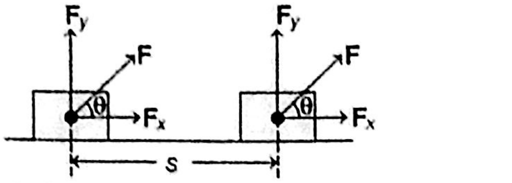
Work done by the force (constant force) is the product of component of force in the direction of the displacement and the magnitude of the displacement. Then, the work done on the body by the force is given by

Work done, $W = \mathbf{F} \cdot \mathbf{s}$

SI unit of work is joule(J).

Its dimensions are $[M^1 L^2 T^{-2}]$.

3. **Work Done When Force and Displacement are Inclined to Each Other**



Work done when force and displacement are inclined to each other

Work done, $W = \mathbf{F} \cdot \mathbf{s} = (F\cos\theta) \cdot s$

Work done, $W = Fs\cos\theta$

Two cases can be considered for the maximum and minimum work.

Case I When **F** and **s** are in the same direction, i.e. $\theta = 0°$, then work done is

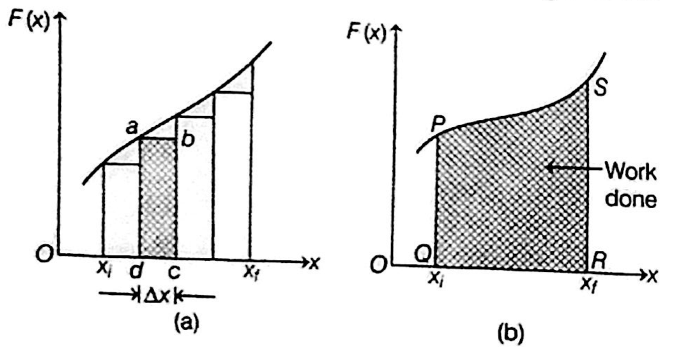
$$W = Fs\cos 0° = Fs(1) = Fs$$

i.e. maximum work done by the force.

Case II When **F** and **s** are perpendicular to each other, then $W = \mathbf{F} \cdot \mathbf{s} = Fs\cos 90° = Fs(0) = 0$, i.e. no work done by the force, when a body moves in a direction perpendicular to the force acting.

4. **Work Done by a Variable Force**

Work done $\Delta W = F(x)\Delta x =$ Area of rectangle *abcd*.



Work done by a variable force

When adding all the rectangles in figure, we get the total work done as

$$W \cong \sum_{x_i}^{x_f} F(x)\,\Delta x$$

$$= \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\,\Delta x$$

$$= \int_{x_i}^{x_f} F(x)\,dx$$

$$W_{x_i \to x_f} = \int_{x_i}^{x_f} \mathbf{F} \cdot d\mathbf{x}$$

$$= \int_{x_i}^{x_f} (F\cos\theta)\,dx$$

$$= \text{Area of } PQRS$$

When the magnitude and direction of a force vary in three dimensions. Then it can be expressed in terms of rectangular component.

So, work done from x_i to x_f

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{x_i}^{x_f} F_y\,dy + \int_{x_i}^{x_f} F_z\,dz$$

where, F_x, F_y and F_z are the rectangular components of force in *x*, *y* and *z*-directions, respectively.

5. **Conservative Force**

If the work done by the force in displacing an object depends only on the initial and final positions of the object and not on the nature of the path followed between the initial and final positions, such a force is known as conservative force. e.g. Gravitational force is a conservative force.

6. **Non-Conservative Force**

If the work done by a force in displacing an object from one position to another, it depends upon the path between the two positions. Such a force is known as non-conservative force.

7. **Energy**

The energy of a body is defined as its capacity or ability for doing work.

(*i*) Like work, energy is a scalar quantity, having magnitude only and no direction.

(*ii*) The dimensions of energy are the same as the dimensions of work, i.e. $[M^1L^2T^{-2}]$.

(*iii*) It is measured in the same unit as work, i.e. joule in SI system and erg in CGS system.

8. **Kinetic Energy**

The energy possessed by a body by virtue of its motion is called kinetic energy. In other words, the amount of work done, by a moving object before coming to rest is equal to its kinetic energy.

$\therefore$ Kinetic energy, $KE = \frac{1}{2}mv^2$

where, *m* is a mass and *v* is the velocity of a body.

The units and dimensions of KE are joule (in SI) and $[ML^2T^{-2}]$ respectively.

Relation between Kinetic Energy and Linear Momentum

$$p = \sqrt{2mK}$$

9. **Work Energy Theorem or Work Energy Principle**

It states that work done by the net force acting on a body is equal to the change, produced in the kinetic energies of the body.

$$K_f - K_i = \int_i^f \mathbf{F}_{net} \cdot d\mathbf{x}$$

$$\therefore \quad K_f - K_i = W$$

where, K_f and K_i are the final and initial kinetic energies of the body.

10. Potential Energy

The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration in some field. So, if configuration of the system changes, then its potential energy changes.

The units and dimensions of potential energy are same as that of kinetic energy given as,

$$\text{Dimensions} = [ML^2T^{-2}]$$

$$\text{SI Unit} = \text{Joule}$$

11. Gravitational Potential Energy

Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

Gravitational potential energy, $U = mgh$

12. Potential Energy of a Spring

For a small stretch or compression, spring obeys Hooke's law, i.e. restoring force $\propto$ stretch or compression

$$-F_s \propto x \quad \Rightarrow \quad F_s = -kx$$

where, k is called **spring constant**. Its SI unit is Nm^{-1}.The negative sign shows F_s acts in the opposite direction of displacement x.

If the block is moved from an initial displacement x_i to final displacement x_f, then work done by spring force is

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_{s(i \to f)} = \frac{1}{2}k(x_i^2 - x_f^2)$$

$\therefore$ Change in potential energy of a spring

$$\Delta U = -W_s = \frac{1}{2}k(x_f^2 - x_i^2).$$

If $\quad x_i = 0, \Delta U = \frac{1}{2}kx_f^2$

13. Conservation of Mechanical Energy

This principle states that, if only the conservative forces are doing work on a body, then its mechanical energy (KE + PE) remains constant.

i.e. $\quad K + U = \text{constant} = E$

$\therefore \quad K_i + U_i = K_f + U_f$

The quantity $K + U$, is called the total **mechanical energy** of the system.

14. Principle of Conservation of Energy

It states that, the energy can neither be created nor be destroyed but can only be converted from one form to another.

15. Power

Power of a person or machine is defined as the rate at which work is done or energy is transferred.

Average power (P_{av}) = rate of doing work

$$= \frac{\text{work done } (W)}{\text{time taken } (t)}$$

Thus, the average power of a force is defined as the ratio of the work, W to the total time t.

The instantaneous power of an agent at any instant is equal to the dot product of its force and velocity vectors at that instant.

$$P = \mathbf{F} \cdot \mathbf{v}$$

The SI unit of power is watt (W).

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}} = 1 \text{ Js}^{-1}$$

Another popular units of power are kilowatt and horsepower.

1 kilowatt = 1000 watt or 1 kW = 10^3 W

1 horse power = 746 watt or 1 HP = 746 W

This unit is used to describe the output of automobiles, motorbikes, engines, etc.

Power is a scalar quantity and its dimensional formula is $[ML^2T^{-3}]$.

16. Collision

A collision is an isolated event in which two or more colliding bodies exert strong forces on each other for a relatively short time. For a collision to take place, the actual physical contact is not necessary.

Collision between particles have been divided into two types which can be differentiate as.

Elastic Collision	Inelastic Collision
A collision in which there is absolutely no loss of kinetic energy.	A collision in which there occurs some loss of kinetic energy.
Forces involved during elastic collision must be conserved in nature.	Some or all forces involved during collision may be non-conservative in nature.
The mechanical energy is not converted into heat, light, sound, etc.	A part of the mechanical energy is converted into heat, light, sound, etc.
e.g. Collision between subatomic particles, collision between glass balls, etc.	e.g. Collision between two vehicles, collision between a ball and floor, etc.

Conservation of Linear Momentum In Collision Total linear momentum is conserved at each instant during collision.

$\therefore \quad p_1 + p_2 =$ constant

Elastic Collision in One Dimension In one-dimensional elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision.

$$u_1 - u_2 = v_2 - v_1$$

17. Velocities of the Bodies After the Collision

Velocity of Ist body after collision,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad \ldots(i)$$

Velocity of IInd body after collision,

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2} \quad \ldots(ii)$$

Eqs. (i) and (ii) give the final velocities of the colliding bodies in terms of their initial velocities.

The two cases under the action of same and different masses can be considered as given below.

Case I When two bodies of equal masses collide.

i.e. $m_1 = m_2 = m$ (say)

From Eq. (i), we get

$$v_1 = \frac{2mu_2}{2m} = u_2$$

= Velocity of body of mass m_2 before collision

From Eq. (ii), we get

$$v_2 = \frac{2mu_1}{2m} = u_1$$

= Velocity of body of mass m_1 before collision.

Case II When a light body collides against a massive stationary body.

Here, $m_1 << m_2$ and $u_2 = 0$

Neglecting m_1 in Eq. (i), we get

$$v_1 = -\frac{m_2u_1}{m_2} = -u_1$$

From Eq. (ii), we get

$$v_2 \simeq 0.$$

18. Perfectly Inelastic Collision in One Dimension

When the two colliding bodies together move as a single body with a common velocity after the collision, then the collision is perfectly inelastic. In perfectly inelastic collision between two bodies of masses m_1 and m_2, the body of mass m_2 happens to be initially at rest ($u_2 = 0$). After the collision, the two bodies move together with common velocity v. The change in their kinetic energies, is

$$\Delta KE = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

$\therefore$ ΔKE is a positive quantity.

Therefore, kinetic energy is lost mainly in the form of light, sound and heat.

19. Inelastic collision in two dimensions

When two bodies travelling initially along the same straight line collide involving some loss of kinetic energy and move after collision along different directions in a plane, then it is called inelastic collision.

20. Coefficient of Restitution or Coefficient of Resilience

It is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is denoted by e.

$$e = \frac{\text{Relative velocity of separation (after collision)}}{\text{Relative velocity of approach (before collision)}}$$

$$e = \frac{|v_2 - v_1|}{|u_2 - u_1|}$$

Coefficient of resilience,

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$

where u_1, u_2 are velocities of two bodies before collision and v_1, v_2 are their respective velocities after collision.

21. Different Types of Collisions

Collision	Kinetic energy	Coefficient of restitution	Main domain
Elastic	Conserved	$e = 1$	Between atomic particles
Inelastic	Non-conserved	$0 < e < 1$	Between ordinary objects
Perfectly inelastic	Maximum loss of KE	$e = 0$	During shooting
Super elastic	KE increases	$e > 1$	In explosions